Pensieve header: Scatter and Glow in OneCo. Continues pensieve://2016-03/.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"]
```

```
C:\drorbn\AcademicPensieve\Projects\OneCo-1604
```

I ought to be able to replace c[i] with a[c,i], and ca with aa, everywhere!

Signs

```
SetSigns[0] := Do[Unset[ϵi], {i, 0, 48}];
```

```
SetSigns[4] :=
 ({ϵ0, ϵ1, ϵ2, ϵ3, ϵ4, ϵ5, ϵ6, ϵ7, ϵ8, ϵ9, ϵ10, ϵ11, ϵ12, ϵ13, ϵ14, ϵ15, ϵ16, ϵ17, ϵ18,
    ϵ19, ϵ20, ϵ21, ϵ22, ϵ23, ϵ24, ϵ25, ϵ26, ϵ27, ϵ28, ϵ29, ϵ30, ϵ31, ϵ32, ϵ33,
    ϵ34, ϵ35, ϵ36, ϵ37, ϵ38, ϵ39, ϵ40, ϵ41, ϵ42, ϵ43, ϵ44, ϵ45, ϵ46, ϵ47, ϵ48} =
   {ϵ1, ϵ1, ϵ1 ϵ10, ϵ5, ϵ10, ϵ5, ϵ6, -ϵ1 ϵ5, ϵ5 ϵ9, ϵ9, ϵ10, ϵ1 ϵ10, ϵ1 ϵ10, -ϵ1 ϵ10, ϵ1 ϵ10,
    ϵ1 ϵ10, -ϵ1 ϵ9 ϵ10, ϵ1 ϵ9 ϵ10, ϵ1^2 ϵ5 ϵ10 / ϵ6, ϵ1 ϵ5, ϵ1 ϵ5, ϵ1^2 ϵ5, ϵ1 ϵ5, ϵ1^2 ϵ10, ϵ1 ϵ10, ϵ1^2 ϵ10,
    ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ5 ϵ10 / ϵ6, ϵ1 ϵ5, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ5 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ5,
    ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10^2, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^3 ϵ10^2, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10, ϵ1^2 ϵ10});
```

```
SetSigns[5] := (
  {ϵ0, ϵ1, ϵ2, ϵ3, ϵ4, ϵ5, ϵ6, ϵ7, ϵ8, ϵ9, ϵ10, ϵ11, ϵ12, ϵ13, ϵ14, ϵ15, ϵ16,
    ϵ17, ϵ18, ϵ19, ϵ20, ϵ21, ϵ22, ϵ23, ϵ24, ϵ25, ϵ26, ϵ27, ϵ28, ϵ29, ϵ30, ϵ31, ϵ32,
    ϵ33, ϵ34, ϵ35, ϵ36, ϵ37, ϵ38, ϵ39, ϵ40, ϵ41, ϵ42, ϵ43, ϵ44, ϵ45, ϵ46, ϵ47, ϵ48} =
   {1, 1, ϵ10, ϵ5, ϵ10, ϵ5, ϵ6, -ϵ5, ϵ5 ϵ9, ϵ9, ϵ10, ϵ10, ϵ10, -ϵ10, ϵ10, ϵ10, -ϵ9 ϵ10,
    ϵ9 ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5, ϵ5, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10, ϵ5 ϵ10 / ϵ6, ϵ5, ϵ5 ϵ10,
    ϵ5 ϵ10, ϵ5 ϵ10, ϵ10, ϵ5, ϵ10, ϵ10, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10^2, ϵ10, ϵ10, ϵ10, ϵ10}
 );
```

SetSigns

