OneCo Implementation Showcase

http://drorbn.net/AcademicPensieve/Projects/OneCo-1604/
Started April 4, 2016; modified April 13, 2016

In $\mathcal{U}(T)\mathcal{U}(H)$ conventions.

I ought to be able to replace `c[i]` with `a[c,i]`, and `ca` with `aa`, everywhere!

SetSigns.

```
Cases[SetSigns[5], ϵ_, ∞] // Union
```

$\{\epsilon_5, \epsilon_6, \epsilon_9, \epsilon_{10}\}$

```
Table[i → ϵ_i, {i, 0, 48}]
```

$$\left\{0\to 1,\ 1\to 1,\ 2\to \epsilon_{10},\ 3\to \epsilon_5,\ 4\to \epsilon_{10},\ 5\to \epsilon_5,\right.$$
$$6\to \epsilon_6,\ 7\to -\epsilon_5,\ 8\to \epsilon_5\,\epsilon_9,\ 9\to \epsilon_9,\ 10\to \epsilon_{10},\ 11\to \epsilon_{10},$$
$$12\to \epsilon_{10},\ 13\to -\epsilon_{10},\ 14\to \epsilon_{10},\ 15\to \epsilon_{10},\ 16\to -\epsilon_9\,\epsilon_{10},$$
$$17\to \epsilon_9\,\epsilon_{10},\ 18\to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 19\to \epsilon_5,\ 20\to \epsilon_5,\ 21\to \epsilon_5,$$
$$22\to \epsilon_5,\ 23\to \epsilon_{10},\ 24\to \epsilon_{10},\ 25\to \epsilon_{10},\ 26\to \epsilon_{10},\ 27\to \epsilon_{10},$$
$$28\to \epsilon_{10},\ 29\to \epsilon_{10},\ 30\to \frac{\epsilon_5\,\epsilon_{10}}{\epsilon_6},\ 31\to \epsilon_5,\ 32\to \epsilon_5\,\epsilon_{10},$$
$$33\to \epsilon_5\,\epsilon_{10},\ 34\to \epsilon_5\,\epsilon_{10},\ 35\to \epsilon_{10},\ 36\to \epsilon_5,\ 37\to \epsilon_{10},$$
$$38\to \epsilon_{10},\ 39\to \epsilon_{10},\ 40\to \epsilon_{10},\ 41\to \epsilon_{10}^2,\ 42\to \epsilon_{10},$$
$$\left.43\to \epsilon_{10},\ 44\to \epsilon_{10}^2,\ 45\to \epsilon_{10},\ 46\to \epsilon_{10},\ 47\to \epsilon_{10},\ 48\to \epsilon_{10}\right\}$$

Generalities.

```
DQ[is___] := (Sort[{is}] === Union[{is}]);
OQ[is___] := OrderedQ[{is}];
(*Also true if {is}={i,i}*)
Kδ /: Kδ_is__ := KroneckerDelta[1, Length[Union[{is}]]];
Simp[expr_] := Expand[expr];
S[expr_] :=
  expr /. (λ_β | λ_a | λ_δβ | λ_δa | λ_c | λ_ca | λ_δaa) :>
    MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r]);
AutoCollecting /@ {β, a, δβ, c, δa, ca, δaa};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];
γ[f_, j_, k_] := δa[f, j, k] - c[ϵ0 bj f, k];
γa[f_, j_, k_, l_, m_] :=
  δaa[f, j, k, l, m] - ca[ϵ0 bj f, k, l, m];
```

δaa relations.

```
UU[expr_] // S := UU[S[expr //. {
      δaa[f_, i_, j_, k_, l_] /; !OQ[j, l] :>
       δaa[f, k, l, i, j],
      δaa[f_, i_, j_, k_, l_] /;
        !OQ[i, k] ⋀ DQ[j, l] ⋀ OQ[j, l] :>
       δaa[f, i, l, k, j] + ca[ϵ1 bk f, l, i, j] +
        ca[-ϵ1 bi f, l, k, j] + ca[-ϵ1 bk f, j, i, l] +
        ca[ϵ1 bi f, j, k, l],
      δaa[f_, i_, k_, j_, k_] /; !OQ[i, j] :>
       δaa[f, j, k, i, k] + δa[-ϵ2 bi f, j, k] +
        δa[ϵ2 bj f, i, k]
     }]];
```

