

Pensieve header: Extraction graphs for braids representing the Rolfsen table knots.

Old program, from Gamma.nb.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OU"];
<< KnotTheory`
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

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```
In[*]:= SetAttributes[VD, Orderless]
```

```
In[*]:= bp[i_, j_] :=  $\sigma_{i,j}$ ; bm[i_, j_] :=  $\bar{\sigma}_{i,j}$ ;
```

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```
In[*]:= Tidy[vd_VD] := Module[{ps = Union@@(List@@@vd)},
  Replace[vd, Thread[ps -> Range@Length@ps], {2}]]
```

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```
In[*]:= R12Reduce1[vd_VD] := Tidy@Module[{R2s, R2}, Which[
  Length[R2s = Cases[vd, X_s_[i_, j_] -> X_-s[i + 1, j + 1]]  $\cap$  (List@@vd)] > 0,
  Complement[vd, VD[R2 = First@R2s, R2 /. X_s_[i_, j_] -> X_-s[i - 1, j - 1]]],
  Length[R2s = Cases[vd, X_s_[i_, j_] -> X_-s[i + 1, j - 1]]  $\cap$  (List@@vd)] > 0,
  Complement[vd, VD[R2 = First@R2s, R2 /. X_s_[i_, j_] -> X_-s[i - 1, j + 1]]],
  True, DeleteCases[vd, X_[i_, j_] /; Abs[i - j] == 1]]]
```

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```
In[*]:= R12Reduce[vd_VD] := FixedPoint[R12Reduce1, vd]
```

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```
In[*]:=  $\gamma$ [vd_VD] := Module[{js, s1, i1, j1, s2, i2, j2},
  js = Cases[vd, X_[_, j_] -> j]  $\cap$  Cases[vd, X_[i_, _] -> i - 1];
  If[Length[js] == 0, vd,
  j1 = RandomChoice[js]; i2 = j1 + 1;
  Cases[vd, X_s_[i_, j1] -> (s1 = s; i1 = i)];
  Cases[vd, X_s_[i2, j_] -> (s2 = s; j2 = j)];
  Tidy@Join[Complement[vd, VD[X_s1[i1, j1], X_s2[i2, j2]]],
  VD[X_s2[j1, j2], X_s1[i1, i2], X_s1s2[i1 - s1/3, j2 + s2/3], X_-s1s2[i1 + s1/3, j2 - s2/3]]]
]]]
```

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```
In[*]:=  $\Gamma$ [vd_VD] := FixedPoint[ $\gamma$ , vd, 216]
```

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```
In[*]:=  $\Gamma$ [T_] /; Head[T] != VD :=  $\Gamma$ [VD[T]]
```

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```
In[*]:=  $\bar{\Gamma}[\text{vd\_VD}] := \text{FixedPoint}[\gamma @ * \text{R12Reduce}, \text{vd}, 2^{16}]$ 
```

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```
In[*]:=  $\bar{\Gamma}[T\_]$  /;  $\text{Head}[T] \neq \text{VD} := \bar{\Gamma}[\text{VD}[T]]$ 
```

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```
In[*]:=  $\text{VPB}[n_, \{\sigma\_\_\_\}] := \text{VPB}[n, \sigma];$ 
```

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```
In[*]:=  $\text{VD} /: \text{vd1\_VD} ** \text{vd2\_VD} := \text{Module}[\{\text{es1}, \text{es2}, \text{m2}\},$   

 $\text{es1} = \text{Cases}[\text{vd1}, \text{EOS}[i\_ ] \rightarrow i];$   

 $\text{m2} = \text{Max}[\text{es2} = \text{Cases}[\text{vd2}, \text{EOS}[i\_ ] \rightarrow i]];$   

 $\text{Tidy}[\text{vd1} \cup \text{Replace}[\text{DeleteCases}[\text{vd2}, \_ \text{EOS}],$   

 $\text{i\_ } \rightarrow i / \text{m2} - 1 + \text{es1}[\text{1} + \text{Count}[\text{es2}, e\_ /; i > e]], \{2\}]]$   

 $]$ 
```

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```
In[*]:=  $\text{VD}[\text{VPB}[n\_]] := \text{VD} @@ (\text{EOS} / @ \text{Range}[n]);$   

 $\text{VD}[\text{VPB}[n_, \sigma_{i,j\_}]] := \text{Tidy} @ \text{Append}[\text{VD} @@ (\text{EOS} / @ \text{Range}[n]), \text{X}_{+1}[i - 0.5, j - 0.5]];$   

 $\text{VD}[\text{VPB}[n_, \bar{\sigma}_{i,j\_}]] := \text{Tidy} @ \text{Append}[\text{VD} @@ (\text{EOS} / @ \text{Range}[n]), \text{X}_{-1}[i - 0.5, j - 0.5]];$   

 $\text{VD}[\text{VPB}[n_, \sigma, \sigma\_\_\_\]] := \text{VD}[\text{VPB}[n, \sigma]] ** \text{VD}[\text{VPB}[n, \sigma\_\_\_\]]$ 
```

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```
In[*]:=  $\text{VPBGenerators}[n\_ ] :=$   

 $\text{VPBGenerators}[n] = \text{Flatten} @ \text{Table}[\{\sigma_{i,j}, \bar{\sigma}_{i,j}\}, \{i, n\}, \{j, \text{DeleteCases}[\text{Range}@n, i]\}];$ 
```

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```
In[*]:=  $\text{ProudFollowers}[n_, \sigma_{i,j\_} ] := \text{ProudFollowers}[n, \sigma_{i,j}] = \text{Module}[\{p, q, s\},$   

 $\text{Flatten} @ \{\sigma_{i,j}, \sigma_{j,i}, \bar{\sigma}_{j,i},$   

 $\text{Table}[\{\sigma_{p,q}, \sigma_{q,p}, \bar{\sigma}_{p,q}, \bar{\sigma}_{q,p}\}, \{p, \{i, j\}\}, \{q, \text{Complement}[\text{Range}[n], \{i, j\}]\}],$   

 $\text{Table}[\{\sigma_{p,q}, \bar{\sigma}_{p,q}\},$   

 $\{p, \text{Complement}[\text{Range}[i + 1, n], \{j\}]\}, \{q, \text{Complement}[\text{Range}[n], \{i, j, p\}]\}\}$   

 $\}];$   

 $\text{ProudFollowers}[n_, \bar{\sigma}_{i,j\_} ] := \text{ProudFollowers}[n, \bar{\sigma}_{i,j}] = \text{ProudFollowers}[n, \sigma_{i,j}] /. \sigma_{i,j} \rightarrow \bar{\sigma}_{i,j}$ 
```

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```
In[*]:=  $\text{ProudVPBs}[n_, 0] := \{\text{VPB}[n]\};$   

 $\text{ProudVPBs}[n_, 1] := \text{VPB}[n, \#] \& / @ \text{VPBGenerators}[n];$   

 $\text{ProudVPBs}[n_, m\_ ] /; m > 1 := \text{Flatten}[$   

 $\text{ProudVPBs}[n, m - 1] /. \text{VPB}[n, \sigma\_\_\_\_, \sigma\_ ] \rightarrow (\text{VPB}[n, \sigma, \sigma, \#] \& / @ \text{ProudFollowers}[n, \sigma])]$ 
```

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```
In[*]:=  $\text{CountOUForms}[n_, m\_ ] := \text{Module}[\{k\},$   

 $\text{Length} @ \text{Union} @ \text{Flatten} @ \text{Table}[\bar{\Gamma} @ \text{vpb}, \{k, 0, m\}, \{\text{vpb}, \text{ProudVPBs}[n, k]\}]]$ 
```

New Code

```

In[ ]:= AllOUs[n_, 0] := {VD@@Flatten@Table[{BT[2 i - 1], EOS[2 i]}, {i, n}]}];
AllOUs[n_, m_] /; m > 0 :=
  Sort@Flatten[AllOUs[n, m - 1] /. vd_VD => Module[{BTs, EOSs, k, max0, s},
    BTs = Sort@Cases[vd, BT[i_] => i];
    EOSs = Sort@Cases[vd, EOS[i_] => i];
    max0 = Max[1, Max[Cases[vd, X_[i_, _] => i]]];
    Table[
      Tidy[Append[vd, Xs[p - 0.5, q + 0.5]]],
      {s, {-1, 1}}, {k, Length[BTs]},
      {q, BTs[[k]], EOSs[[k] - 1]}, {p, Select[BTs, (# >= max0) &]}
    ]]]];

In[ ]:= AllROUs[n_, m_] :=
  Select[AllOUs[n, m] /. vd_VD => Tidy@DeleteCases[vd, _BT], (# === R12Reduce[#]) &]

In[ ]:=  $\xi$ [vd_VD] := Count[ $\bar{\Gamma}$ [vd], X[_ , _]]

In[ ]:= VD /: ( $\sigma_{i,j}$  | vd_VD) := Switch[Order[ $\xi$ [vd],  $\xi$ [VD[VPB[Count[vd, _EOS],  $\bar{\sigma}_{i,j}$ ]] ** vd]],
  0, Print["OMG, Trouble!"],
  1, False, -1, True];
VD /: ( $\bar{\sigma}_{i,j}$  | vd_VD) := Switch[Order[ $\xi$ [vd],  $\xi$ [VD[VPB[Count[vd, _EOS],  $\sigma_{i,j}$ ]] ** vd]],
  0, Print["OMG, Trouble!"],
  1, False, -1, True];

In[ ]:= VD /: Divisors[vd_VD] := Select[VPBGenerators[Count[vd, _EOS]], (# | vd) &];
VD /: Quotients[vd_VD] :=
   $\bar{\Gamma}$ [VD[VPB[Count[vd, _EOS], # /. { $\sigma \rightarrow \bar{\sigma}$ ,  $\bar{\sigma} \rightarrow \sigma$ }] ** vd] & /@ Divisors[vd];

In[ ]:= OUGraph[n_, m_] := Module[{gens, OUs, k, d, g, q, m1, m2},
  gens = VPBGenerators[n];
  OUs = Flatten@Table[AllROUs[n, k], {k, 0, m}];
  OURule = Dispatch@Thread[OUs -> Range@Length@OUs];
  Graph[
    Range@Length@OUs,
    Union@Flatten@Table[
      m1 = Count[d, X[_ , _]];
      m2 = Count[q =  $\bar{\Gamma}$ [VD[VPB[n, g]] ** d], X[_ , _]];
      If[m2 < m1, Labeled[(d -> q) /. OURule, g], Nothing],
      {d, OUs}, {g, gens}
    ]
  ]
]

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In[*]:= ExtractVPB[vd_VD] := Module[{n, ds, d},
  n = Count[vd, _EOS];
  If[Length[ds = Divisors[vd]] == 0, VPB[n],
    d = First@Sort[ds];
    q =  $\overline{\Gamma}$ [VD[VPB[n, d /. { $\sigma \rightarrow \overline{\sigma}$ ,  $\overline{\sigma} \rightarrow \sigma$ }]]] ** vd];
    Insert[ExtractVPB[q], d, 2]
  ]];
CF[vpb_VPB] := ExtractVPB[ $\overline{\Gamma}$ [vpb]];