```
Cases[SetSigns[5], ϵ_, ∞] // Union
```

SetSigns

```
{ϵ5, ϵ6, ϵ9, ϵ10}
```

SetSigns

```
Table[i → ϵ_i, {i, 0, 48}]
```

SetSigns

$$\left\{0 \to 1,\ 1 \to 1,\ 2 \to \epsilon_{10},\ 3 \to \epsilon_5,\ 4 \to \epsilon_{10},\ 5 \to \epsilon_5,\ 6 \to \epsilon_6,\ 7 \to -\epsilon_5,\ 8 \to \epsilon_5\,\epsilon_9,\right.$$
$$9 \to \epsilon_9,\ 10 \to \epsilon_{10},\ 11 \to \epsilon_{10},\ 12 \to \epsilon_{10},\ 13 \to -\epsilon_{10},\ 14 \to \epsilon_{10},\ 15 \to \epsilon_{10},\ 16 \to -\epsilon_9\,\epsilon_{10},$$
$$17 \to \epsilon_9\,\epsilon_{10},\ 18 \to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 19 \to \epsilon_5,\ 20 \to \epsilon_5,\ 21 \to \epsilon_5,\ 22 \to \epsilon_5,\ 23 \to \epsilon_{10},\ 24 \to \epsilon_{10},$$
$$25 \to \epsilon_{10},\ 26 \to \epsilon_{10},\ 27 \to \epsilon_{10},\ 28 \to \epsilon_{10},\ 29 \to \epsilon_{10},\ 30 \to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 31 \to \epsilon_5,\ 32 \to \epsilon_5\,\epsilon_{10},$$
$$33 \to \epsilon_5\,\epsilon_{10},\ 34 \to \epsilon_5\,\epsilon_{10},\ 35 \to \epsilon_{10},\ 36 \to \epsilon_5,\ 37 \to \epsilon_{10},\ 38 \to \epsilon_{10},\ 39 \to \epsilon_{10},\ 40 \to \epsilon_{10},$$
$$\left.41 \to \epsilon_{10}^2,\ 42 \to \epsilon_{10},\ 43 \to \epsilon_{10},\ 44 \to \epsilon_{10}^2,\ 45 \to \epsilon_{10},\ 46 \to \epsilon_{10},\ 47 \to \epsilon_{10},\ 48 \to \epsilon_{10}\right\}$$

Generalities

Generalities

```
DQ[is___] := (Sort[{is}] === Union[{is}]);
OQ[is___] := OrderedQ[{is}];(*Also true if {is}={i,i}*)
Kδ /: Kδ_is__ := KroneckerDelta[1, Length[Union[{is}]]];
Simp[expr_] := Expand[expr];
S[expr_] :=
  expr /. (λ_β | λ_a | λ_δβ | λ_δa | λ_c | λ_ca | λ_δaa) :> MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r]);
AutoCollecting /@ {β, a, δβ, c, δa, ca, δaa};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];
γ[f_, j_, k_] := δa[f, j, k] - c[ϵ0 bj f, k];
γa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[ϵ0 bj f, k, l, m];
```

Bases

```
UUBasis[T_List, H_List, f_] := Module[
    {ff, n = 0, h, t, h1, h2},
    ff := f_{++n} @@ Table[b_t, {t, T}];
    UU /@ Flatten@{
         β[ff],
         Table[{a[ff, t, h], δa[ff, t, h]}, {t, T}, {h, H}],
         δβ[ff],
         Table[c[ff, h], {h, H}],
         Table[ca[ff, h1, t, h2], {h1, H}, {t, T}, {h2, H}],
         Table[δaa[ff, T[[i]], H[[j]], T[[k]], H[[l]]],
          {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
        } /. 1_[___] → 1
  ];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];
```

δaa relations

Switch from thth to tthh indexing? (not for the moment)