Definition of *tm*.

```
UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
      a[f_, x, j_] :> a[f, z, j] + ϵ3 γ[∂by f, z, j],
      a[f_, y, j_] :> a[f, z, j],
      δa[f_, x | y, j_] :> δa[f, z, j],
      ca[f_, i_, x | y, j_] :> ca[f, i, z, j],
      δaa[f_, i_, j_, k_, l_] :>
       δaa[f, i // Replace[x | y → z], j,
        k // Replace[x | y → z], l]
     } /. bx|y → bz]]];
```

Definition of *hm*.

```
UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
      a[f_, i_, x | y] :> a[f, i, z],
      c[f_, x | y] :> c[f, z],
      δa[f_, i_, x | y] :> δa[f, i, z],
      ca[f_, y, j_, x] :> ca[f, z, j, z] + ϵ4 γ[f, j, z],
      ca[f_, i_, j_, k_] :>
       ca[f, i // Replace[x | y → z], j,
        k // Replace[x | y → z]],
      δaa[f_, i_, y, k_, x] :> δaa[f, k, z, i, z],
      δaa[f_, i_, j_, k_, l_] :>
       δaa[f, i, j // Replace[x | y → z], k,
        l // Replace[x | y → z]]
     }]]];
```

Definition of *hts*.

```
UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
      a[f_, i_, j_] :> a[f, i, j] - ϵ5 Kδj,y γ[∂bx f, i, y] -
        Kδi,x Kδj,y (ϵ6 β[f bx] + ϵ7 c[f, y] - ϵ8 δβ[bx ∂bx f]),
      δa[f_, x, y] :> δa[f, x, y] - ϵ9 δβ[f bx],
      ca[f_, i_, j_, k_] :>
       ca[f, i, j, k] + ϵ10 Kδi,y Kδj,x γ[f, x, k] +
        Kδj,x Kδk,y c[-ϵ11 f bx, i],
      δaa[f_, i_, j_, k_, l_] :>
       δaa[f, i, j, k, l] + ϵ12 Kδi,x Kδj,y δa[-bx f, k, l] +
        ϵ13 Kδi,x Kδl,y (-δa[bk f, x, j] + δa[bx f, k, j]) +
        ϵ14 Kδk,x Kδj,y (δa[bi f, x, l] - δa[bx f, i, l]) +
        ϵ15 Kδk,x Kδl,y δa[-bx f, i, j] +
        ϵ16 Kδi,x Kδj,l,y δβ[bx bk f] +
        2 ϵ17 Kδx,i,k Kδy,j,l δβ[bx bx f]
     }]]];
```

Definition of *dm*.

```
dm[x_, y_, z_][expr_] :=
 expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

Renaming operations.

```
tσ[x_List, y_List][expr_] := Module[{r = Thread[x → y]},
   S[expr /. b_i :↦ b_i/.r /. {
      a[f_, i_, j_] :↦ a[f, i /. r, j],
      δa[f_, i_, j_] :↦ δa[f, i /. r, j],
      ca[f_, i_, j_, k_] :↦ ca[f, i, j /. r, k],
      δaa[f_, i_, j_, k_, l_] :↦ δaa[f, i /. r, j, k /. r, l]
     }]];
tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];
hσ[x_List, y_List][expr_] :=
  Module[{r = Thread[x → y]},
   S[expr /. {
      a[f_, i_, j_] :↦ a[f, i, j /. r],
      c[f_, i_] :↦ c[f, i /. r],
      δa[f_, i_, j_] :↦ δa[f, i, j /. r],
      ca[f_, i_, j_, k_] :↦ ca[f, i /. r, j, k /. r],
      δaa[f_, i_, j_, k_, l_] :↦ δaa[f, i, j /. r, k, l /. r]
     }]];
hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
dσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];
```

Definition of *tb*.

```
tb[x_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _c | _δa | _ca | _δaa,
          _β | _δβ | _c | _δa | _ca | _δaa] → 0,
        p[u_β | u_δβ | u_c | u_δa | u_ca | u_δaa, v_a] :↦
         -p[v, u]
       } /. {
       p[a[f_, x, j_], u_] :↦ (u /. {
           β[g_] :↦ ϵ18 γ[f ∂bx g, x, j],
           a[g_, k_, l_] :↦ ϵ19 γa[f ∂bx g, x, j, k, l] +
             Kδx,k (-γa[ϵ20 g ∂bk f, k, l, x, j] +
                ca[ϵ21 f g, l, x, j] - ca[ϵ21 f g, j, k, l]),
           _ → 0
          }),
       p[a[f_, j_, k_], a[g_, x, l_]] /; DQ[j, x] :↦
        -γa[ϵ22 g ∂bx f, x, l, j, k],
       p[_, _] → 0
      }]]]];
```

Definition of *thb*.