```

```

In[*]:= ExtractionGraph[obj_] := Module[{vd, n, gs, vs, es, p, m1, m2, g, q, k},
  gs = VPBGenerators[n = Count[vd =  $\overline{\Gamma}$ [obj], _EOS]];
  vs = {vd}; es = {}; p = 0;
  While[p < Length[vs],
    m1 = Count[vd = vs[ $[[++p]]$ ], X[_[_]];
    Do[
      m2 = Count[q =  $\overline{\Gamma}$ [VD[VPB[n, g /. { $\sigma \rightarrow \overline{\sigma}$ ,  $\overline{\sigma} \rightarrow \sigma$ }]]] ** vd], X[_[_]];
      If[m2 < m1,
        If[! MemberQ[vs, q], AppendTo[vs, q]];
        k = Position[vs, q][[1, 1]];
        AppendTo[es, Labeled[p  $\leftrightarrow$  k, g]]
      ],
      {g, gs}
    ]
  ];
  Graph[Table[Labeled[k, kLength[vs[[k]]-n], {k, p}], es]
]

```

```

In[*]:= VPB[BR[n_, is_List]] := VPB[n, Module[{ $\pi$ , i},
   $\pi$  = Range[n];
  Sequence@@Table[
    If[i > 0,
       $\pi$ [[{i, i + 1}]] =  $\pi$ [[{i + 1, i}]];  $\sigma_{\pi[[i+1]], \pi[[i]]}$ ,
       $\pi$ [[{-i, -i + 1}]] =  $\pi$ [[{-i + 1, -i}]];  $\overline{\sigma}_{\pi[[i]], \pi[[i+1]]}$ 
    ],
    {i, is}
  ] ]];
VD[br_BR] := VD[VPB@br]

```

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In[*]:= RandomBraid[n_, m_] := BR[n, Table[RandomChoice[Range[n - 1]  $\cup$  (-Range[n - 1])], {m}]]

```

Extraction Graphs for All Knots

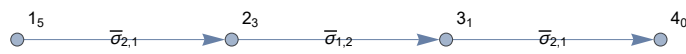
```

In[*]:= (Print[#  $\rightarrow$  (# // BR)]; Print[# // BR // ExtractionGraph]) & /@ AllKnots[];
Knot[0, 1]  $\rightarrow$  BR[1, {}]

