Pensieve header: Scatter and Glow in OneCo. Before ϵs removal.

In the U(T)⊗U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"]
```

```
C:\drorbn\AcademicPensieve\Projects\OneCo-1604
```

Signs

```
SetSigns[0] := Do[Unset[ϵi], {i, 0, 48}];
```

```
SetSigns[4] :=
  (
   {ϵ0, ϵ1, ϵ2, ϵ3, ϵ4, ϵ5, ϵ6, ϵ7, ϵ8, ϵ9, ϵ10, ϵ11, ϵ12, ϵ13, ϵ14, ϵ15, ϵ16, ϵ17, ϵ18,
    ϵ19, ϵ20, ϵ21, ϵ22, ϵ23, ϵ24, ϵ25, ϵ26, ϵ27, ϵ28, ϵ29, ϵ30, ϵ31, ϵ32, ϵ33,
    ϵ34, ϵ35, ϵ36, ϵ37, ϵ38, ϵ39, ϵ40, ϵ41, ϵ42, ϵ43, ϵ44, ϵ45, ϵ46, ϵ47, ϵ48} =
   {ϵ1, ϵ1, ϵ1 ϵ10, ϵ5, ϵ10, ϵ5, ϵ6, -ϵ1 ϵ5, ϵ5 ϵ9, ϵ9, ϵ10, ϵ1 ϵ10, ϵ1 ϵ10, -ϵ1 ϵ10, ϵ1 ϵ10,
    ϵ1 ϵ10, -ϵ1 ϵ9 ϵ10, ϵ1 ϵ9 ϵ10, ϵ1^2 ϵ5 ϵ10 / ϵ6, ϵ1 ϵ5, ϵ1 ϵ5, ϵ1^2 ϵ5, ϵ1 ϵ5, ϵ1^2 ϵ10, ϵ1 ϵ10, ϵ1^2 ϵ10,
    ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ5 ϵ10 / ϵ6, ϵ1 ϵ5, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ5,
    ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10^2, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^3 ϵ10^2, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10}
  );
```

```
SetSigns[5] := (
   {ϵ0, ϵ1, ϵ2, ϵ3, ϵ4, ϵ5, ϵ6, ϵ7, ϵ8, ϵ9, ϵ10, ϵ11, ϵ12, ϵ13, ϵ14, ϵ15, ϵ16,
    ϵ17, ϵ18, ϵ19, ϵ20, ϵ21, ϵ22, ϵ23, ϵ24, ϵ25, ϵ26, ϵ27, ϵ28, ϵ29, ϵ30, ϵ31, ϵ32,
    ϵ33, ϵ34, ϵ35, ϵ36, ϵ37, ϵ38, ϵ39, ϵ40, ϵ41, ϵ42, ϵ43, ϵ44, ϵ45, ϵ46, ϵ47, ϵ48} =
   {1, 1, ϵ10, ϵ5, ϵ10, ϵ5, ϵ6, -ϵ5, ϵ5 ϵ9, ϵ9, ϵ10, ϵ10, ϵ10, -ϵ10, ϵ10, ϵ10, -ϵ9 ϵ10,
    ϵ9 ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5, ϵ5, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5 ϵ10,
    ϵ5 ϵ10, ϵ5 ϵ10, ϵ10, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10, ϵ10}
  );
```

```
SetSigns[6] := (
   {ϵ0, ϵ1, ϵ2, ϵ3, ϵ4, ϵ5, ϵ6, ϵ7, ϵ8, ϵ9, ϵ10, ϵ11, ϵ12, ϵ13, ϵ14, ϵ15, ϵ16, ϵ17,
    ϵ18, ϵ19, ϵ20, ϵ21, ϵ22, ϵ23, ϵ24, ϵ25, ϵ26, ϵ27, ϵ28, ϵ29, ϵ30, ϵ31, ϵ32, ϵ33,
    ϵ34, ϵ35, ϵ36, ϵ37, ϵ38, ϵ39, ϵ40, ϵ41, ϵ42, ϵ43, ϵ44, ϵ45, ϵ46, ϵ47, ϵ48, ϵ49} =
   {1, 1, ϵ10, ϵ5, ϵ10, ϵ5, ϵ6, -ϵ5, ϵ5 ϵ9, ϵ9, ϵ10, ϵ10, ϵ10, -ϵ10, ϵ10, ϵ10, -ϵ9 ϵ10,
    ϵ9 ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5, ϵ5, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5 ϵ10,
    ϵ5 ϵ10, ϵ5 ϵ10, ϵ10, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10}
  );
```

SetSigns