```
UU[expr_] // S := UU[S[expr]]; (* Temporary fixture! *)
```

delta-aa

```
UU[expr_] // S := UU[S[expr //. {
       δaa[f_, i_, j_, k_, l_] /; !OQ[j, l] :↦ δaa[f, k, l, i, j],
       δaa[f_, i_, j_, k_, l_] /; !OQ[i, k] ⋀ DQ[j, l] ⋀ OQ[j, l] :↦
        δaa[f, i, l, k, j] + ca[ϵ1 b_k f, l, i, j] + ca[-ϵ1 b_i f, l, k, j] +
         ca[-ϵ1 b_k f, j, i, l] + ca[ϵ1 b_i f, j, k, l],
       δaa[f_, i_, k_, j_, k_] /; !OQ[i, j] :↦ δaa[f, j, k, i, k] +
         δa[-ϵ2 b_i f, j, k] + δa[ϵ2 b_j f, i, k]
      }]];
```

tm, hm, hts, dm

tm-def

```
UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
        a[f_, x, j_] :> a[f, z, j] + ϵ3 γ[∂b_y f, z, j],
        a[f_, y, j_] :> a[f, z, j],
        δa[f_, x | y, j_] :> δa[f, z, j],
        ca[f_, i_, x | y, j_] :> ca[f, i, z, j],
        δaa[f_, i_, j_, k_, l_] :>
         δaa[f, i // Replace[x | y → z], j, k // Replace[x | y → z], l]
       } /. b_{x|y} → b_z]]];
```

hm-def

```
UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
       a[f_, i_, x | y] :> a[f, i, z],
       c[f_, x | y] :> c[f, z],
       δa[f_, i_, x | y] :> δa[f, i, z],
       ca[f_, y, j_, x] :> ca[f, z, j, z] + ϵ4 γ[f, j, z],
       ca[f_, i_, j_, k_] :>
        ca[f, i // Replace[x | y → z], j, k // Replace[x | y → z]],
       δaa[f_, i_, y, k_, x] :> δaa[f, k, z, i, z],
       δaa[f_, i_, j_, k_, l_] :>
        δaa[f, i, j // Replace[x | y → z], k, l // Replace[x | y → z]]
      }]]];
```

hts-def

```
UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
       a[f_, i_, j_] :> a[f, i, j] - ϵ5 Kδ_{j,y} γ[∂b_x f, i, y] -
         Kδ_{i,x} Kδ_{j,y} (ϵ6 β[f b_x] + ϵ7 c[f, y] - ϵ8 δβ[b_x ∂b_x f]),
       δa[f_, x, y] :> δa[f, x, y] - ϵ9 δβ[f b_x],
       ca[f_, i_, j_, k_] :>
        ca[f, i, j, k] + ϵ10 Kδ_{i,y} Kδ_{j,x} γ[f, x, k] + Kδ_{j,x} Kδ_{k,y} c[-ϵ11 f b_x, i],
       δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j, k, l] + ϵ12 Kδ_{i,x} Kδ_{j,y} δa[-b_x f, k, l] +
         ϵ13 Kδ_{i,x} Kδ_{l,y} (-δa[b_k f, x, j] + δa[b_x f, k, j]) +
         ϵ14 Kδ_{k,x} Kδ_{j,y} (δa[b_i f, x, l] - δa[b_x f, i, l]) + ϵ15 Kδ_{k,x} Kδ_{l,y} δa[-b_x f, i, j] +
         ϵ16 Kδ_{i,x} Kδ_{j,l,y} δβ[b_x b_k f] + 2 ϵ17 Kδ_{x,i,k} Kδ_{y,j,l} δβ[b_x b_x f]
      }]]];
```

dm-def

```
dm[x_, y_, z_][expr_] := expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

tσ, hσ, dσ on {β, a, δβ, c, δa, ca, δaa}

sigma-def

```
tσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. b_i_ :> b_(i /. r) /. {
      a[f_, i_, j_] :> a[f, i /. r, j],
      δa[f_, i_, j_] :> δa[f, i /. r, j],
      ca[f_, i_, j_, k_] :> ca[f, i, j /. r, k],
      δaa[f_, i_, j_, k_, l_] :> δaa[f, i /. r, j, k /. r, l]
     }]];
tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];
hσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. {
      a[f_, i_, j_] :> a[f, i, j /. r],
      c[f_, i_] :> c[f, i /. r],
      δa[f_, i_, j_] :> δa[f, i, j /. r],
      ca[f_, i_, j_, k_] :> ca[f, i /. r, j, k /. r],
      δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j /. r, k, l /. r]
     }]];
hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
dσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];
```