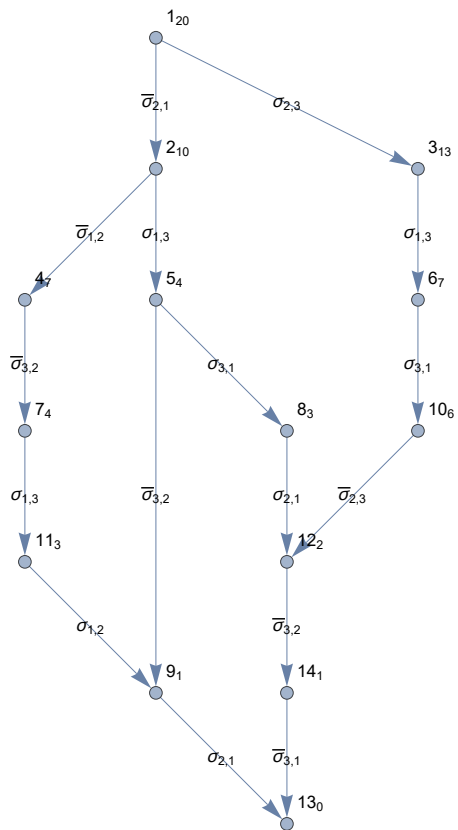
```

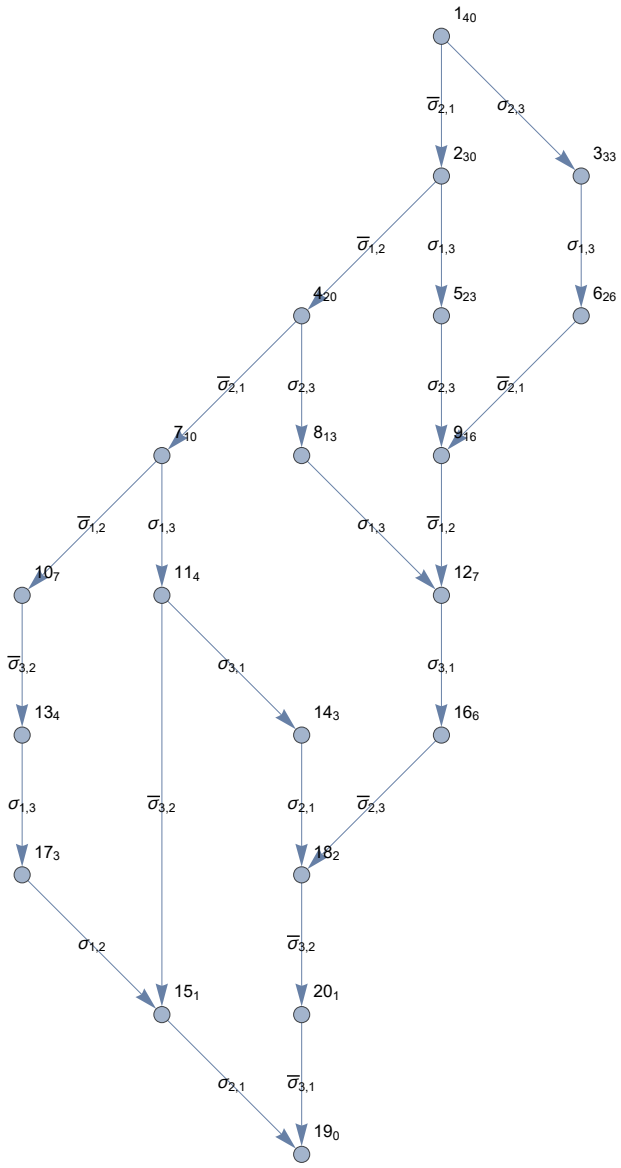



Knot [3, 1] → BR [2, {-1, -1, -1}]

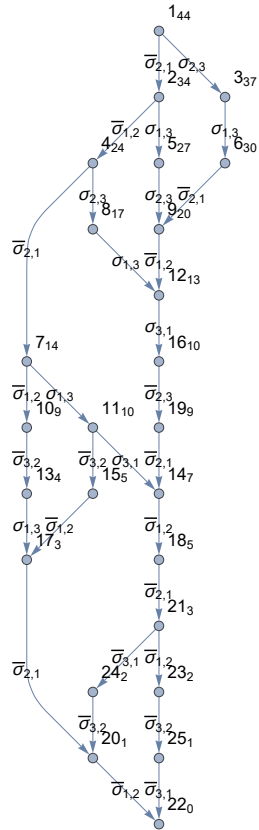


Knot [4, 1] → BR [3, {-1, 2, -1, 2}]

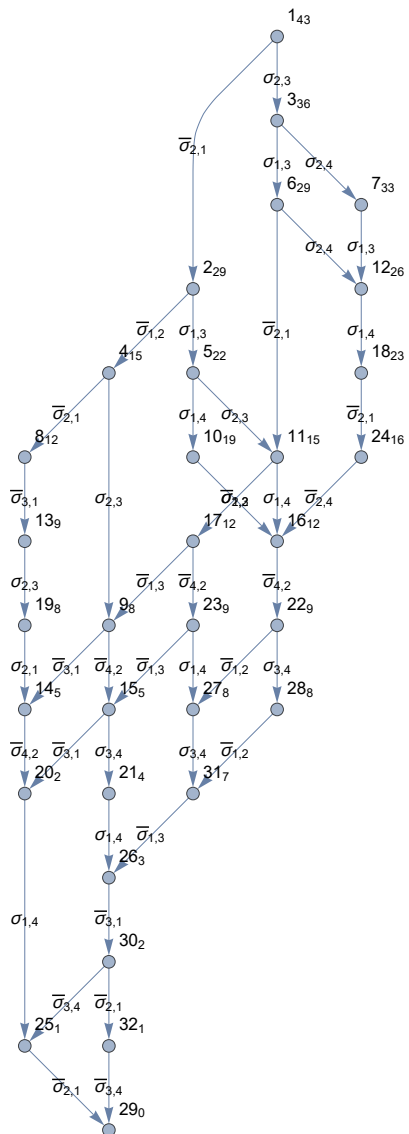




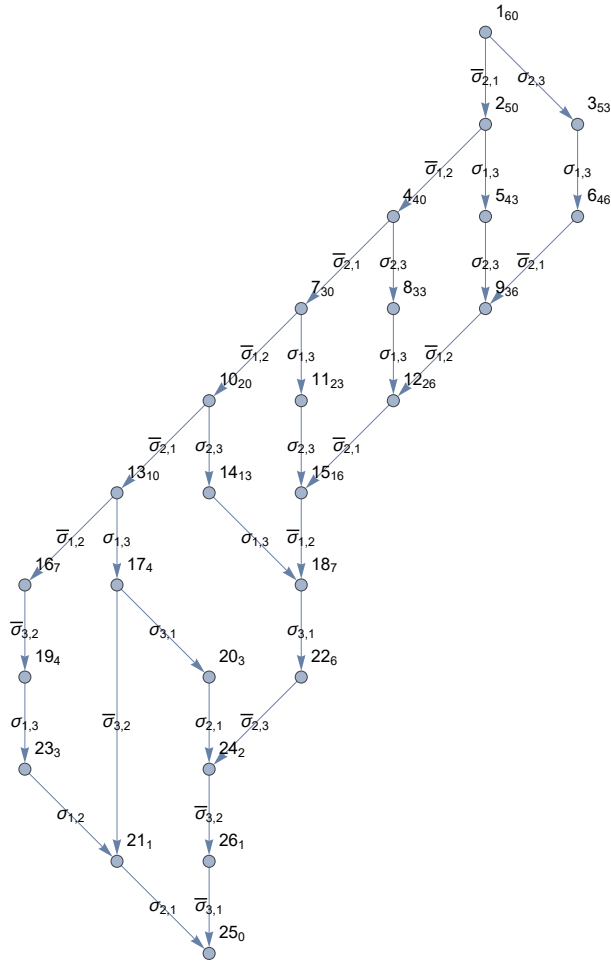
Knot [6, 3] → BR[3, {-1, -1, 2, -1, 2, 2}]



Knot [7, 6] → BR [4, {-1, -1, 2, -1, -3, 2, -3}]



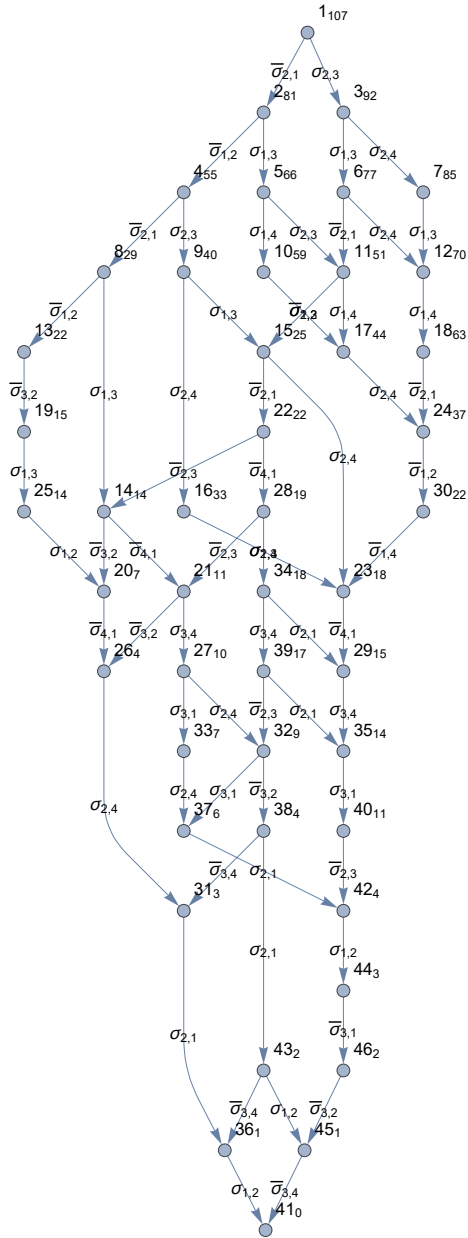
Knot [7, 7] → BR[4, {1, -2, 1, -2, 3, -2, 3}]



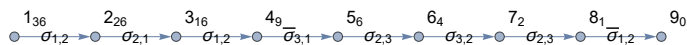
Knot [8, 3] \rightarrow BR[5, {-1, -1, -2, 1, 3, -2, 3, 4, -3, 4}]



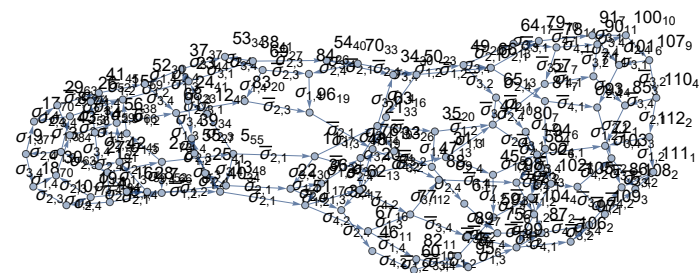
Knot [8, 4] → BR [4, {-1, -1, -1, 2, -1, 2, 3, -2, 3}]



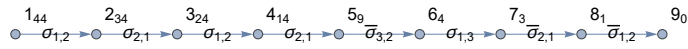
Knot [8, 5] \rightarrow BR [3, {1, 1, 1, -2, 1, 1, 1, -2}]



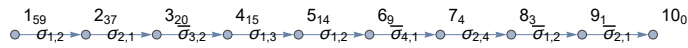
Knot [8, 6] \rightarrow BR [4, {-1, -1, -1, -1, -2, 1, 3, -2, 3}]



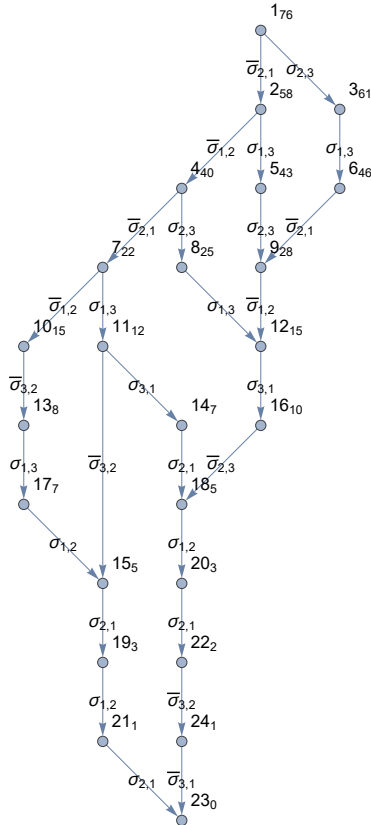
Knot [8, 7] → BR[3, {1, 1, 1, 1, -2, 1, -2, -2}]



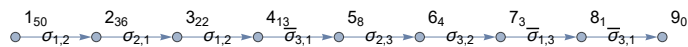
Knot [8, 8] → BR[4, {1, 1, 1, 2, -1, -3, 2, -3, -3}]



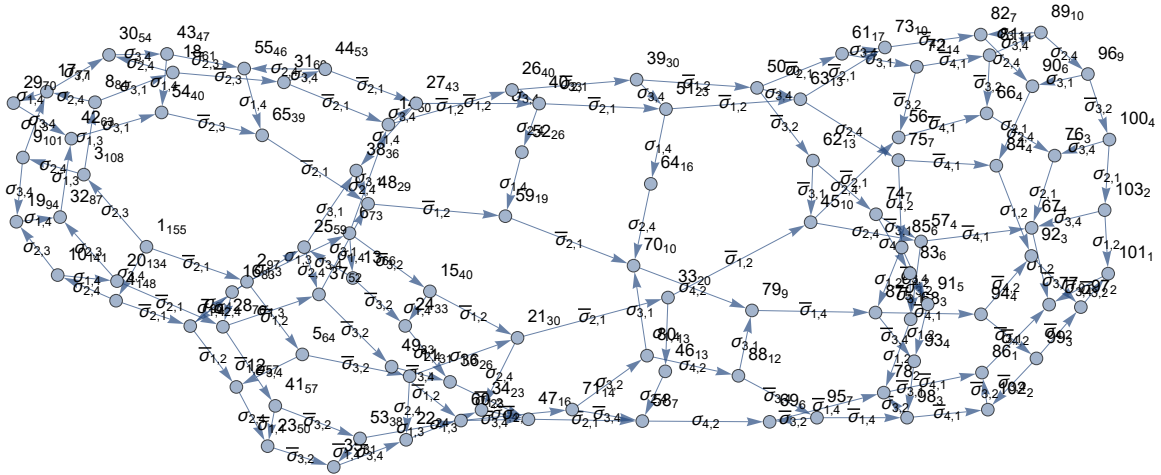
Knot [8, 9] → BR[3, {-1, -1, -1, 2, -1, 2, 2, 2}]



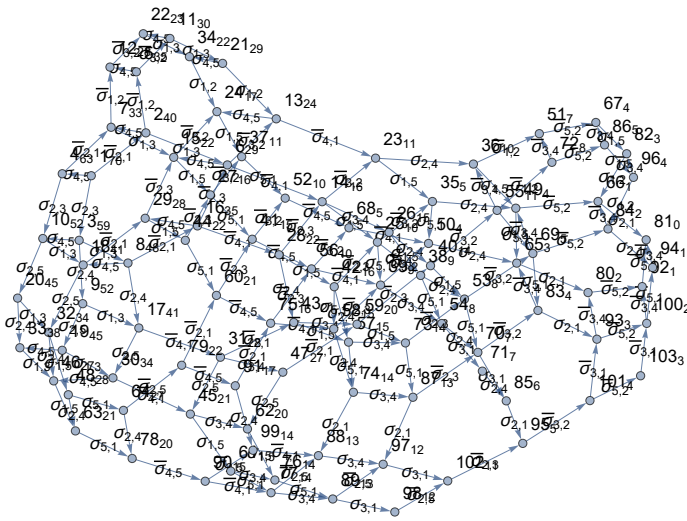
Knot [8, 10] → BR[3, {1, 1, 1, -2, 1, 1, -2, -2}]



Knot [8, 11] → BR[4, {-1, -1, -2, 1, -2, -2, 3, -2, 3}]