```
Cases[SetSigns[6], ϵ_, ∞] // Union
```

SetSigns

$$\{\epsilon_5, \epsilon_6, \epsilon_9, \epsilon_{10}\}$$

SetSigns

```
Table[i → ϵ_i, {i, 0, 49}]
```

SetSigns

$$\{0 \to 1,\ 1 \to 1,\ 2 \to \epsilon_{10},\ 3 \to \epsilon_5,\ 4 \to \epsilon_{10},\ 5 \to \epsilon_5,\ 6 \to \epsilon_6,\ 7 \to -\epsilon_5,\ 8 \to \epsilon_5\,\epsilon_9,\ 9 \to \epsilon_9,$$
$$10 \to \epsilon_{10},\ 11 \to \epsilon_{10},\ 12 \to \epsilon_{10},\ 13 \to -\epsilon_{10},\ 14 \to \epsilon_{10},\ 15 \to \epsilon_{10},\ 16 \to -\epsilon_9\,\epsilon_{10},\ 17 \to \epsilon_9\,\epsilon_{10},$$
$$18 \to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 19 \to \epsilon_5,\ 20 \to \epsilon_5,\ 21 \to \epsilon_5,\ 22 \to \epsilon_5,\ 23 \to \epsilon_{10},\ 24 \to \epsilon_{10},\ 25 \to \epsilon_{10},$$
$$26 \to \epsilon_{10},\ 27 \to \epsilon_{10},\ 28 \to \epsilon_{10},\ 29 \to \epsilon_{10},\ 30 \to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 31 \to \epsilon_5,\ 32 \to \epsilon_5\,\epsilon_{10},\ 33 \to \epsilon_5\,\epsilon_{10},$$
$$34 \to \epsilon_5\,\epsilon_{10},\ 35 \to \epsilon_{10},\ 36 \to \epsilon_5,\ 37 \to \epsilon_{10},\ 38 \to \epsilon_{10},\ 39 \to \epsilon_{10},\ 40 \to \epsilon_{10},\ 41 \to \epsilon_{10}^2,$$
$$42 \to \epsilon_{10},\ 43 \to \epsilon_{10},\ 44 \to \epsilon_{10}^2,\ 45 \to \epsilon_{10},\ 46 \to \epsilon_{10},\ 47 \to \epsilon_{10},\ 48 \to \epsilon_{10},\ 49 \to \epsilon_{10}\}$$

Generalities

Generalities

```
Simp[expr_] := Expand[expr];
S[expr_] := expr /. (λ_β | λ_a | λ_δβ | λ_δa | λ_δaa) :↦ MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r]);
AutoCollecting /@ {β, a, δβ, δa, δaa};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];
b_ç = 1; ca[f_, i_, j_, k_] := δaa[f, ç, i, j, k];
γ[f_, j_, k_] := δa[f, j, k] - δa[ϵ_0 b_j f, ç, k];
γa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[ϵ_0 b_j f, k, l, m];
Kδ /: Kδ_is__ := KroneckerDelta[1, Length[Union[{is}]]];
```

δaa relations

delta-aa

```
i_ ≤ j_ := OrderedQ[{i, j}]; i_ ≺ j_ := ! OrderedQ[{j, i}];
S[UU[expr_]] := UU[S[expr /. δaa[f_, i_, j_, k_, l_] :> Which[
        k === ç, δaa[f, ç, l, i, j] + Kδ_jl γ[ϵ49 f, i, j],
        (i === ç) ⋁ (i ≤ k ⋀ j ≤ l), δaa[f, i, j, k, l],
        k ≺ i ⋀ j ≺ l, δaa[f, k, j, i, l] + ca[-f b_i ϵ1, l, k, j] +
         ca[f b_i ϵ1, j, k, l] + ca[-f b_k ϵ1, j, i, l] + ca[f b_k ϵ1, l, i, j],
        k ≺ i ⋀ j === l, δa[-f b_i ϵ2, k, j] + δa[f b_k ϵ2, i, j] + δaa[f, k, j, i, j],
        i ≤ k ⋀ l ≺ j, δaa[f, i, l, k, j] + ca[-f b_i ϵ1, l, k, j] +
         ca[f b_i ϵ1, j, k, l] + ca[-f b_k ϵ1, j, i, l] + ca[f b_k ϵ1, l, i, j],
        k ≺ i ⋀ l ≺ j, δaa[f, k, l, i, j]
       ]]];
```

Bases

