

Pensieve header: OneCo, the local picture. Branched from pensieve://Projects/OneCo-1604/.

```
Print["In the U(T)⊗U(H) conventions and with renormalized
      arrows at lco level. Internal use symbols: ", {rr, pp}]
```

```
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      renormalized arrows at lco level. Internal use symbols: {rr, pp}
```

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604\\AoBranch"]
```

```
C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1605
```

Temporary

```
ToOld = ToNew = Identity;
```

Generalities

Generalities

```
Simp[ε_] := Simplify[ε];
CF[ε_] := ε /. {λ_β | λ_a | λ_δβ | λ_c | λ_ao | λ_ca => MapAt[Simp, λ, 1],
              aao[f_, i_, j_, k_, l_] => aao[Simp@f,
              Sort[{i, k}][[1], Sort[{j, l}][[1], Sort[{i, k}][[2], Sort[{j, l}][[2]]]]];
AutoCollecting[λ_] := (λ /: λ[0, ___] = 0;
                      λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f+g], r];
                      λ /: g_*λ[f_, r___] := λ[Simp[gf], r]);
AutoCollecting /@ {β, a, δβ, c, ao, ca, aao};
UU /: UU[x_] + UU[y_] := UU[x+y];
UU /: a_*UU[x_] := UU[Expand[a x]];
UU /: D[u_UU, vs_] :=
  CF[u /. _β | λ_a | λ_δβ | λ_c | λ_ao | λ_ca | λ_aao => MapAt[D[#, vs] &, λ, 1]];
hb[f_, i_, j_, k_] := a[b_j f, i, k] - a[b_i f, j, k] (* = f · [a_ik, a_jk] *);
UU /: Coefficient[u_UU, λ[js___]] := Total[Cases[u, λ[f_, js] => f, ∞]];
Kδ /: Kδ[is_] := KroneckerDelta[1, Length[Union[{is}]]];
```

Bases

```

UUBasis[T_List, H_List, f_] := Module[
  {ff, n = 0, h, t, h1, h2, i, j, k, l},
  ff := f_{++n} @@ Table[b_t, {t, T}];
  CF /@ UU /@ Flatten@{
     $\beta$ [ff],
    Table[a[ff, t, h], {t, T}, {h, H}],
     $\delta\beta$ [ff],
    Table[c[ff, h], {h, H}],
    Table[ao[ff, t, h], {t, T}, {h, H}],
    Table[ca[ff, h1, t, h2], {t, T}, {h1, H}, {h2, H}],
    Table[aao[ff, T[[i]], H[[j]], T[[k]], H[[l]],
      {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
  } /. 1_[___]  $\rightarrow$  1
];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];
hooBasis[S_List, f_] := Module[{n = 0},
  Select[UUBasis[S, S  $\cup$  {hoo}, f], Count[#, hoo, {2}] == 1 &] /. f_[bs___]  $\Rightarrow$  f_{++n}[bs]
];

```

NonCommutativeMultiply

NonCommutativeMultiply

```

UU /: UU[L_] ** UU[R_] := CF@UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _]  $\rightarrow$  0, pp[_ , 0]  $\rightarrow$  0,
  pp[ $\beta$ [f_],  $\lambda_\beta$  |  $\lambda_a$  |  $\lambda_{\delta\beta}$  |  $\lambda_c$  |  $\lambda_{ao}$  |  $\lambda_{ca}$  |  $\lambda_{aao}$ ]  $\Rightarrow$  MapAt[f # &,  $\lambda$ , 1],
  pp[_  $\delta\beta$  | _c | _ao | _ca | _aao, _ $\delta\beta$  | _c | _ao | ca | _aao]  $\rightarrow$  0,
  pp[ao[f_, i_, j_], a[g_, k_, l_]]  $\Rightarrow$ 
    (* Not done! *) aao[fg, i, j, k, l] - K $\delta_{jk}$   $\delta$ hb[fg, i, j, l],
  pp[ $\delta\beta$ [f_],  $\beta$ [g_]]  $\Rightarrow$   $\delta\beta$ [fg],
  pp[ $\delta\beta$ [f_], a[g_, i_, j_]]  $\Rightarrow$  ao[fg, i, j]
}]];

```

Ufinished, untested!

$t\sigma, h\sigma, d\sigma$ on $\{\beta, a, \delta\beta, c, ao, ca, aao\}$

sigma-def