```
thb[x_, y_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
       p[0, _] → 0, p[_, 0] → 0,
       p[_β | _δβ | _c | _δa | _ca | _δaa,
         _β | _δβ | _c | _δa | _ca | _δaa] → 0,
       p[_a, _β | _δβ] → 0,
       p[β[f_], a[g_, i_, j_]] :↦
        Kδy,j γ[ϵ30 g ∂bx f, i, y],
       p[a[f_, i_, j_], a[g_, k_, l_]] :↦ Kδy,l (
          γa[ϵ31 g ∂bx f, k, l, i, j] + Kδx,i (
            γ[-ϵ32 bk g ∂bx f, i, j] + δa[ϵ33 bk g ∂bx f,
              i, j] - δa[ϵ34 bi g ∂bx f, k, j] - a[
              ϵ35 bk f g, i, j] + a[ϵ35 bi f g, k,
              j] + ca[ϵ36 f g, j, k, l] - ca[ϵ36 f g,
              l, k, j])),
       p[a[f_, i_, j_], c[g_, k_]] :↦
        -ϵ37 Kδi,x Kδk,y γ[f g, i, j],
       p[a[f_, i_, j_], δa[g_, k_, l_]] :↦
        ϵ38 Kδx,i Kδy,l (-δa[bk f g, i, j] + δa[bi f g, k, j]),
       p[a[f_, i_, j_], ca[g_, k_, l_, m_]] :↦ Kδx,i (
          -ϵ39 Kδy,k γa[f g, i, j, l, m] +
           ϵ40 Kδy,m (-ca[bl f g, k, i, j] + ca[bi f g,
              k, l, j]) - ϵ41 Kδy,k,m γ[bl f g, x, j]),
       p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] :↦ Kδx,i (
          ϵ42 Kδy,l (-δaa[bk f g, i, j, m, n] + δaa[
              bi f g, k, j, m, n]) +
           ϵ43 Kδy,n (-δaa[bm f g, k, l, i, j] + δaa[
              bi f g, k, l, m, j]) +
           ϵ44 Kδy,l,n (δa[bx bm f g, k, j] - δa[bk bm f g,
              x, j])),
       p[_δβ | _c, _a] → 0,
       p[δa[f_, i_, j_], a[g_, k_, l_]] :↦
        ϵ45 Kδx,i Kδy,l (-δa[bk f g, i, j] + δa[bi f g, k, j]),
       p[ca[f_, m_, i_, j_], a[g_, k_, l_]] :↦
        ϵ46 Kδx,i Kδy,l
         (-ca[bk f g, m, i, j] + ca[bi f g, m, k, j]),
       p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :↦
        ϵ47 Kδx,i Kδy,l (-δaa[bk f g, i, j, m, n] +
            δaa[bi f g, k, j, m, n]) +
         ϵ48 Kδx,m Kδy,l
          (-δaa[bk f g, i, j, m, n] + δaa[bm f g, i, j, k, n])
      }]]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

Definition of *hb*.

```
hb[y_][UU[L_], UU[R_]] :=
  Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ, _] → 0,
        p[_, _β | _δβ] → 0,
        p[_c | _δa | _ca | _δaa, _c | _δa | _ca | _δaa] → 0,
        p[u_c | u_δa | u_ca | u_δaa, v_a] :→ -p[v, u]
       } /. {
       p[a[f_, i_, y], u_] :→ (u /. {
            a[g_, j_, k_] :→
             ϵ23 Kδy,k (a[bj f g, i, y] - a[bi f g, j, k]),
            c[g_, j_] :→ ϵ24 Kδy,j γ[f g, i, j],
            δa[g_, j_, k_] :→
             ϵ25 Kδy,k (δa[bj f g, i, y] - δa[bi f g, j, k]),
            ca[g_, j_, k_, l_] :→
             Kδy,j γa[ϵ26 f g, i, j, k, l] +
              Kδy,l (ca[ϵ27 bk f g, j, i, y] -
                 ca[ϵ27 bi f g, j, k, l]),
            δaa[g_, j_, k_, l_, m_] :→
             ϵ28 Kδy,k (δaa[bj f g, i, y, l, m] -
                  δaa[bi f g, j, k, l, m]) +
              ϵ29 Kδy,m (δaa[bl f g, j, k, i, y] -
                  δaa[bi f g, j, k, l, m])
           }),
        _p → 0
       }]]]];
```

Definition of *db*.