```
UUBasis[T_List, H_List, f_] := Module[
   {ff, n = 0, h, t, h1, h2},
   ff := f_{++n} @@ Table[b_t, {t, T}];
   S /@ UU /@ Flatten@{
         β[ff], δβ[ff],
         Table[a[ff, t, h], {t, T}, {h, H}],
         Table[δa[ff, t, h], {t, T ⋃ {ç}}, {h, H}],
         Table[ca[ff, h1, t, h2], {t, T}, {h1, H}, {h2, H}],
         Table[δaa[ff, T[[i]], H[[j]], T[[k]], H[[l]],
          {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
        } /. 1_[___] → 1
  ];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];
```

tm, hm, hts, dm

tm-def

```
UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
         a[f_, x, j_] :> a[f, z, j] + ϵ3 γ[∂_{b_y} f, z, j],
         a[f_, y, j_] :> a[f, z, j],
         δa[f_, x | y, j_] :> δa[f, z, j],
         δaa[f_, i_, j_, k_, l_] :>
          δaa[f, i // Replace[x | y → z], j, k // Replace[x | y → z], l]
        } /. b_{x|y} → b_z]]];
```

hm-def

```
UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
        a[f_, i_, x | y] :> a[f, i, z],
        δa[f_, i_, x | y] :> δa[f, i, z],
        δaa[f_, i_, y, k_, x] :> δaa[f, k, z, i, z],
        δaa[f_, i_, j_, k_, l_] :>
         δaa[f, i, j // Replace[x | y → z], k, l // Replace[x | y → z]]
       }]]];
```

hts-def

```
UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
        a[f_, i_, j_] :> a[f, i, j] - ϵ5 Kδ_jy γ[∂_bx f, i, y] -
          Kδ_ix Kδ_jy (ϵ6 β[f b_x] + ϵ7 δa[f, ç, y] - ϵ8 δβ[b_x ∂_bx f]),
        δa[f_, x, y] :> δa[f, x, y] - ϵ9 δβ[f b_x],
        δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j, k, l] +
          ϵ12 Kδ_ix Kδ_jy δa[-b_x f, k, l] + ϵ13 Kδ_ix Kδ_ly (-δa[b_k f, x, j] + δa[b_x f, k, j]) +
          ϵ14 Kδ_kx Kδ_jy (δa[b_i f, x, l] - δa[b_x f, i, l]) + ϵ15 Kδ_kx Kδ_ly δa[-b_x f, i, j] +
          ϵ16 Kδ_ix Kδ_jly δβ[b_x b_k f] + 2 ϵ17 Kδ_xik Kδ_yjl δβ[b_x b_x f]
       }]]];
```

dm-def

```
dm[x_, y_, z_][expr_] := expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

tσ, hσ, dσ on {β, a, δβ, δa, δaa}

sigma-def

```
tσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. b_i_ :> b_i /. r /. {
      a[f_, i_, j_] :> a[f, i /. r, j],
      δa[f_, i_, j_] :> δa[f, i /. r, j],
      δaa[f_, i_, j_, k_, l_] :> δaa[f, i /. r, j, k /. r, l]
     }]];
tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];
hσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. {
      a[f_, i_, j_] :> a[f, i, j /. r],
      δa[f_, i_, j_] :> δa[f, i, j /. r],
      δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j /. r, k, l /. r]
     }]];
hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
dσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];
```

tb, hb, thb, htb, db, bb on {β, a, δβ, δa, δaa}

tb-def

```
tb[x_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
          p[u_β | u_δβ | u_δa | u_δaa, v_a] :→ -p[v, u]
         } /. {
         p[a[f_, x, j_], u_] :→ (u /. {
              β[g_] :→ ϵ18 γ[f ∂b_x g, x, j],
              a[g_, k_, l_] :→ ϵ19 γa[f ∂b_x g, x, j, k, l] + Kδxk (-γa[ϵ20 g ∂b_k f, k,
                      l, x, j] + ca[ϵ21 f g, l, x, j] - ca[ϵ21 f g, j, k, l]),
              _ → 0
             }),
         p[a[f_, j_, k_], a[g_, x, l_]] /; j =!= x :→ -γa[ϵ22 g ∂b_x f, x, l, j, k],
         p[_, _] → 0
        }]]]];
```