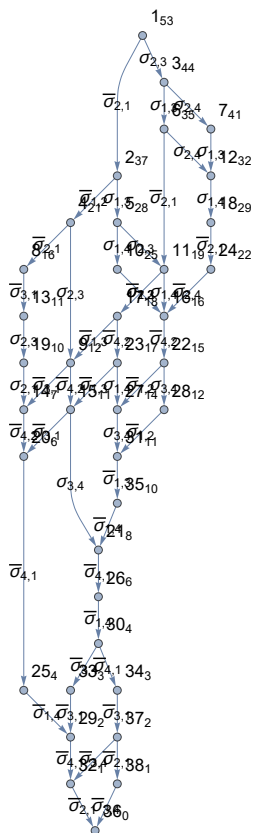


Knot [8, 12] → BR [5, {-1, 2, -1, -3, 2, 4, -3, 4}]

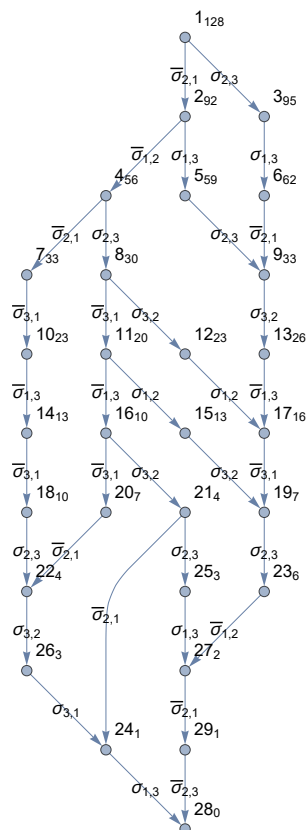


Knot [8, 13] → BR [4, {-1, -1, 2, -1, 2, 2, 3, -2, 3}]

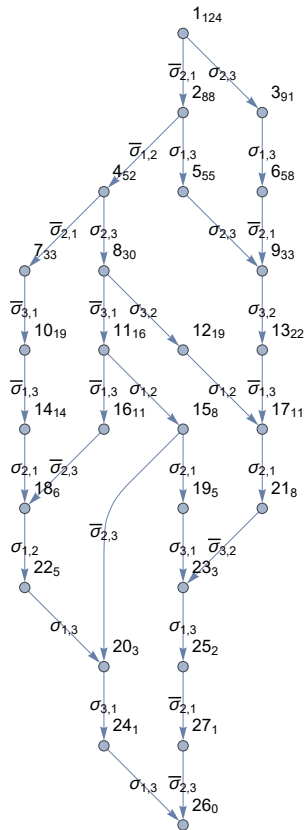
Knot [8, 15] → BR [4, {-1, -1, 2, -1, -3, -2, -2, -2, -3}]



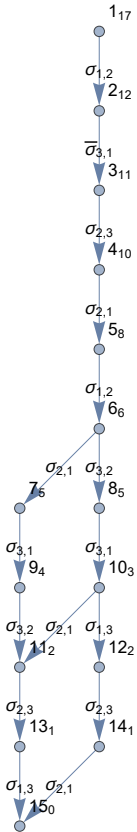
Knot [8, 16] → BR [3, {-1, -1, 2, -1, -1, 2, -1, 2}]



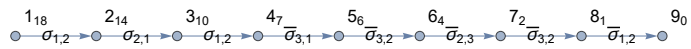
$\text{Knot}[8, 17] \rightarrow \text{BR}[3, \{-1, -1, 2, -1, 2, -1, 2, 2\}]$



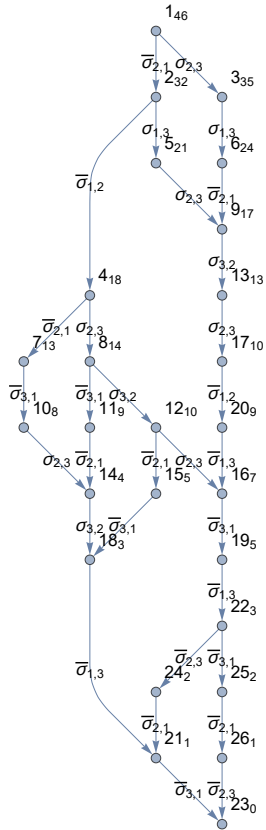
Knot [8, 18] → BR [3, { -1, 2, -1, 2, -1, 2, -1, 2 }]



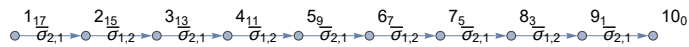
Knot [8, 20] → BR [3, {1, 1, 1, -2, -1, -1, -1, -2}]



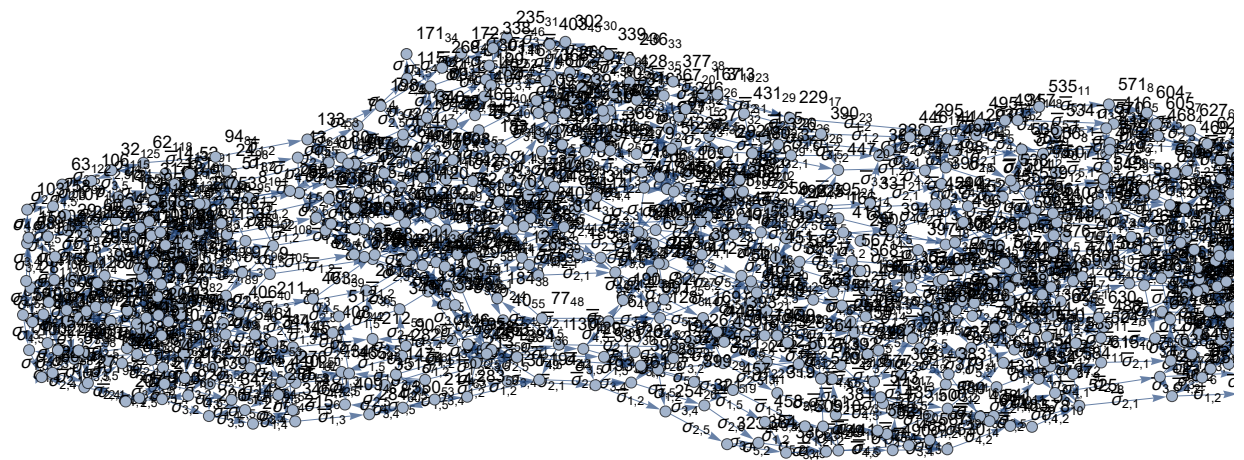
Knot [8, 21] → BR [3, {-1, -1, -1, -2, 1, 1, -2, -2}]



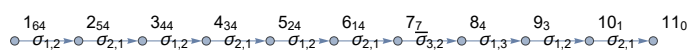
Knot [9, 1] → BR [2, { -1, -1, -1, -1, -1, -1, -1, -1, -1 }]



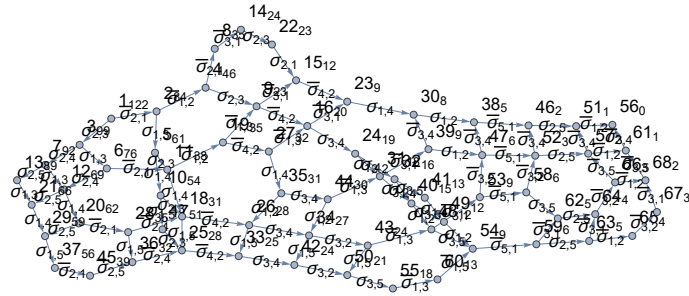
Knot [9, 2] → BR [5, { -1, -1, -1, -2, 1, -2, -3, 2, -3, -4, 3, -4 }]



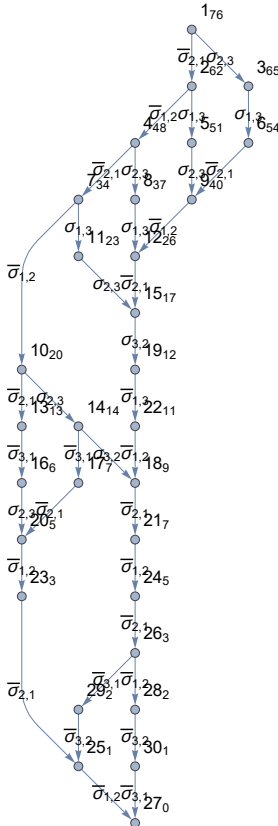
Knot [9, 3] → BR [3, { 1, 1, 1, 1, 1, 1, 1, 2, -1, 2 }]



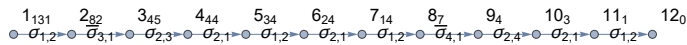
Knot [9, 4] → BR [4, { -1, -1, -1, -1, -1, -2, 1, -2, -3, 2, -3 }]



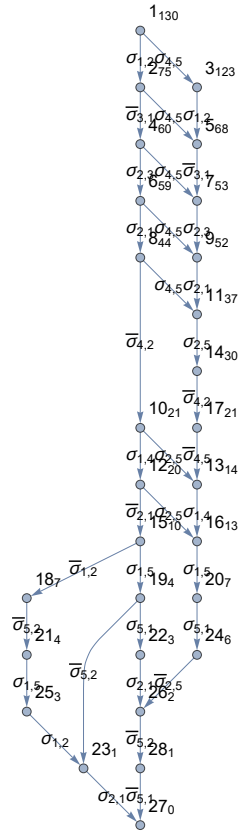
Knot [9, 9] → BR [3, { -1, -1, -1, -1, -1, -2, 1, -2, -2, -2 }]