```

tσ[x_List, y_List][ε_] := (rr = Replace[Thread[x → y]]];
  CF[ε /. b_i_ => b_rr@i /. {
    a[f_, i_, j_] => a[f, rr@i, j],
    ao[f_, i_, j_] => ao[f, rr@i, j],
    ca[f_, i_, j_, k_] => ca[f, i, rr@j, k],
    aao[f_, i_, j_, k_, l_] => aao[f, rr@i, j, rr@k, l]
  }]);
tσ[x_, y_][ε_] := tσ[{x}, {y}][ε];
hσ[x_List, y_List][ε_] := (rr = Replace[Thread[x → y]]];
  CF[ε /. {
    a[f_, i_, j_] => a[f, i, rr@j],
    c[f_, i_] => c[f, rr@i],
    ao[f_, i_, j_] => ao[f, i, rr@j],
    ca[f_, i_, j_, k_] => ca[f, rr@i, j, rr@k],
    aao[f_, i_, j_, k_, l_] => aao[f, i, rr@j, k, rr@l]
  }]);
hσ[x_, y_][ε_] := hσ[{x}, {y}][ε];
dσ[x_, y_][ε_] := ε // tσ[x, y] // hσ[x, y];

```

tm, hm, hts, dm

tm-def

```

UU[ε_] // tm[x_, y_, z_] := (rr = Replace[x | y → z]);
  CF[UU[Expand[ε /. {
    a[f_, x, j_] => a[f, z, j] + ao[∂by]f, z, j],
    a[f_, y, j_] => a[f, z, j],
    ao[f_, x | y, j_] => ao[f, z, j],
    ca[f_, i_, x | y, k_] => ca[f, i, z, k],
    aao[f_, i_, j_, k_, l_] => aao[f, rr@i, j, rr@k, l]
  } /. b_x|y → b_z]]]);

```

hm-def

```

UU[ε_] // hm[x_, y_, z_] := (rr = Replace[x | y → z];
  CF[UU[Expand[ε /. {
    a[f_, i_, x | y] := a[f, i, z],
    c[f_, x | y] := c[f, z],
    ao[f_, i_, x | y] := ao[f, i, z],
    ca[f_, y, j_, x] := ca[f, z, j, z] + ao[f, j, z],
    ca[f_, i_, j_, k_] := ca[f, rr@i, j, rr@k],
    aao[f_, i_, y, k_, x] := aao[f, k, z, i, z],
    aao[f_, i_, j_, k_, l_] := aao[f, i, rr@j, k, rr@l]
  }]]]);

```

hts-def

```

UU[ε_] // hts[y_, x_] := CF[UU[Expand[ε /. {
  a[f_, i_, j_] :=
    a[f, i, j] - Kδjy ao[∂bx f, i, y] + Kδix Kδjy (β[-bx f] + δβ[bx ∂bx f] + c[f, y]),
  ao[f_, x, y] := ao[f, x, y] - δβ[f bx],
  ca[f_, i_, j_, k_] :=
    ca[f, i, j, k] + Kδi,y Kδj,x ao[f, x, k] + Kδj,x Kδk,y c[-f bx, i],
  aao[f_, i_, j_, k_, l_] := aao[f, i, j, k, l] + Kδix Kδjy ao[-bx f, k, l] +
    Kδix Kδiy ao[-bx f, k, j] + Kδkx Kδjy ao[-bx f, i, l] +
    Kδkx Kδiy ao[-bx f, i, j] + 2 Kδxik Kδyjl δβ[bx bx f]
}]]];

```

dm-def

```

dm[x_, y_, z_][ε_] := ε // hts[x, y] // tm[x, y, z] // hm[x, y, z]

```

tb, hb, thb, htb, db, bb on $\{\beta, a, \delta\beta, ao, aao\}$

tb-def