hb-def

```
hb[y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ, _] → 0,
          p[_, _β | _δβ] → 0,
          p[_δa | _δaa, _δa | _δaa] → 0,
          p[u_δa | u_δaa, v_a] :→ -p[v, u]
         } /. {
         p[a[f_, i_, y], u_] :→ (u /. {
              a[g_, j_, k_] :→ ϵ23 Kδyk (a[bj f g, i, y] - a[bi f g, j, k]),
              δa[g_, j_, k_] :→ ϵ25 Kδyk (δa[bj f g, i, y] - δa[bi f g, j, k]),
              δaa[g_, j_, k_, l_, m_] :→ ϵ28 Kδyk (δaa[bj f g, i, y, l, m] - δaa[bi f g, j, k,
                     l, m]) + ϵ29 Kδym (δaa[bl f g, j, k, i, y] - δaa[bi f g, j, k, l, m])
             }),
         _p → 0
        }]]]];
```

thb-def

```
thb[x_, y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _δa | _δaa, _β | _δβ | _δa | _δaa] → 0,
        p[_a, _β | _δβ] → 0,
        p[β[f_], a[g_, i_, j_]] :↦ Kδ_yj γ[ϵ30 g ∂_bx f, i, y],
        p[a[f_, i_, j_], a[g_, k_, l_]] :↦ Kδ_yl (
            γa[ϵ31 g ∂_bx f, k, l, i, j] + Kδ_xi (
                γ[-ϵ32 b_k g ∂_bx f, i, j] + δa[ϵ33 b_k g ∂_bx f, i,
                  j] - δa[ϵ34 b_i g ∂_bx f, k, j] - a[ϵ35 b_k f g, i, j] + a[ϵ35 b_i f g,
                  k, j] + ca[ϵ36 f g, j, k, l] - ca[ϵ36 f g, l, k, j])),
        p[a[f_, i_, j_], δa[g_, k_, l_]] :↦ ϵ38 Kδ_xi Kδ_yl
          (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] :↦ Kδ_xi (
            ϵ42 Kδ_yl (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
             ϵ43 Kδ_yn (-δaa[b_m f g, k, l, i, j] + δaa[b_i f g, k, l, m, j]) +
             ϵ44 Kδ_yln (δa[b_x b_m f g, k, j] - δa[b_k b_m f g, x, j])),
        p[_δβ, _a] → 0,
        p[δa[f_, i_, j_], a[g_, k_, l_]] :↦
         ϵ45 Kδ_xi Kδ_yl (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :↦
         ϵ47 Kδ_xi Kδ_yl (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
          ϵ48 Kδ_xm Kδ_yl (-δaa[b_k f g, i, j, m, n] + δaa[b_m f g, i, j, k, n])
       }]]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

$t_1\, h_1\, t_2\, h_2 \to t_1\, t_2\, h_1\, h_2 \to t_2\, t_1\, h_1\, h_2 \to t_2\, t_1\, h_2\, h_1 \to t_2\, h_2\, t_1\, h_1$:

db-def

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
    htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
    tb[x][u, v // hσ[x, h]] // hm[x, h, x],
    hb[x][u, v // tσ[x, t]] // tm[t, x, x],
    thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x]
   ]];
```

bb-def

```
bb[S_List] := Module[{w, bar, t, n = 0},
   bar[x_] := -x;
   w = #2 // dσ[S, bar /@ S];
   Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]]], {i, k + 1, Length@S}];
    t,
    {k, Length@S}
   ]
  ] &
bb[S___] := bb[{S}]
```

ct (contract)

```
ct::usage =
  "ct[h,t][L,R] contracts the head h in L with the tail t in R. ct[s][L,R] takes
    h=t=s, and ct[][L,R] takes s=0. When ambiguous, L is placeed below R.";
```