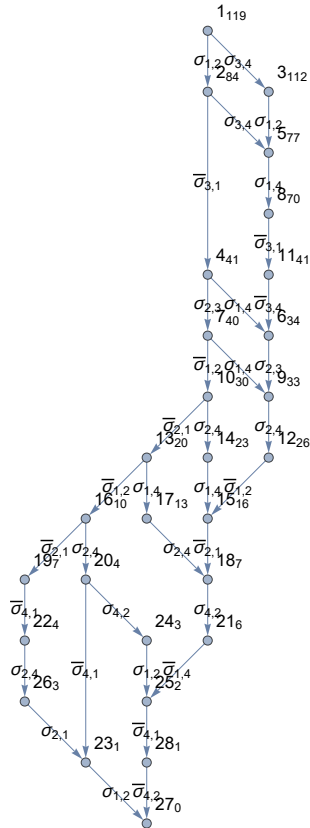
Knot [9, 10] → BR [4, { 1, 1, 2, -1, 2, 2, 2, 2, 3, -2, 3 }]



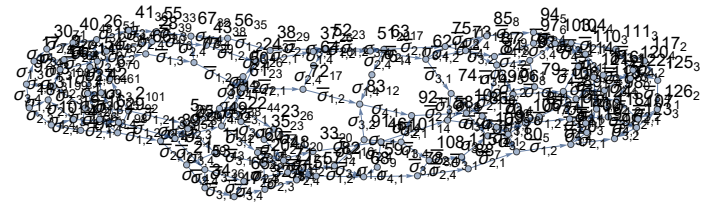
Knot [9, 11] → BR [4, { 1, 1, 1, 1, -2, 1, 3, -2, 3 }]



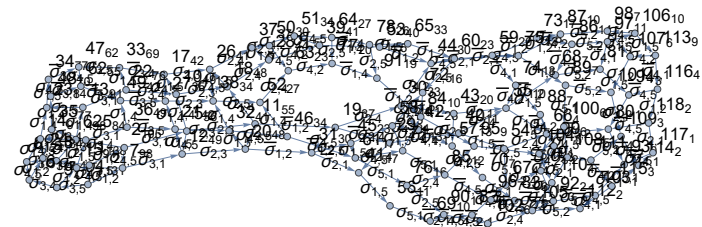
Knot [9, 15] → BR [5, {1, 1, 1, 2, -1, -3, 2, 4, -3, 4}]



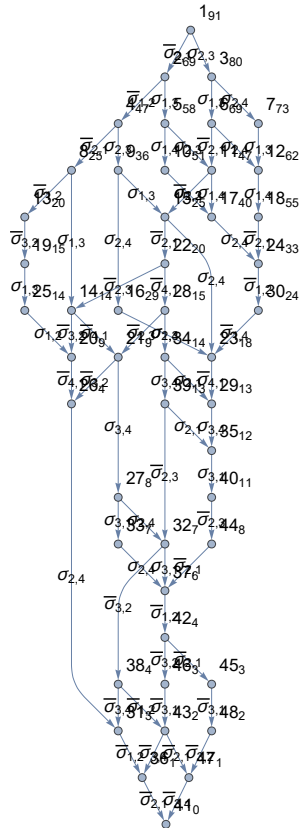
Knot [9, 18] → BR [4, {-1, -1, -1, -2, 1, -2, -2, -2, -3, 2, -3}]



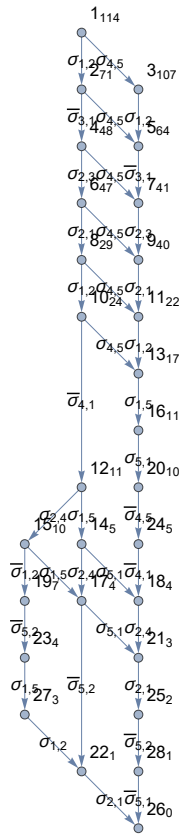
Knot [9, 19] → BR [5, {1, -2, 1, -2, -2, -3, 2, 4, -3, 4}]



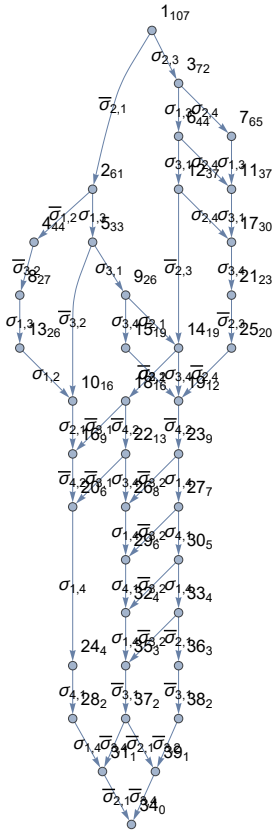
Knot [9, 20] → BR [4, {-1, -1, -1, 2, -1, -3, 2, -3, -3}]



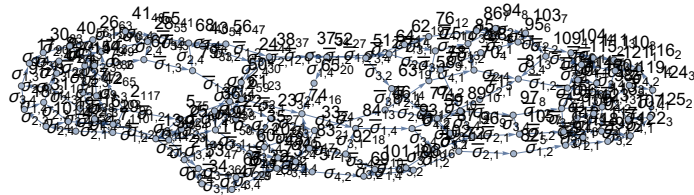
Knot [9, 21] → BR [5, {1, 1, 2, -1, 2, -3, 2, 4, -3, 4}]



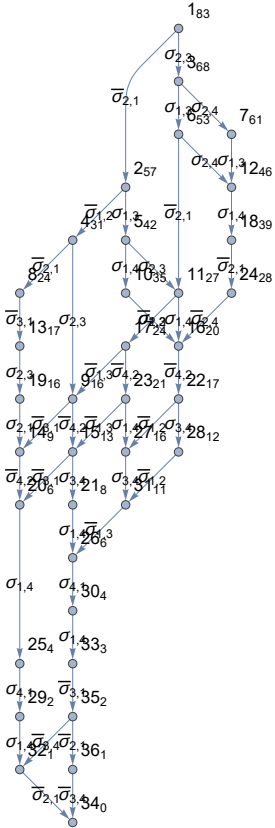
Knot [9, 22] → BR [4, {-1, 2, -1, 2, -3, 2, 2, 2, -3}]



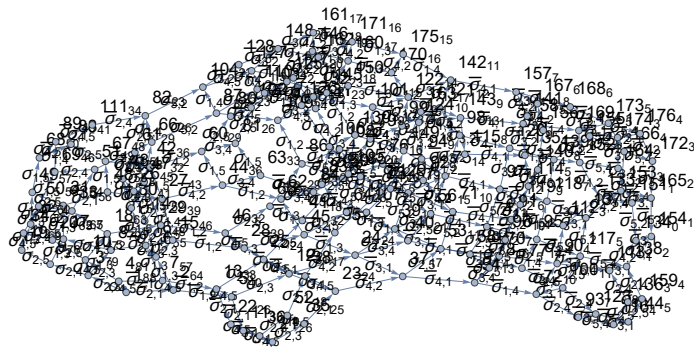
Knot [9, 23] → BR [4, {-1, -1, -1, -2, 1, -2, -2, -3, 2, -3, -3}]



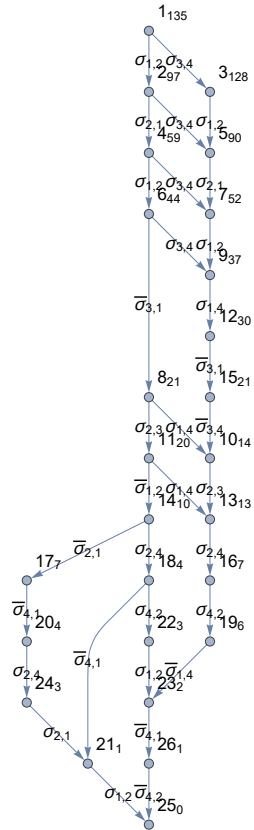
Knot [9, 24] → BR [4, {-1, -1, 2, -1, -3, 2, 2, 2, -3}]



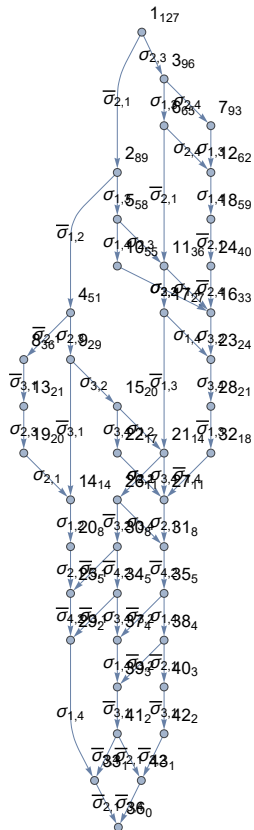
Knot [9, 25] \rightarrow BR [5, { -1, -1, 2, -1, -3, -2, -2, 4, -3, 4 }]



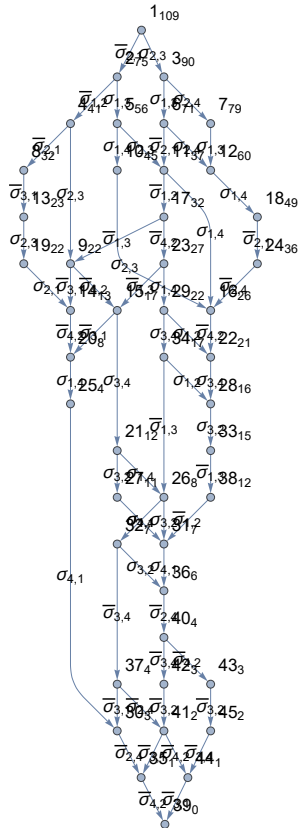
Knot [9, 26] \rightarrow BR [4, { 1, 1, 1, -2, 1, -2, 3, -2, 3 }]



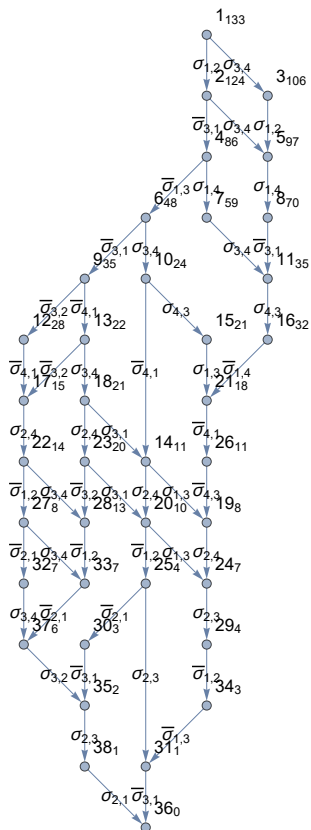
$\text{Knot}[9, 27] \rightarrow \text{BR}[4, \{-1, -1, 2, -1, 2, 2, -3, 2, -3\}]$