tb, hb, thb, htb, db, bb on {β, a, δβ, c, δa, ca, δaa}

tb-def

```
tb[x_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
          p[u_β | u_δβ | u_c | u_δa | u_ca | u_δaa, v_a] :> -p[v, u]
         } /. {
         p[a[f_, x, j_], u_] :> (u /. {
             β[g_] :> ϵ18 γ[f ∂b_x g, x, j],
             a[g_, k_, l_] :> ϵ19 γa[f ∂b_x g, x, j, k, l] + Kδ_x,k (-γa[ϵ20 g ∂b_k f, k,
                    l, x, j] + ca[ϵ21 f g, l, x, j] - ca[ϵ21 f g, j, k, l]),
             _ → 0
            }),
         p[a[f_, j_, k_], a[g_, x, l_]] /; DQ[j, x] :> -γa[ϵ22 g ∂b_x f, x, l, j, k],
         p[_, _] → 0
        }]]]];
```

hb-def

```
hb[y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ, _] → 0,
          p[_, _β | _δβ] → 0,
          p[_c | _δa | _ca | _δaa, _c | _δa | _ca | _δaa] → 0,
          p[u_c | u_δa | u_ca | u_δaa, v_a] :↦ -p[v, u]
         } /. {
         p[a[f_, i_, y], u_] :↦ (u /. {
              a[g_, j_, k_] :↦ ϵ23 Kδ_{y,k} (a[b_j f g, i, y] - a[b_i f g, j, k]),
              c[g_, j_] :↦ ϵ24 Kδ_{y,j} γ[f g, i, j],
              δa[g_, j_, k_] :↦ ϵ25 Kδ_{y,k} (δa[b_j f g, i, y] - δa[b_i f g, j, k]),
              ca[g_, j_, k_, l_] :↦ Kδ_{y,j} γa[ϵ26 f g, i, j, k, l] +
                Kδ_{y,l} (ca[ϵ27 b_k f g, j, i, y] - ca[ϵ27 b_i f g, j, k, l]),
              δaa[g_, j_, k_, l_, m_] :↦ ϵ28 Kδ_{y,k} (δaa[b_j f g, i, y, l, m] - δaa[b_i f g, j,
                    k, l, m]) + ϵ29 Kδ_{y,m} (δaa[b_l f g, j, k, i, y] - δaa[b_i f g, j, k, l, m])
             }),
          _p → 0
         }]]]];
```