Using $h_1h_2t_1t_2 \to h_1h_2t_1t_2 \to h_1h_2t_2t_1 \to h_2h_1t_2t_1 \to h_2h_1t_2t_1$:

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
    htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] //
     hm[x, h, x],
    tb[x][u, v // hσ[x, h]] // hm[x, h, x],
    hb[x][u, v // tσ[x, t]] // tm[t, x, x],
    thb[x, x][u // hσ[x, h], v // tσ[x, t]] //
      tm[t, x, x] // hm[x, h, x]
   ]];
```

The bracket.

```
bb[S_List] := Module[{w, bar, t, n = 0},
   bar[x_] := -x;
   w = #2 // dσ[S, bar /@ S];
   Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]]],
     {i, k + 1, Length@S}];
    t,
    {k, Length@S}
   ]
  ] &
bb[S___] := bb[{S}]
```

AutoAd.

```
AutoAd[B_, x_][y_] :=
  Module[{pows, states, i, s, seq, sh = 5, dseq,
    sf1, sf2, sf, t1, n},
   pows = NestList[B[x, #] &, y, 20];
   states =
    Union[Cases[pows,
      s_β | s_δβ | s_a | s_c | s_δa | s_ca | s_δaa :→
       ReplacePart[s, 1 → _], ∞]];
   UU@Sum[
     seq = Cases[{#}, states[[i]], ∞] & /@ pows;
     seq = Replace[seq, {{_[f_, ___]} :→ f, {} → 0}, {1}];
     dseq = Drop[seq, sh];
     If[Union[Length[MonomialList[#]] & /@ dseq] === {1} ∧
       Union[Length[FactorTermsList[#]] & /@ dseq] ===
        {2},
       sf1 = FindSequenceFunction[
         FactorTermsList[#][[1]] & /@ dseq];
       sf2 = FindSequenceFunction[
         FactorTermsList[#][[2]] & /@ dseq];
       sf = (sf1[#] sf2[#] &),
       (*Else*)
       sf = FindSequenceFunction[dseq,
         FunctionSpace → {"ConstantRecursive",
           "HolonomicSequence", "Polynomial",
           "RationalFunction", "HypergeometricTerm"}]];
     ReplacePart[states[[i]],
      1 → Simplify[Sum[seq[[n + 1]]/n!, {n, 0, sh - 1}] +
         Sum[sf[n + 1 - sh]/n!, {n, sh, ∞}]],
     {i, Length@states}]];
(* Hint: Perhaps improve using Variables,
CoefficientList,
FromCoefficientList *)
```

Convert to definition + verification

The scattering of a tail by an exponential of tails.

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 1, 0]]
```

$$UU\left[a[1,1,0] + ca[\epsilon_5,0,1,2] + ca\left[-\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},2,1,0\right] + \delta aa\left[-\frac{\epsilon_5\left(-1+e^{-b_1\epsilon_{10}}\epsilon_{10}+b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},1,0,1,2\right]\right]$$

The scattering of a tail by an exponential of heads.

```
AutoAd[bb[1, 2], UU@a[1, 1, 2]][UU@a[1, 2, 0]]
```

$$\begin{aligned}
&UU\left[a[e^{b_1\epsilon_{10}},2,0] + a\left[-\frac{\left(-1+e^{b_1\epsilon_{10}}\right)b_2}{b_1},1,0\right] + \right.\\
&c\left[\frac{b_2\epsilon_5\left(1+e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1},0\right] + ca\left[\frac{\left(1-e^{-b_1\epsilon_{10}}\right)\epsilon_5}{b_1\epsilon_{10}},2,1,0\right] + \\
&ca\left[-\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1\epsilon_{10}},0,2,2\right] + \\
&ca\left[\frac{\epsilon_5\left(-2\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(1-e^{b_1\epsilon_{10}}+\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right)}{b_1^2\epsilon_{10}},0,1,2\right] + \\
&\delta a\left[-\frac{b_2\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2},1,0\right] + \\
&\delta aa\left[\frac{\epsilon_5\left(1-e^{b_1\epsilon_{10}}+e^{b_1\epsilon_{10}}b_1\epsilon_{10}\right)}{b_1^2\epsilon_{10}},1,0,2,2\right] + \delta aa\left[-\frac{1}{b_1^3\epsilon_{10}}\right. \\
&\left.\left.e^{-b_1\epsilon_{10}}\epsilon_5\left(-2e^{b_1\epsilon_{10}}\left(-1+e^{b_1\epsilon_{10}}\right)b_2+b_1\left(-\left(-1+e^{b_1\epsilon_{10}}\right)^2+e^{b_1\epsilon_{10}}\left(1+e^{b_1\epsilon_{10}}\right)b_2\epsilon_{10}\right)\right),1,0,1,2\right]\right]
\end{aligned}$$