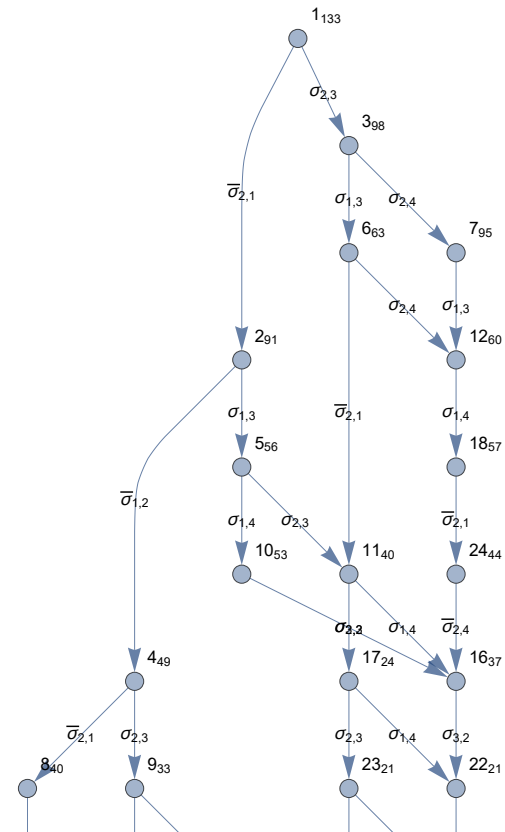
Knot [9, 28] → BR [4, {-1, -1, 2, -1, -3, 2, 2, -3, -3}]

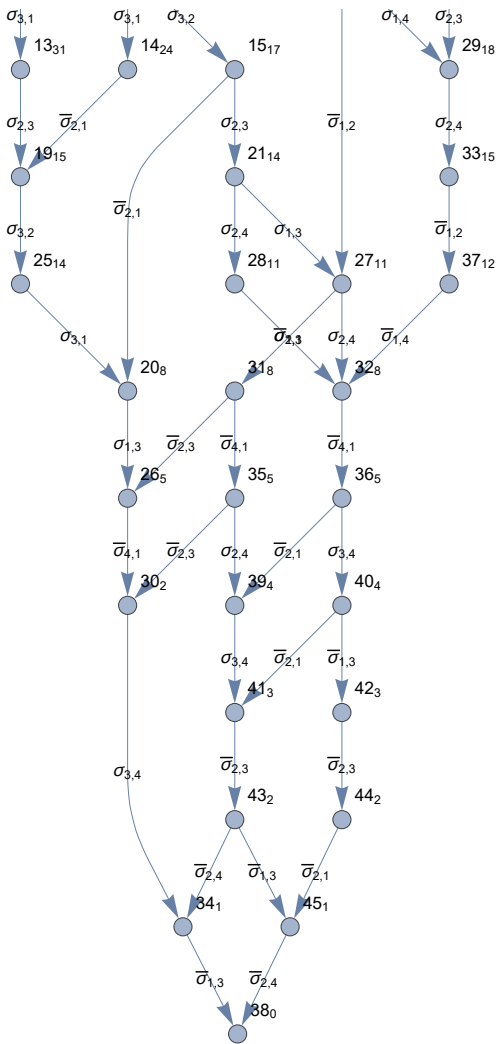


Knot [9, 29] → BR [4, {1, -2, -2, 3, -2, 1, -2, 3, -2}]

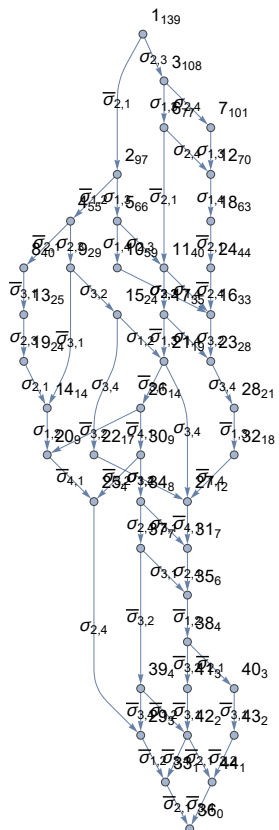


Knot [9, 30] → BR [4, {-1, -1, 2, 2, -1, 2, -3, 2, -3}]

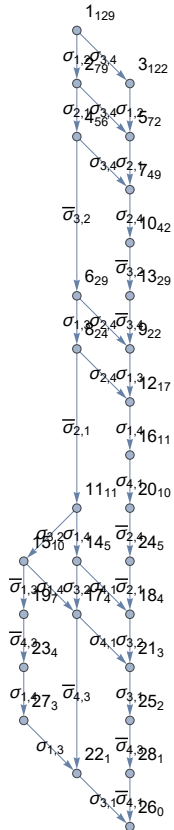




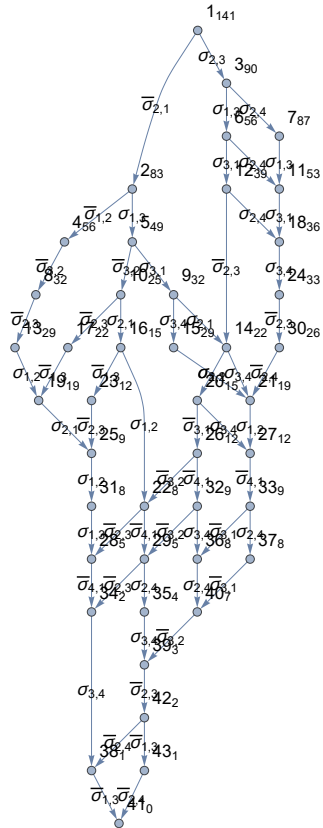
Knot [9, 31] → BR [4, {-1, -1, 2, -1, 2, -3, 2, -3, -3}]



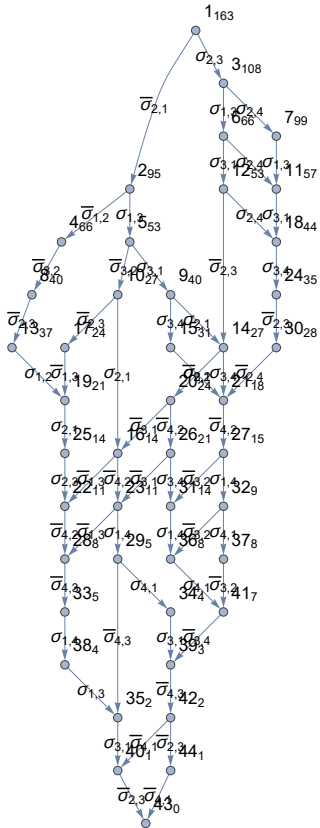
Knot [9, 32] \rightarrow BR [4, {1, 1, -2, 1, -2, 1, 3, -2, 3}]



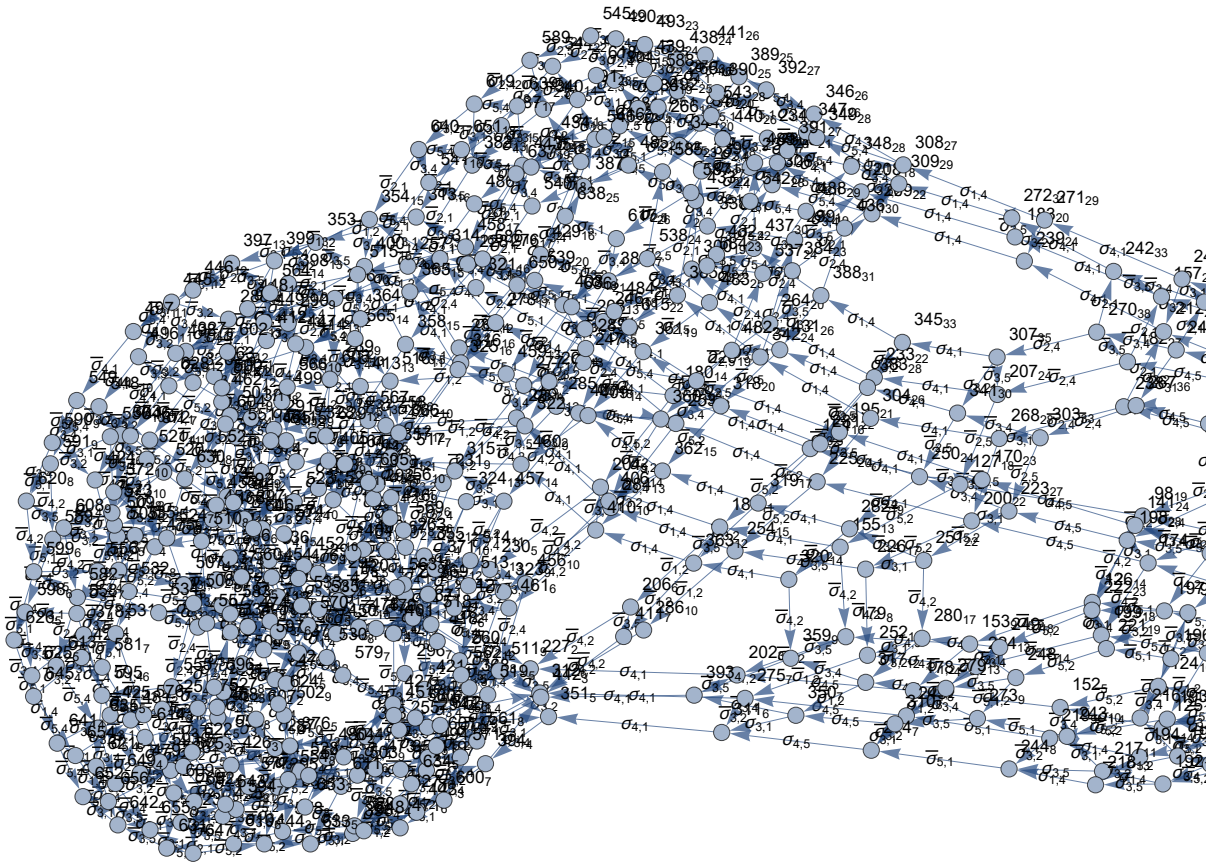
Knot [9, 33] → BR [4, {-1, 2, -1, 2, 2, -1, -3, 2, -3}]



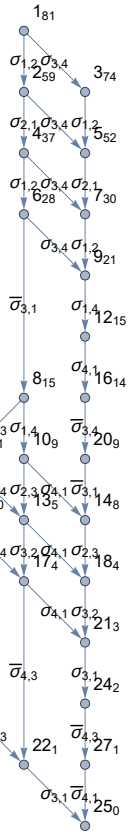
Knot [9, 34] → BR [4, {-1, 2, -1, 2, -3, 2, -1, 2, -3}]



Knot [9, 35] → BR [5, {-1, -1, -2, 1, -2, -2, -3, 2, 2, -4, 3, -2, -4, -3}]



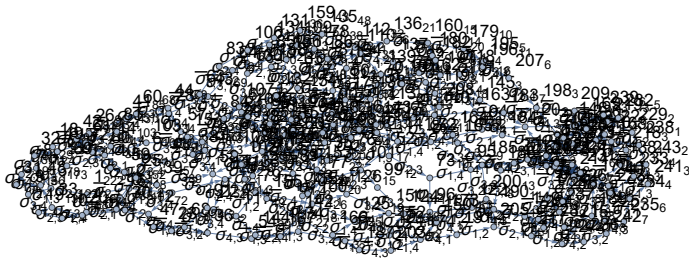
Knot [9, 36] → BR [4, {1, 1, 1, -2, 1, 1, 3, -2, 3}]



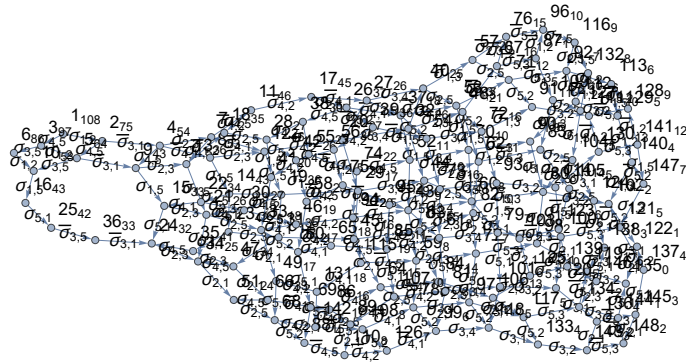
Knot [9, 37] → BR [5, { -1, -1, 2, -1, -3, 2, 1, 4, -3, 2, -3, 4 }]



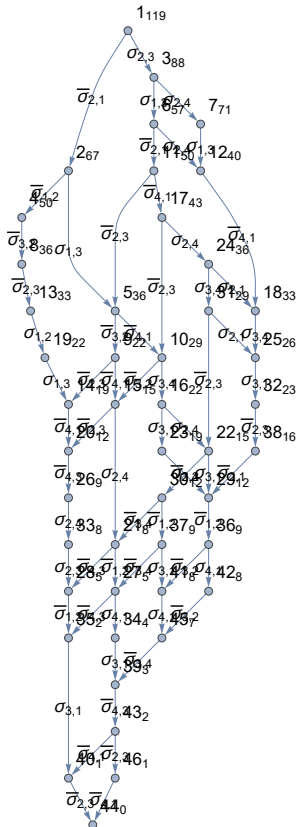
Knot [9, 38] → BR [4, { -1, -1, -2, -2, 3, -2, 1, -2, -3, -3, -2 }]



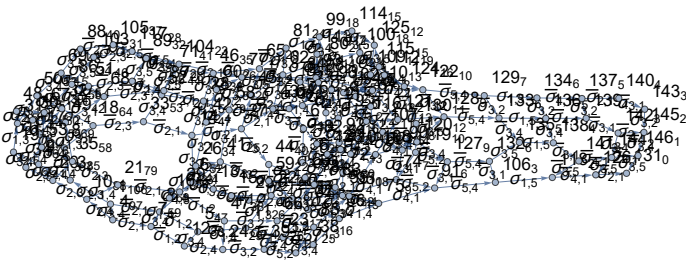
Knot [9, 39] → BR [5, { 1, 1, 2, -1, -3, -2, 1, 4, 3, -2, 3, 4 }]



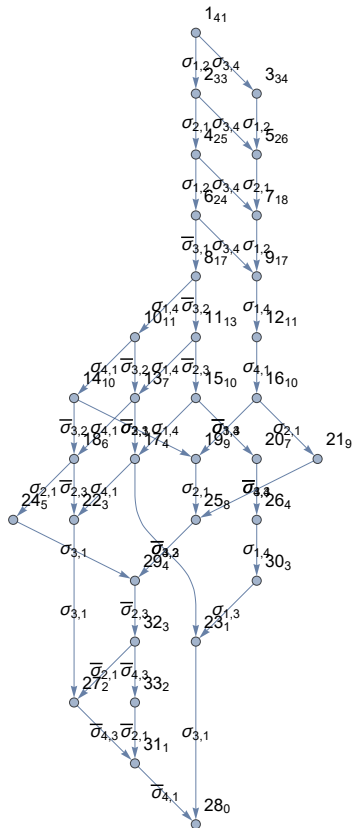