```

tb[x_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ | _c | _ao | _ca | _aao, _β | _δβ | _c | _ao | _ca | _aao] → 0,
  pp[u_β | u_δβ | u_c | u_ao | u_ca | u_aao, v_a] ⇒ -pp[v, u]
} /. {
  pp[a[f_, x, j_], u_] ⇒ (u /. {
    β[g_] ⇒ ao[f ∂bxg, x, j],
    a[g_, k_, l_] ⇒
      aao[f ∂bxg, x, j, k, l] + ca[bkf ∂bxg, l, x, j] + Kδxk(ca[-fg - bxg ∂bxf,
        j, x, l] + ca[fg, l, x, j] + aao[-g ∂bxf, x, j, x, l]),
    _ → 0
  })],
  pp[a[f_, j_, k_], a[g_, x, l_]]
  (*; j!=x*) ⇒ -aao[g ∂bxf, x, l, j, k] - ca[bjg ∂bxf, k, x, l],
  pp[_ , _] → 0
}]]];

```

hb-def

```

hb[y_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ, _] → 0,
  pp[_ , _β | _δβ] → 0,
  pp[_c | _ao | _ca | _aao, _c | _ao | _ca | _aao] → 0,
  pp[u_c | u_ao | u_ca | u_aao, v_a] ⇒ -pp[v, u]
} /. {
  pp[a[f_, i_, y], u_] ⇒ (u /. {
    a[g_, j_, k_] ⇒ Kδykhb[fg, i, j, k],
    c[g_, j_] ⇒ Kδyjao[fg, i, y],
    ao[g_, j_, k_] ⇒ Kδykao[-fg bi, j, y],
    ca[g_, j_, k_, l_] ⇒ Kδyj(ca[fg bk, l, i, y] + aao[fg, i, l, k, y]) +
      Kδyl(ca[-fg bi, j, k, y] + ca[fg bk, j, i, y]) + Kδyjlao[fg bk, i, y],
    aao[g_, j_, k_, l_, m_] ⇒ Kδyk(ca[-fg bj bl, m, i, y] +
      aao[-fg bi, j, m, l, y]) + Kδym(ca[-fg bj bl, k, i, y] +
      aao[-fg bi, j, k, l, y]) + Kδykmao[-fg bj bl, i, y]
  })],
  _pp → 0
}]]];

```

thb-def

```

thb[x_, y_][UU[L_], UU[R_]] := CF[UU[Expand[Distribute[pp[L, R]] /. {
  pp[0, _] → 0, pp[_ , 0] → 0,
  pp[_β | _δβ | _c | _ao | _ca | _aao, _β | _δβ | _c | _ao | _ca | _aao] → 0,
  pp[_a, _β | _δβ] → 0,
  pp[β[f_], a[g_, i_, j_]] ⇒ Kδyj ao[g ∂bx f, i, y],
  pp[a[f_, i_, j_], a[g_, k_, l_]] ⇒ Kδyl (
    aao[g ∂bx f, k, l, i, j] + ca[g bi ∂bx f, j, k, y] + Kδxi
      (a[-g f bk, x, j] + a[g f bx, k, j] + aao[-g ∂bx f, x, j, k, y] + aao[g ∂bx f, k,
        j, x, y] + ao[-g bx ∂bx f, k, j] + ca[-g f, y, k, j] + ca[g f, j, k, y])),
  pp[a[f_, i_, j_], c[g_, k_]] ⇒ Kδix Kδky ao[-f g, x, j],
  pp[a[f_, i_, j_], ao[g_, k_, l_]] ⇒ Kδxi Kδyl ao[f g bx, k, j],
  pp[a[f_, i_, j_], ca[g_, k_, l_, m_]] ⇒ Kδxi (
    Kδyk (aao[-f g, l, j, x, m] + ca[-f g bl, m, x, j]) +
    Kδym (ca[-f g bl, k, x, j] + ca[f g bx, k, l, j]) - Kδy,k,m ao[f g bl, x, j]),
  pp[a[f_, i_, j_], aao[g_, k_, l_, m_, n_]] ⇒ Kδxi (
    Kδyl (aao[f g bx, k, j, m, n] + ca[f g bk bm, n, x, j]) + Kδyn (aao[f g bx, k,
      j, m, l] + ca[f g bk bm, l, x, j]) + Kδy,l,n ao[f g bk bm, x, j]),
  pp[_δβ | _c, _a] → 0,
  pp[ao[f_, i_, j_], a[g_, k_, l_]] ⇒
    Kδxi Kδyl (-ao[bk f g, i, j] + ao[bi f g, k, j]),
  pp[ca[f_, m_, i_, j_], a[g_, k_, l_]] ⇒
    Kδxi Kδyl (ca[-f g bk, m, x, j] + ca[f g bx, m, k, j]),
  pp[aao[f_, x, j_, x, n_], a[g_, k_, y]] ⇒
    aao[-2 f g bk, x, j, x, n] + aao[2 f g bx, k, j, x, n],
  pp[aao[f_, i_, j_, m_, n_], a[g_, k_, l_]] ⇒
    Kδxi Kδyl (aao[-f g bk, m, j, x, n] + aao[f g bx, k, j, m, n]) +
    Kδxm Kδyl (aao[-f g bk, i, j, x, n] + aao[f g bx, i, j, k, n])}]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];

```

$$t_1 h_1 t_2 h_2 \rightarrow t_1 t_2 h_1 h_2 \rightarrow t_2 t_1 h_1 h_2 \rightarrow t_2 t_1 h_2 h_1 \rightarrow t_2 h_2 t_1 h_1 :$$

db-def