Definition of *ct*.

```
ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
    p[_β | _δβ, _] → 0,
    p[a[f_, i_, h], β[g_]] :↦ β[f b_i ((∂_{b_t} g) /. b_t → 0)],
    p[a[f_, i_, h], a[g_, t, j_]] :↦ a[f (g /. b_t → 0), i, j],
    p[a[f_, i_, h], a[g_, j_, k_]] :↦ a[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[a[f_, i_, h], c[g_, j_]] :↦ c[f b_i ((∂_{b_t} g) /. b_t → 0), j],
    p[a[f_, i_, h], δa[g_, t, j_]] :↦ δa[f (g /. b_t → 0), i, j],
    p[a[f_, i_, h], δa[g_, j_, k_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[a[f_, i_, h], ca[g_, k_, t, j_]] :↦ ca[f (g /. b_t → 0), k, i, j],
    p[a[f_, i_, h], ca[g_, l_, j_, k_]] :↦
     ca[f b_i ((∂_{b_t} g) /. b_t → 0), l, j, k],
    p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
    p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] :↦
     δaa[f (g /. b_t → 0), i, j, k, l],
    p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] :↦
     δaa[f (g /. b_t → 0), j, k, i, l],
    p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :↦
     δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    p[a[__], _] → 0,
    p[c[f_, h], β[g_]] :↦ δβ[f ((∂_{b_t} g) /. b_t → 0)],
    p[_c, _β] → 0,
    p[c[f_, h], a[g_, t, j_]] :↦ c[f (g /. b_t → 0), j],
    p[c[f_, h], a[g_, j_, k_]] :↦ δa[f ((∂_{b_t} g) /. b_t → 0), j, k],
    p[_c, _a] → 0,
    p[_c | _δa | _ca | _δaa, _δβ | _c | _δa | _ca | _δaa] → 0,
    p[δa[f_, i_, h], β[g_]] :↦ δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
    p[δa[f_, i_, h], a[g_, t, j_]] :↦ δa[f (g /. b_t → 0), i, j],
    p[δa[f_, i_, h], a[g_, j_, k_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[_δa, _] → 0,
    p[ca[_, h, _, h], _] → 0,
    p[ca[f_, h, i_, j_], β[g_]] :↦ δa[f ((∂_{b_t} g) /. b_t → 0), i, j],
    p[ca[f_, i_, j_, h], β[g_]] :↦ c[f b_j ((∂_{b_t} g) /. b_t → 0), i],
    p[ca[f_, h, i_, j_], a[g_, t, k_]] :↦ ca[f (g /. b_t → 0), k, i, j],
    p[ca[f_, h, i_, j_], a[g_, k_, l_]] :↦
     δaa[f ((∂_{b_t} g) /. b_t → 0), i, j, k, l],
    p[ca[f_, i_, j_, h], a[g_, t, k_]] :↦ ca[f (g /. b_t → 0), i, j, k],
    p[ca[f_, i_, j_, h], a[g_, k_, l_]] :↦
     ca[f b_j ((∂_{b_t} g) /. b_t → 0), i, k, l],
    p[_ca, _] → 0,
    p[δaa[_, _, h, _, h], _] → 0,
    p[δaa[f_, i_, h, j_, k_], β[g_]] :↦ δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
    p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] :↦
     δaa[f (g /. b_t → 0), i, l, j, k],
    p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :↦
     δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
    p[δaa[f_, i_, j_, k_, h], β[g_]] :↦ δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
    p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] :↦
     δaa[f (g /. b_t → 0), i, j, k, l],
    p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :↦
     δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
    p[_δaa, _] → 0
   }]]];
```