thb-def

```
thb[x_, y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
        p[_a, _β | _δβ] → 0,
        p[β[f_], a[g_, i_, j_]] :↦ Kδ_{y,j} γ[ϵ30 g ∂_{b_x} f, i, y],
        p[a[f_, i_, j_], a[g_, k_, l_]] :↦ Kδ_{y,l} (
          γa[ϵ31 g ∂_{b_x} f, k, l, i, j] + Kδ_{x,i} (
            γ[-ϵ32 b_k g ∂_{b_x} f, i, j] + δa[ϵ33 b_k g ∂_{b_x} f, i,
              j] - δa[ϵ34 b_i g ∂_{b_x} f, k, j] - a[ϵ35 b_k f g, i, j] + a[ϵ35 b_i f g,
              k, j] + ca[ϵ36 f g, j, k, l] - ca[ϵ36 f g, l, k, j])),
        p[a[f_, i_, j_], c[g_, k_]] :↦ -ϵ37 Kδ_{i,x} Kδ_{k,y} γ[f g, i, j],
        p[a[f_, i_, j_], δa[g_, k_, l_]] :↦
         ϵ38 Kδ_{x,i} Kδ_{y,l} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[a[f_, i_, j_], ca[g_, k_, l_, m_]] :↦ Kδ_{x,i} (
          -ϵ39 Kδ_{y,k} γa[f g, i, j, l, m] + ϵ40 Kδ_{y,m}
            (-ca[b_l f g, k, i, j] + ca[b_i f g, k, l, j]) - ϵ41 Kδ_{y,k,m} γ[b_l f g, x, j]),
        p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] :↦ Kδ_{x,i} (
          ϵ42 Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
           ϵ43 Kδ_{y,n} (-δaa[b_m f g, k, l, i, j] + δaa[b_i f g, k, l, m, j]) +
           ϵ44 Kδ_{y,l,n} (δa[b_x b_m f g, k, j] - δa[b_k b_m f g, x, j])),
        p[_δβ | _c, _a] → 0,
        p[δa[f_, i_, j_], a[g_, k_, l_]] :↦
         ϵ45 Kδ_{x,i} Kδ_{y,l} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[ca[f_, m_, i_, j_], a[g_, k_, l_]] :↦ ϵ46 Kδ_{x,i} Kδ_{y,l}
          (-ca[b_k f g, m, i, j] + ca[b_i f g, m, k, j]),
        p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :↦
         ϵ47 Kδ_{x,i} Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
          ϵ48 Kδ_{x,m} Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_m f g, i, j, k, n])
        }]]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

$t_1\, h_1\, t_2\, h_2 \to t_1\, t_2\, h_1\, h_2 \to t_2\, t_1\, h_1\, h_2 \to t_2\, t_1\, h_2\, h_1 \to t_2\, h_2\, t_1\, h_1$:

db-def

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
     htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
     tb[x][u, v // hσ[x, h]] // hm[x, h, x],
     hb[x][u, v // tσ[x, t]] // tm[t, x, x],
     thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x]
    ]];
```

bb-def

```
bb[S_List] := Module[{w, bar, t, n = 0},
   bar[x_] := -x;
   w = #2 // dσ[S, bar /@ S];
   Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]]], {i, k + 1, Length@S}];
    t,
    {k, Length@S}
   ]
  ] &
bb[S___] := bb[{S}]
```

ct (contract)

```
ct::usage =
  "ct[h,t][L,R] contracts the head h in L with the tail t in R. ct[s][L,R] takes
    h=t=s, and ct[][L,R] takes s=0. When ambiguous, L is placeed below R.";
```

ct-def

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
       p[_β | _δβ, _] → 0,
       p[a[f_, i_, h], β[g_]] :> β[f bi ((∂bt g) /. bt → 0)],
       p[a[f_, i_, h], a[g_, t, j_]] :> a[f (g /. bt → 0), i, j],
       p[a[f_, i_, h], a[g_, j_, k_]] :> a[f bi ((∂bt g) /. bt → 0), j, k],
       p[a[f_, i_, h], c[g_, j_]] :> c[f bi ((∂bt g) /. bt → 0), j],
       p[a[f_, i_, h], δa[g_, t, j_]] :> δa[f (g /. bt → 0), i, j],
       p[a[f_, i_, h], δa[g_, j_, k_]] :> δa[f bi ((∂bt g) /. bt → 0), j, k],
       p[a[f_, i_, h], ca[g_, k_, t, j_]] :> ca[f (g /. bt → 0), k, i, j],
       p[a[f_, i_, h], ca[g_, l_, j_, k_]] :> ca[f bi ((∂bt g) /. bt → 0), l, j, k],
       p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
       p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] :> δaa[f (g /. bt → 0), i, j, k, l],
       p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] :> δaa[f (g /. bt → 0), j, k, i, l],
       p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :>
        δaa[f bi ((∂bt g) /. bt → 0), j, k, l, m],
       p[a[__], _] → 0,
       p[c[f_, h], β[g_]] :> δβ[f ((∂bt g) /. bt → 0)],
       p[_c, _β] → 0,
       p[c[f_, h], a[g_, t, j_]] :> c[f (g /. bt → 0), j],
       p[c[f_, h], a[g_, j_, k_]] :> δa[f ((∂bt g) /. bt → 0), j, k],
       p[_c, _a] → 0,
       p[_c | _δa | _ca | _δaa, _δβ | _c | _δa | _ca | _δaa] → 0,
```