```

db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
  htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
  tb[x][u, v // hσ[x, h]] // hm[x, h, x],
  hb[x][u, v // tσ[x, t]] // tm[t, x, x],
  thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x]];

```

bb-def

```

bb[S_List] := Module[{w, bar, t, n = 0, i, k},
  w = #2 // do[S, bar /@ S];
  Sum[t = db[S[[k]]] [#1, w // do[bar[S[[k]]], S[[k]]]];
  Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]], {i, 1, k - 1}];
  Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]], {i, k + 1, Length@S}];
  t, {k, Length@S}] &
bb[S_++] := bb[{S}]

```

ct (contract)

ct::usage =

"ct[h,t][L,R] contracts the head h in L with the tail t in R. ct[s][L,R] takes h=t=s, and ct[][L,R] takes s=0. When ambiguous, L is placed below R."

ct-def

```

ct[s_] := ct[s, s]; ct[] = ct[0, 0];
ct[h_, t_] [UU[L_], UU[R_]] := CF[UU[Distribute[pp[L, R]] /. {
  pp[_β | _δβ, _] → 0,
  pp[a[f_, i_, h], β[g_]] ⇒ β[f bi ((∂btg) /. bt → 0)],
  pp[a[f_, i_, h], a[g_, t, j_]] ⇒ a[f (g /. bt → 0), i, j],
  pp[a[f_, i_, h], a[g_, j_, k_]] ⇒ a[f bi ((∂btg) /. bt → 0), j, k],
  pp[a[f_, i_, h], c[g_, j_]] ⇒ c[f bi ((∂btg) /. bt → 0), j],
  pp[a[f_, i_, h], ao[g_, t, j_]] ⇒ ao[f (g /. bt → 0), i, j],
  pp[a[f_, i_, h], ao[g_, j_, k_]] ⇒ ao[f bi ((∂btg) /. bt → 0), j, k],
  pp[a[f_, i_, h], ca[g_, k_, t, j_]] ⇒ ca[f (g /. bt → 0), k, i, j],
  pp[a[f_, i_, h], ca[g_, l_, j_, k_]] ⇒ ca[f bi ((∂btg) /. bt → 0), l, j, k],
  pp[a[f_, i_, h], aao[g_, t, j_, t, k_]] → 0,
  pp[a[f_, i_, h], aao[g_, t, j_, k_, l_]] ⇒ aao[f (g /. bt → 0), i, j, k, l],
  pp[a[f_, i_, h], aao[g_, j_, k_, t, l_]] ⇒ aao[f (g /. bt → 0), j, k, i, l],
  pp[a[f_, i_, h], aao[g_, j_, k_, l_, m_]] ⇒
    aao[f bi ((∂btg) /. bt → 0), j, k, l, m],
  pp[a[_], _] → 0,
  pp[c[f_, h], β[g_]] ⇒ δβ[f ((∂btg) /. bt → 0)],
  pp[_c, _β] → 0,
  pp[c[f_, h], a[g_, t, j_]] ⇒ c[f (g /. bt → 0), j],
  pp[c[f_, h], a[g_, j_, k_]] ⇒
    ao[f ((∂btg) /. bt → 0), j, k] + c[f bj ((∂btg) /. bt → 0), k],
  pp[_c, _a] → 0,
  pp[_c | _ao | _ca | _aao, _c | _ao | _ca | _aao] → 0,
  pp[ao[f_, i_, h], β[g_]] ⇒ 0,
  pp[ao[f_, i_, h], a[g_, t, j_]] ⇒ ao[f (g /. bt → 0), i, j],
  pp[ao[f_, i_, h], a[g_, j_, k_]] ⇒ 0,
  pp[_ao, _] → 0,

```