ct-def

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
        p[_β | _δβ, _] → 0,
        p[a[f_, i_, h], β[g_]] :↦ β[f b_i ((∂_{b_t} g) /. b_t → 0)],
        p[a[f_, i_, h], a[g_, t, j_]] :↦ a[f (g /. b_t → 0), i, j],
        p[a[f_, i_, h], a[g_, j_, k_]] :↦ a[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
        p[a[f_, i_, h], δa[g_, t, j_]] :↦ δa[f (g /. b_t → 0), i, j],
        p[a[f_, i_, h], δa[g_, j_, k_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
        p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
        p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] :↦ δaa[f (g /. b_t → 0), i, j, k, l],
        p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] :↦ δaa[f (g /. b_t → 0), j, k, i, l],
        p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :↦
         δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
        p[a[__], _] → 0, p[_δa | _δaa, _δβ | _δa | _δaa] → 0,
        p[δa[f_, i_, h], β[g_]] :↦ δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
        p[δa[f_, i_, h], a[g_, t, j_]] :↦ δa[f (g /. b_t → 0), i, j],
        p[δa[f_, i_, h], a[g_, j_, k_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
        p[_δa, _] → 0, p[δaa[_, _, h, _, h], _] → 0,
        p[δaa[f_, i_, h, j_, k_], β[g_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
        p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] :↦ δaa[f (g /. b_t → 0), i, l, j, k],
        p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :↦
         δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
        p[δaa[f_, i_, j_, k_, h], β[g_]] :↦ δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
        p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] :↦ δaa[f (g /. b_t → 0), i, j, k, l],
        p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :↦
         δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
        p[_δaa, _] → 0}]]];
```

dect (de-contract)

```
dect::usage =
  "dect[h,t][uu] returns a pair {L,R} such that ct[h,t][L,R]=uu. Similarly
    for dect[s] and dect[]. uu is assumed to be atomic.";
dect[s_] := dect[s, s];
dect[] = dect[0, 0];
dect[h_, t_][β[f_]] := {};
dect[h_, t_][δβ[f_]] := TBD;
```

Ad

AutoAd

```
AutoAd[B_, x_][y_] :=
  Module[{pows, states, i, s, seq, sh = 5, dseq, sf1, sf2, sf, t1, n},
   pows = NestList[B[x, #] &, y, 20];
   states =
    Union[Cases[pows, s_β | s_δβ | s_a | s_δa | s_δaa :> ReplacePart[s, 1 → _], ∞]];
   UU@Sum[
     seq = Cases[{#}, states[[i]], ∞] & /@ pows;
     seq = Replace[seq, {{_[f_, ___]} :> f, {} → 0}, {1}];
     dseq = Drop[seq, sh];
     If[Union[Length[MonomialList[#]] & /@ dseq] === {1} ⋀
       Union[Length[FactorTermsList[#]] & /@ dseq] === {2},
      sf1 = FindSequenceFunction[FactorTermsList[#][[1]] & /@ dseq];
      sf2 = FindSequenceFunction[FactorTermsList[#][[2]] & /@ dseq];
      sf = (sf1[#] sf2[#] &),
      (*Else*) sf = FindSequenceFunction[dseq,
        FunctionSpace → {"ConstantRecursive", "HolonomicSequence",
          "Polynomial", "RationalFunction", "HypergeometricTerm"}]];
     ReplacePart[states[[i]], 1 → Simplify[∑_{n=0}^{sh-1} seq[[n + 1]]/n! + ∑_{n=sh}^{∞} sf[n + 1 - sh]/n!]],
     {i, Length@states}]];
```

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 0, 1]]
```

$$\begin{aligned}
&UU\Big[a[1,0,1]+a\Big[1-e^{-b_1\epsilon_{10}},0,2\Big]+a\Big[\frac{\left(-1+e^{-b_1\epsilon_{10}}\right)b_0}{b_1},1,2\Big]+\\
&\quad\delta a\Big[b_0\epsilon_5\left(\frac{-1+e^{-b_1\epsilon_{10}}}{b_1}+\epsilon_{10}\right),ç,2\Big]+\delta a\Big[\epsilon_5\left(\frac{1-e^{-2b_1\epsilon_{10}}}{b_1}+\left(-1-e^{-b_1\epsilon_{10}}\right)\epsilon_{10}\right),0,2\Big]+\\
&\quad\delta a\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},1,2\Big]+\\
&\quad\delta aa\Big[\frac{\epsilon_5\left(b_1+\frac{-1+e^{-b_1\epsilon_{10}}}{\epsilon_{10}}\right)}{b_1^2},0,1,1,2\Big]+\delta aa\Big[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},ç,1,0,2\Big]+\\
&\quad\delta aa\Big[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},ç,2,0,1\Big]+\delta aa\Big[\frac{b_0\epsilon_5\left(1-e^{-b_1\epsilon_{10}}-b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},ç,1,1,2\Big]+\\
&\quad\delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^3\epsilon_{10}},1,2,1,2\Big]+\\
&\quad\delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}\left(1+e^{b_1\epsilon_{10}}\right)\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},0,2,1,2\Big]+\\
&\quad\delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},ç,2,0,2\Big]+\\
&\quad\delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}\left(-2+e^{b_1\epsilon_{10}}\right)b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},ç,2,1,2\Big]\Big]
\end{aligned}$$