Knot [9, 40] → BR [4, {-1, 2, -1, -3, 2, -1, -3, 2, -3}]



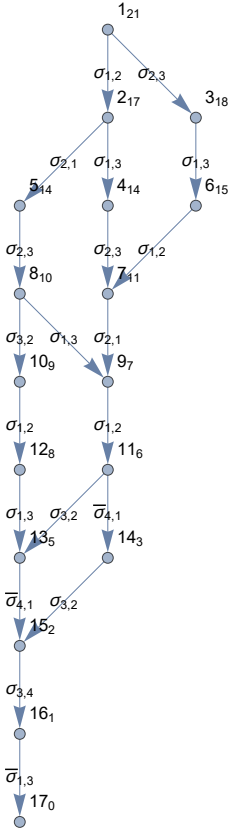
Knot [9, 41] → BR [5, {-1, -1, -2, 1, 3, 2, 2, -4, -3, 2, -3, -4}]



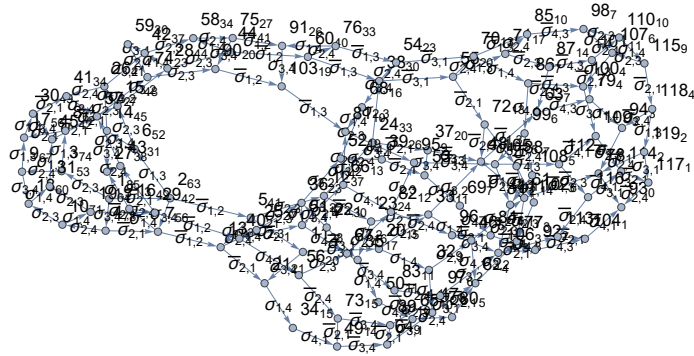
Knot [9, 42] → BR [4, {1, 1, 1, -2, -1, -1, 3, -2, 3}]



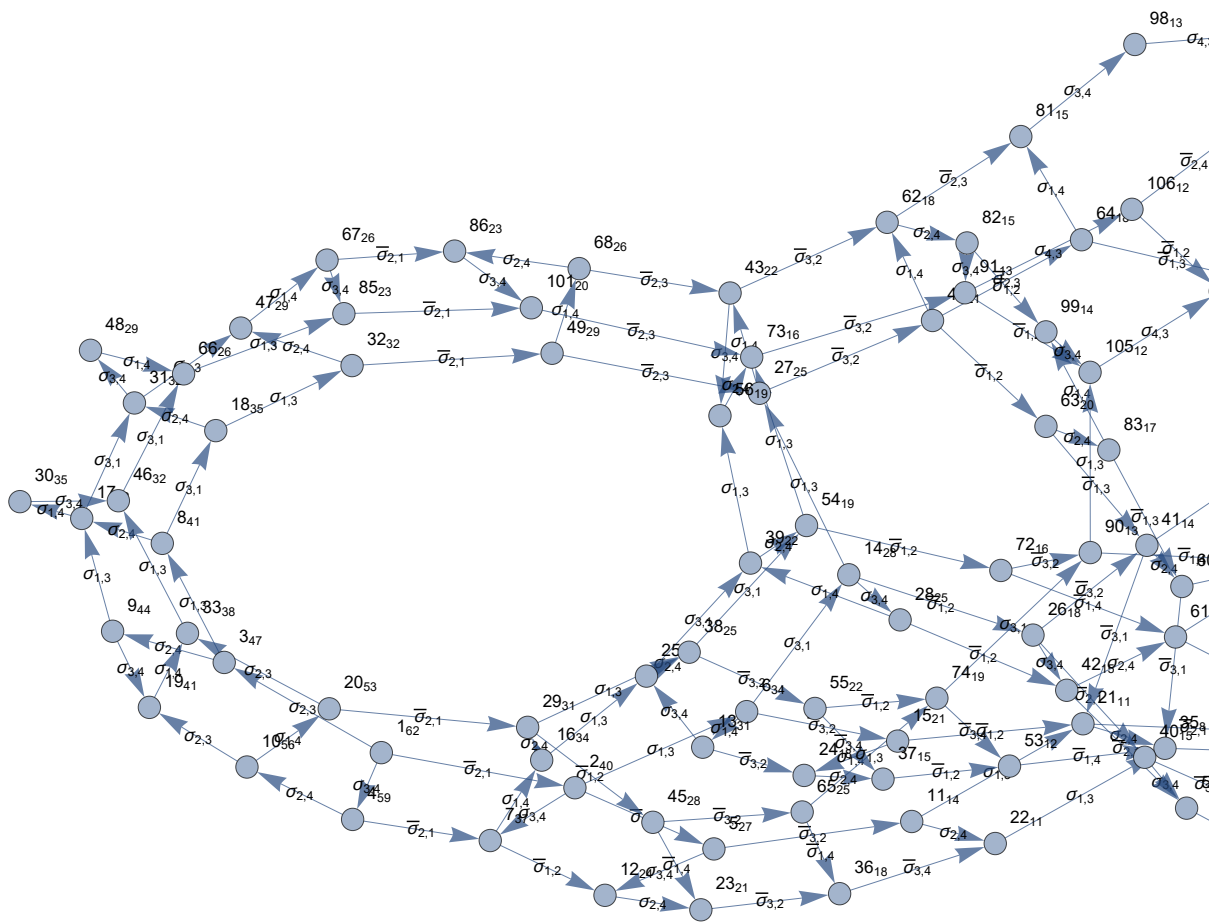
Knot [9, 43] → BR [4, {1, 1, 1, 2, 1, 1, -3, 2, -3}]



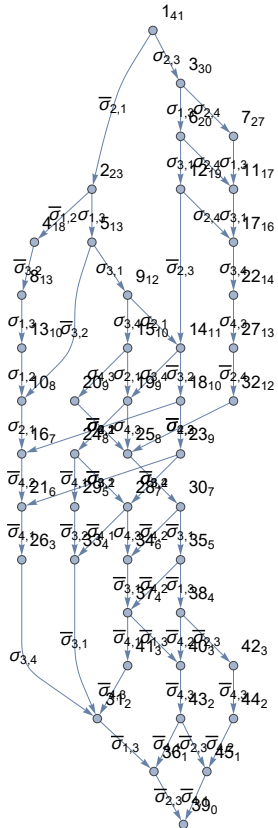
Knot [9, 44] → BR [4, {-1, -1, -1, -2, 1, 1, 3, -2, 3}]



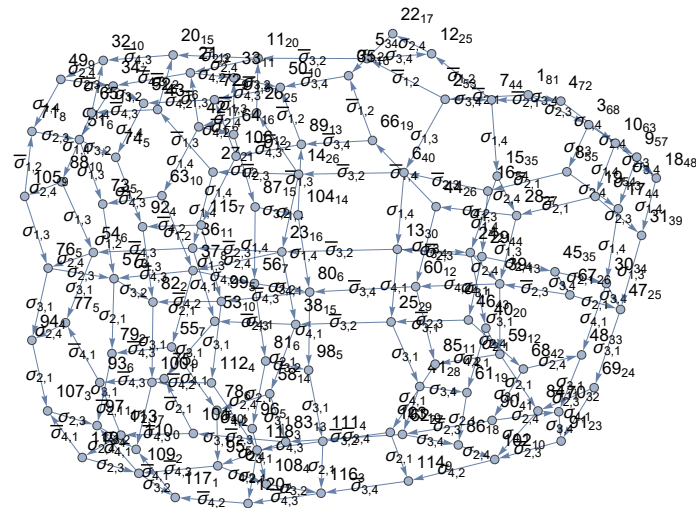
Knot [9, 45] → BR [4, {-1, -1, -2, 1, -2, -1, -3, 2, -3}]



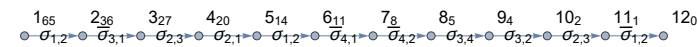
Knot[9, 46] → BR[4, {-1, 2, -1, 2, -3, -2, 1, -2, -3}]



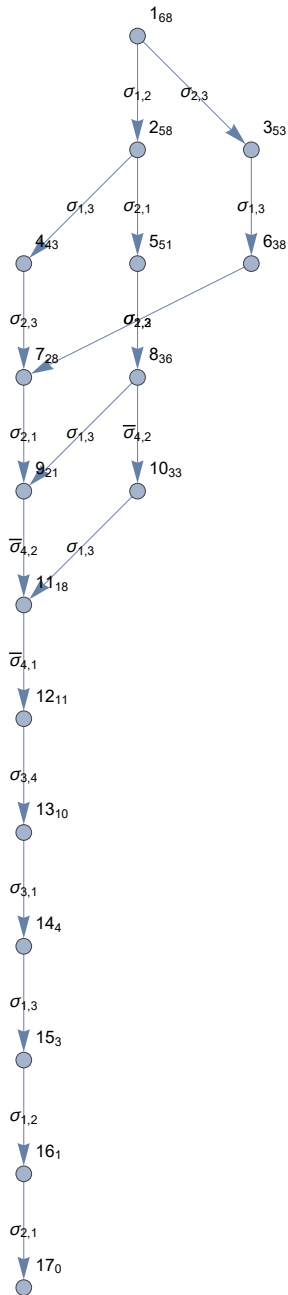
Knot [9, 47] → BR [4, { -1, 2, -1, 2, 3, 2, -1, 2, 3 }]




Knot [9, 48] → BR [4, { 1, 1, 2, -1, 2, 1, -3, 2, -1, 2, -3 }]



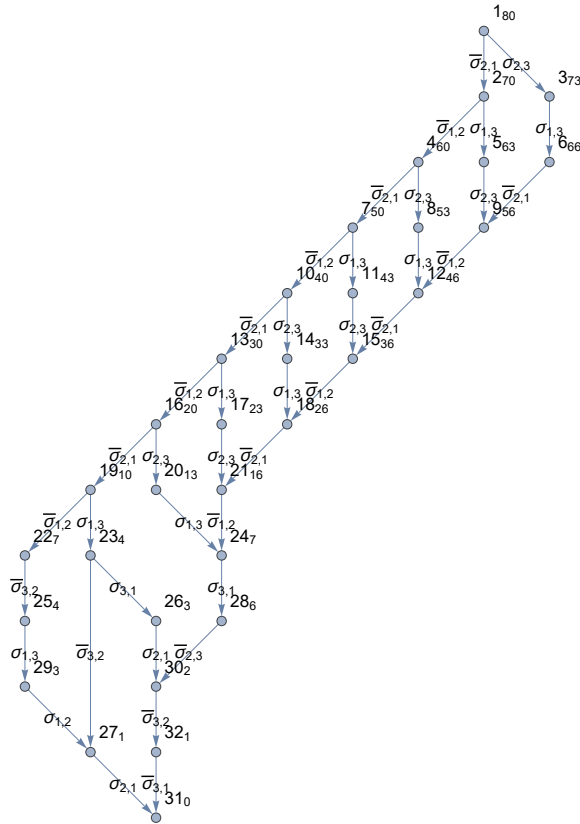
Knot [9, 49] → BR [4, { 1, 1, 2, 1, 1, -3, 2, -1, 2, 3, 3 }]



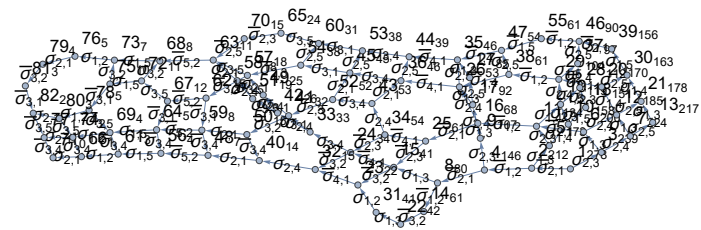
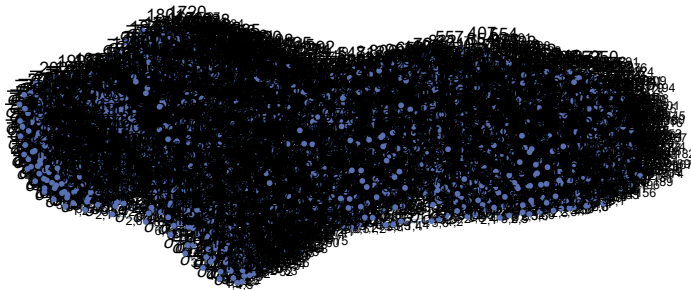
Knot [10, 1] → BR [6, {-1, -1, -2, 1, -2, -3, 2, -3, -4, 3, 5, -4, 5}]

Graph [ Vertex count: 4284
Edge count: 10971]

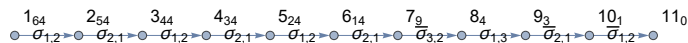
Knot [10, 2] → BR [3, {-1, -1, -1, -1, -1, -1, -1, 2, -1, 2}]



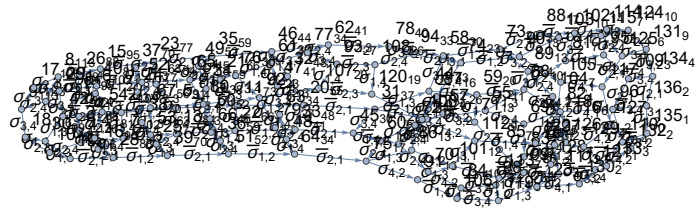
Knot [10, 3] → BR [6, { -1, -1, -2, 1, -2, -3, 2, 4, -3, 4, 5, -4, 5 }]