```

pp[ca[_ , h, _ , h], _] → 0,
pp[ca[f_, h, i_, j_], β[g_]] ⇒
  ao[f ((∂btg) /. bt → 0), i, j] + c[f bi ((∂btg) /. bt → 0), j],
pp[ca[f_, i_, j_, h], β[g_]] ⇒ c[f bj ((∂btg) /. bt → 0), i],
pp[ca[f_, h, i_, j_], a[g_, t, k_]] ⇒ ca[f (g /. bt → 0), k, i, j],
pp[ca[f_, h, i_, j_], a[g_, k_, l_]] ⇒ aao[f ((∂btg) /. bt → 0), i, j, k, l] +
  ca[f bi ((∂btg) /. bt → 0), j, k, l] + ca[f bk ((∂btg) /. bt → 0), l, i, j],
pp[ca[f_, i_, j_, h], a[g_, t, k_]] ⇒ ca[f (g /. bt → 0), i, j, k],
pp[ca[f_, i_, j_, h], a[g_, k_, l_]] ⇒ ca[f bj ((∂btg) /. bt → 0), i, k, l],
pp[_ca, _] → 0,
pp[aao[_ , _ , h, _ , h], _] → 0,
pp[aao[f_, i_, h, j_, k_], β[g_]] ⇒ c[-f bi bj ((∂btg) /. bt → 0), k],
pp[aao[f_, i_, h, j_, k_], a[g_, t, l_]] ⇒ aao[f (g /. bt → 0), i, l, j, k],
pp[aao[f_, i_, h, j_, k_], a[g_, l_, m_]] ⇒
  ca[-f bi bj ((∂btg) /. bt → 0), k, l, m],
pp[aao[f_, i_, j_, k_, h], β[g_]] ⇒ c[-f bi bk ((∂btg) /. bt → 0), j],
pp[aao[f_, i_, j_, k_, h], a[g_, t, l_]] ⇒ aao[f (g /. bt → 0), i, j, k, l],
pp[aao[f_, i_, j_, k_, h], a[g_, l_, m_]] ⇒
  ca[-f bi bk ((∂btg) /. bt → 0), j, l, m],
pp[_aao, _] → 0 }]];

```

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```

SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1605"];

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
    DeleteFile[temp],
    (* else *) Print["Exporting " <> fname <> "..."];
    If[exists, DeleteFile[fname]];
    RenameFile[temp, fname]
  ];
  fname
]

Button["Export",
  SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
  TagProperties[_] := {};

```



```

TagProperties["ct-def"] = {PageWidth → 8/0.65};
Options[CellExport] = {
  PageWidth → 4/0.65, CellFilter → Identity,
  ExportDirectory → "Snips", ExportBaseFilename → Automatic,
  ExportFormat → ".pdf", ExportOptions → {}, Split → False
};
CellExport[tag_String, opts___Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
    OptionValue[ExportDirectory] /. Automatic → Directory[],
    OptionValue[ExportBaseFilename] /. Automatic → tag
  }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
    nb, c_Cell /; FreeQ[List@@c, Cell] && !FreeQ[c, CellTags → tag],
    Infinity
  ]];
  If[!OptionValue[Split],
    If[Length[cells] ≥ 1,
      If[Length[cells] == 1,
        cells = Append[First[cells], PageWidth → 1.2 × 72 OptionValue[PageWidth]],
        cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
      ];
    ConditionalExport[
      filename <> format, cells,
      ImageResolution → 300,
      OptionValue[ExportOptions]
    ]
  ],
  k = 0;
  Table[
    ++k;
    ConditionalExport[
      filename <> "-" <> ToString[k] <> format,
      Append[cell, PageWidth → 72 OptionValue[PageWidth]],
      ImageResolution → 300,
      OptionValue[ExportOptions]
    ]
  ],

```

```
        {cell, cells}
      ]
    ]
];
nb = NotebookGet[EvaluationNotebook[]];
tags = Cases[nb, (CellTags -> tag_) :-> tag, Infinity] // Union;
CellExport /@ tags;
Print["Done."]
]
```

Export

Exporting Snips\ct-def.pdf...

Done.