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 0, 2]]
```

$$\begin{aligned}
&UU\Big[\\
&\quad a\Big[e^{-b_1\epsilon_{10}},0,2\Big]+a\Big[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)b_0}{b_1},1,2\Big]+\delta a\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},1,2\Big]+\\
&\quad\delta a\Big[\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1},0,2\Big]+\\
&\quad\delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^3\epsilon_{10}},1,2,1,2\Big]+\\
&\quad\delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},ç,2,1,2\Big]+\\
&\quad\delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},0,2,1,2\Big]+\\
&\quad\delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},ç,2,0,2\Big]\Big]
\end{aligned}$$

ScatterTailByTails

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 1, 0]]
```

ScatterTailByTails

$$\begin{aligned}
&UU\Big[a[1,1,0]+\delta aa[\epsilon_5,ç,0,1,2]+\\
&\quad\delta aa\Big[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},ç,2,1,0\Big]+\delta aa\Big[-\frac{\epsilon_5\left(-1+e^{-b_1\epsilon_{10}}+b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},1,0,1,2\Big]\Big]
\end{aligned}$$

ScatterTailByHeads

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 2, 0]]
```

ScatterTailByHeads

$$\begin{aligned}
&UU\Big[a\Big[e^{b_1\epsilon_{10}},\,2,\,0\Big]+a\Big[-\frac{\left(-1+e^{b_1\epsilon_{10}}\right)b_2}{b_1},\,1,\,0\Big]+\\
&\quad\delta a\Big[-\frac{b_2\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},\,1,\,0\Big]+\delta a\Big[\frac{b_2\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1},\,ç,\,0\Big]+\\
&\quad\delta aa\Big[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},\,ç,\,2,\,1,\,0\Big]+\delta aa\Big[\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},\,1,\,0,\,2,\,2\Big]+\\
&\quad\delta aa\Big[-\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},\,ç,\,0,\,2,\,2\Big]+\\
&\quad\delta aa\Big[\frac{1}{b_1^2\epsilon_{10}}\epsilon_5\left(-2\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(1-e^{b_1\epsilon_{10}}+\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right),\,ç,\,0,\,1,\,2\Big]+\delta aa\Big[\\
&\quad-\frac{1}{b_1^3\epsilon_{10}}e^{-b_1\epsilon_{10}}\epsilon_5\left(-2\,e^{b_1\epsilon_{10}}\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(-\left(-1+e^{b_1\epsilon_{10}}\right)^2+e^{b_1\epsilon_{10}}\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right),\\
&\quad 1,\,0,\,1,\,2\Big]\Big]
\end{aligned}$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"];

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]

Button["Export",
 SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
 TagProperties[_] := {};
 TagProperties["ct-def"] = {PageWidth → 6 / 0.65};
 Options[CellExport] = {
   PageWidth → 4 / 0.65, CellFilter → Identity,
   ExportDirectory → "Snips", ExportBaseFilename → Automatic,
```

```
  ExportFormat → ".pdf", ExportOptions → {}, Split → False
 };
CellExport[tag_String, opts___Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
 ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
     OptionValue[ExportDirectory] /. Automatic → Directory[],
     OptionValue[ExportBaseFilename] /. Automatic → tag
    }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
     nb, c_Cell /; FreeQ[List @@ c, Cell] && ! FreeQ[c, CellTags → tag],
     Infinity
    ]];
  If[! OptionValue[Split],
   If[Length[cells] ≥ 1,
    If[Length[cells] == 1,
     cells = Append[First[cells], PageWidth → 1.2 × 72 OptionValue[PageWidth]],
     cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
    ];
    ConditionalExport[
     filename <> format, cells,
     ImageResolution → 300,
     OptionValue[ExportOptions]
    ]
   ],
   k = 0;
   Table[
    ++k;
    ConditionalExport[
     filename <> "-" <> ToString[k] <> format,
     Append[cell, PageWidth → 72 OptionValue[PageWidth]],
     ImageResolution → 300,
     OptionValue[ExportOptions]
    ],
    {cell, cells}
   ]
  ]
 ];
```

```
 nb = NotebookGet[EvaluationNotebook[]];
 tags = Cases[nb, (CellTags → tag_) :↦ tag, Infinity] // Union;
 CellExport /@ tags;
 Print["Done."]
]
```

Export

```
Exporting Snips\db-def.pdf...
Done.
```