```
  p[δa[f_, i_, h], β[g_]] :> δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
  p[δa[f_, i_, h], a[g_, t, j_]] :> δa[f (g /. b_t → 0), i, j],
  p[δa[f_, i_, h], a[g_, j_, k_]] :> δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
  p[_δa, _] → 0,
  p[ca[_, h, _, h], _] → 0,
  p[ca[f_, h, i_, j_], β[g_]] :> δa[f ((∂_{b_t} g) /. b_t → 0), i, j],
  p[ca[f_, i_, j_, h], β[g_]] :> c[f b_j ((∂_{b_t} g) /. b_t → 0), i],
  p[ca[f_, h, i_, j_], a[g_, t, k_]] :> ca[f (g /. b_t → 0), k, i, j],
  p[ca[f_, h, i_, j_], a[g_, k_, l_]] :> δaa[f ((∂_{b_t} g) /. b_t → 0), i, j, k, l],
  p[ca[f_, i_, j_, h], a[g_, t, k_]] :> ca[f (g /. b_t → 0), i, j, k],
  p[ca[f_, i_, j_, h], a[g_, k_, l_]] :> ca[f b_j ((∂_{b_t} g) /. b_t → 0), i, k, l],
  p[_ca, _] → 0,
  p[δaa[_, _, h, _, h], _] → 0,
  p[δaa[f_, i_, h, j_, k_], β[g_]] :> δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
  p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] :> δaa[f (g /. b_t → 0), i, l, j, k],
  p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :>
   δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
  p[δaa[f_, i_, j_, k_, h], β[g_]] :> δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
  p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] :> δaa[f (g /. b_t → 0), i, j, k, l],
  p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :>
   δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
  p[_δaa, _] → 0
 }]]];
```

dect (de-contract)

```
dect::usage =
  "dect[h,t][uu] returns a pair {L,R} such that ct[h,t][L,R]=uu. Similarly
    for dect[s] and dect[]. uu is assumed to be atomic.";
dect[s_] := dect[s, s];
dect[] = dect[0, 0];
dect[h_, t_][β[f_]] := {};
dect[h_, t_][δβ[f_]] := TBD;
```

Ad

AutoAd

```
AutoAd[B_, x_][y_] :=
  Module[{pows, states, i, s, seq, sh = 5, dseq, sf1, sf2, sf, t1, n},
   pows = NestList[B[x, #] &, y, 20];
   states = Union[Cases[pows,
      s_β | s_δβ | s_a | s_c | s_δa | s_ca | s_δaa :> ReplacePart[s, 1 → _], ∞]];
   UU@Sum[
     seq = Cases[{#}, states[[i]], ∞] & /@ pows;
     seq = Replace[seq, {{_[f_, ___]} :> f, {} → 0}, {1}];
     dseq = Drop[seq, sh];
     If[Union[Length[MonomialList[#]] & /@ dseq] === {1} ⋀
       Union[Length[FactorTermsList[#]] & /@ dseq] === {2},
      sf1 = FindSequenceFunction[FactorTermsList[#][[1]] & /@ dseq];
      sf2 = FindSequenceFunction[FactorTermsList[#][[2]] & /@ dseq];
      sf = (sf1[#] sf2[#] &),
      (*Else*) sf = FindSequenceFunction[dseq,
        FunctionSpace → {"ConstantRecursive", "HolonomicSequence",
          "Polynomial", "RationalFunction", "HypergeometricTerm"}]];
     ReplacePart[states[[i]], 1 → Simplify[∑_{n=0}^{sh-1} seq[[n + 1]]/n! + ∑_{n=sh}^{∞} sf[n + 1 - sh]/n!]],
     {i, Length@states}]];
(* Hint: Perhaps improve using Variables, CoefficientList, FromCoefficientList *)
```

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 0, 1]]
```

$$\begin{aligned}
&UU\Big[a[1,0,1]+a\Big[1-e^{-b_1\epsilon_{10}},0,2\Big]+a\Big[\frac{\left(-1+e^{-b_1\epsilon_{10}}\right)b_0}{b_1},1,2\Big]+c\Big[b_0\epsilon_5\left(\frac{-1+e^{-b_1\epsilon_{10}}}{b_1}+\epsilon_{10}\right),2\Big]+\\
&\quad ca\Big[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},1,0,2\Big]+ca\Big[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},2,0,1\Big]+\\
&\quad ca\Big[\frac{b_0\epsilon_5\left(1-e^{-b_1\epsilon_{10}}-b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},1,1,2\Big]+ca\Big[-\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},2,0,2\Big]+\\
&\quad ca\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}\left(-2+e^{b_1\epsilon_{10}}\right)b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},2,1,2\Big]+\\
&\quad \delta a\Big[\epsilon_5\left(\frac{1-e^{-2b_1\epsilon_{10}}}{b_1}+\left(-1-e^{-b_1\epsilon_{10}}\right)\epsilon_{10}\right),0,2\Big]+\\
&\quad \delta a\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},1,2\Big]+\delta aa\Big[\frac{\epsilon_5\left(b_1+\frac{-1+e^{-b_1\epsilon_{10}}}{\epsilon_{10}}\right)}{b_1^2},0,1,1,2\Big]+\\
&\quad \delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^3\epsilon_{10}},1,2,1,2\Big]+\\
&\quad \delta aa\Big[\frac{e^{-2b_1\epsilon_{10}}\left(1+e^{b_1\epsilon_{10}}\right)\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},0,2,1,2\Big]\Big]
\end{aligned}$$

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 0, 2]]
```

$$\begin{aligned}
&UU\Big[a\Big[e^{-b_1\epsilon_{10}},0,2\Big]+a\Big[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)b_0}{b_1},1,2\Big]+\\
&\quad ca\Big[\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},2,1,2\Big]+\\
&\quad ca\Big[\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},2,0,2\Big]+\\
&\quad \delta a\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},1,2\Big]+\\
&\quad \delta a\Big[\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1},0,2\Big]+\\
&\quad \delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}b_0\epsilon_5\left(-1+e^{2b_1\epsilon_{10}}-2e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^3\epsilon_{10}},1,2,1,2\Big]+\\
&\quad \delta aa\Big[-\frac{e^{-2b_1\epsilon_{10}}\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},0,2,1,2\Big]\Big]
\end{aligned}$$

ScatterTailByTails

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 1, 0]]
```

ScatterTailByTails

$$\begin{aligned}
&UU\Big[a[1,1,0]+ca[\epsilon_5,0,1,2]+\\
&\quad ca\Big[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},2,1,0\Big]+\delta aa\Big[-\frac{\epsilon_5\left(-1+e^{-b_1\epsilon_{10}}+b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},1,0,1,2\Big]\Big]
\end{aligned}$$

ScatterTailByHeads

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 2, 0]]
```

ScatterTailByHeads

$$\begin{aligned}
&UU\left[a\left[e^{b_1 \epsilon_{10}}, 2, 0\right] + a\left[-\frac{\left(-1+e^{b_1 \epsilon_{10}}\right) b_2}{b_1}, 1, 0\right] + c\left[\frac{b_2 \epsilon_5 \left(1-e^{b_1 \epsilon_{10}}+e^{b_1 \epsilon_{10}} b_1 \epsilon_{10}\right)}{b_1}, 0\right] + \right. \\
&\quad ca\left[\frac{\left(1-e^{-b_1 \epsilon_{10}}\right) \epsilon_5}{b_1 \epsilon_{10}}, 2, 1, 0\right] + ca\left[-\frac{\epsilon_5 \left(1-e^{b_1 \epsilon_{10}}+e^{b_1 \epsilon_{10}} b_1 \epsilon_{10}\right)}{b_1 \epsilon_{10}}, 0, 2, 2\right] + \\
&\quad ca\left[\frac{1}{b_1^2 \epsilon_{10}} \epsilon_5 \left(-2\left(-1+e^{b_1 \epsilon_{10}}\right) b_2 + b_1 \left(1-e^{b_1 \epsilon_{10}}+\left(1+e^{b_1 \epsilon_{10}}\right) b_2 \epsilon_{10}\right)\right), 0, 1, 2\right] + \\
&\quad \delta a\left[-\frac{b_2 \epsilon_5 \left(1-e^{b_1 \epsilon_{10}}+e^{b_1 \epsilon_{10}} b_1 \epsilon_{10}\right)}{b_1^2}, 1, 0\right] + \\
&\quad \delta aa\left[\frac{\epsilon_5 \left(1-e^{b_1 \epsilon_{10}}+e^{b_1 \epsilon_{10}} b_1 \epsilon_{10}\right)}{b_1^2 \epsilon_{10}}, 1, 0, 2, 2\right] + \delta aa\left[-\frac{1}{b_1^3 \epsilon_{10}} e^{-b_1 \epsilon_{10}} \epsilon_5 \right. \\
&\qquad \left.\left.\left(-2 e^{b_1 \epsilon_{10}}\left(-1+e^{b_1 \epsilon_{10}}\right) b_2 + b_1 \left(-\left(-1+e^{b_1 \epsilon_{10}}\right)^2 + e^{b_1 \epsilon_{10}}\left(1+e^{b_1 \epsilon_{10}}\right) b_2 \epsilon_{10}\right)\right), 1, 0, 1, 2\right]\right]
\end{aligned}$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"];

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]

Button["Export",
 SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
 TagProperties[_] := {};
 TagProperties["ct-def"] = {PageWidth → 5/0.65};
 Options[CellExport] = {
   PageWidth → 4/0.65, CellFilter → Identity,
   ExportDirectory → "Snips", ExportBaseFilename → Automatic,
   ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
```

```
CellExport[tag_String, opts___Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
 ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
     OptionValue[ExportDirectory] /. Automatic → Directory[],
     OptionValue[ExportBaseFilename] /. Automatic → tag
    }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
     nb, c_Cell /; FreeQ[List @@ c, Cell] && ! FreeQ[c, CellTags → tag],
     Infinity
    ]];
  If[! OptionValue[Split],
   If[Length[cells] ≥ 1,
    If[Length[cells] == 1,
     cells = Append[First[cells], PageWidth → 1.2 × 72 OptionValue[PageWidth]],
     cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
    ];
    ConditionalExport[
     filename <> format, cells,
     ImageResolution → 300,
     OptionValue[ExportOptions]
    ]
   ],
   k = 0;
   Table[
    ++k;
    ConditionalExport[
     filename <> "-" <> ToString[k] <> format,
     Append[cell, PageWidth → 72 OptionValue[PageWidth]],
     ImageResolution → 300,
     OptionValue[ExportOptions]
    ],
    {cell, cells}
   ]
  ]
 ];
nb = NotebookGet[EvaluationNotebook[]];
tags = Cases[nb, (CellTags → tag_) :↦ tag, Infinity] // Union;
```

```
 CellExport /@ tags;
 Print["Done."]
]
```

Export

```
Exporting Snips\Generalities.pdf...
Exporting Snips\hb-def.pdf...
Exporting Snips\hts-def.pdf...
Exporting Snips\tb-def.pdf...
Exporting Snips\thb-def.pdf...
Done.
```