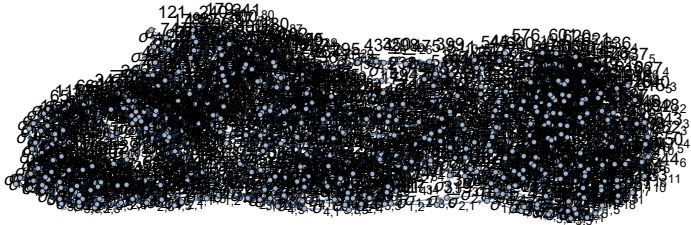
Knot [10, 5] → BR [3, { 1, 1, 1, 1, 1, 1, -2, 1, -2, -2 }]



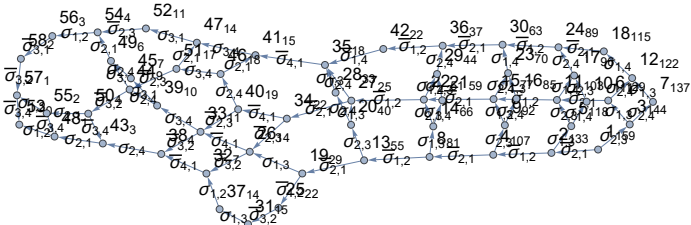
Knot [10, 6] → BR [4, { -1, -1, -1, -1, -1, -1, -2, 1, 3, -2, 3 }]



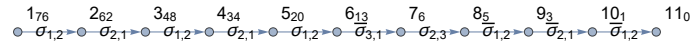
Knot [10, 7] → BR [5, {-1, -1, -2, 1, -2, -3, 2, -3, -3, 4, -3, 4}]



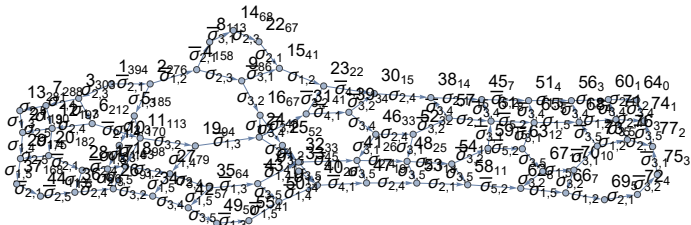
Knot [10, 8] → BR [4, {-1, -1, -1, -1, -1, 2, -1, 2, 3, -2, 3}]



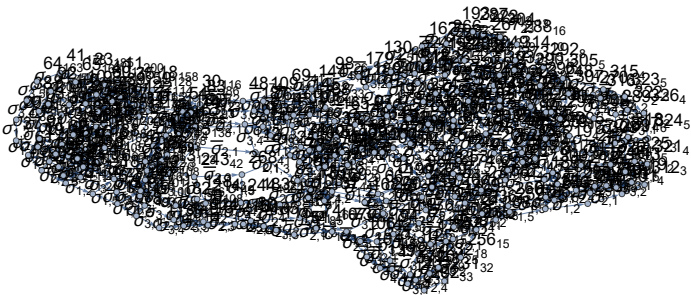
Knot [10, 9] → BR [3, {1, 1, 1, 1, 1, -2, 1, -2, -2, -2}]



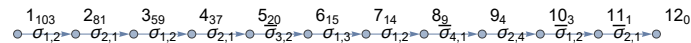
Knot [10, 10] → BR [5, {-1, -1, 2, -1, 2, 2, 3, -2, 3, 4, -3, 4}]



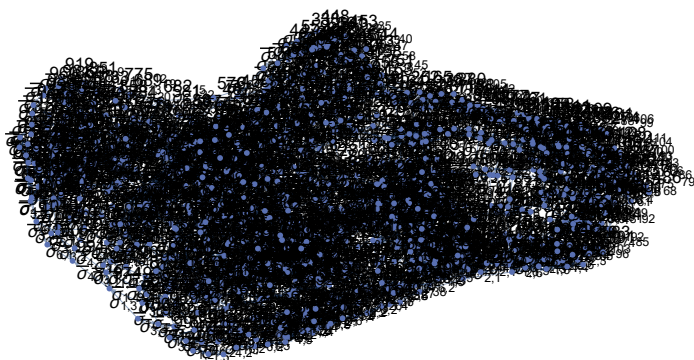
Knot [10, 11] → BR [5, {-1, -1, -1, -1, -2, 1, 3, -2, 3, 4, -3, 4}]



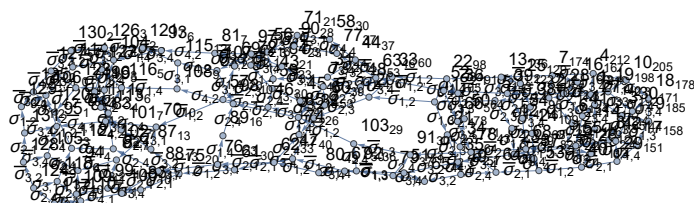
Knot [10, 12] → BR [4, {1, 1, 1, 1, 1, 2, -1, -3, 2, -3, -3}]



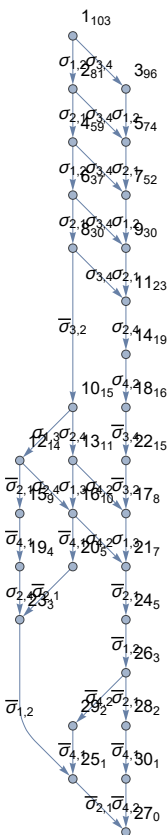
Knot [10, 13] → BR [6, {-1, -1, -2, 1, 3, -2, -4, 3, 5, -4, 5}]



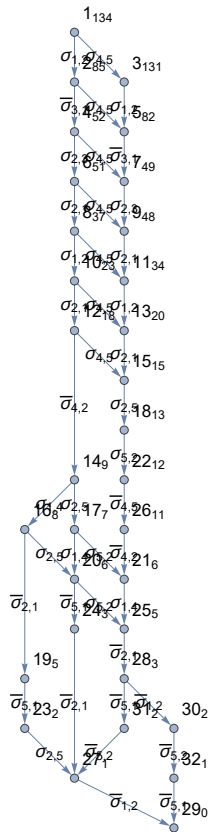
Knot [10, 14] → BR [4, {-1, -1, -1, -1, -1, -2, 1, -2, 3, -2, 3}]



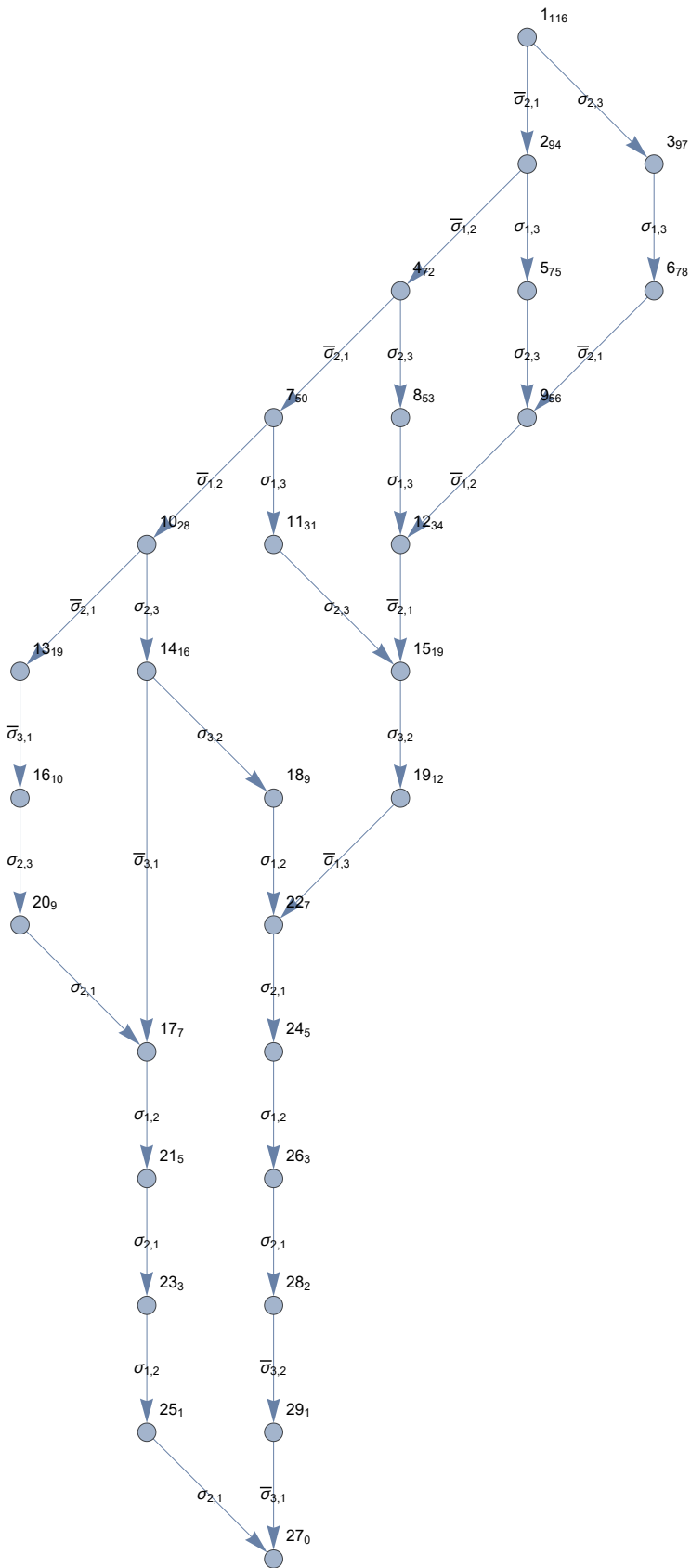
Knot [10, 15] → BR [4, {1, 1, 1, 1, -2, 1, -2, -3, 2, -3, -3}]



Knot [10, 16] → BR [5, {1, 1, 2, -1, 2, 2, -3, 2, -3, -4, 3, -4}]



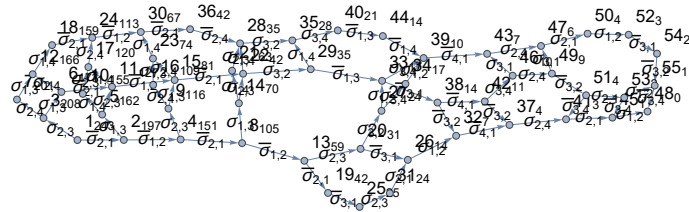
Knot [10, 17] → BR [3, {-1, -1, -1, -1, 2, -1, 2, 2, 2, 2}]



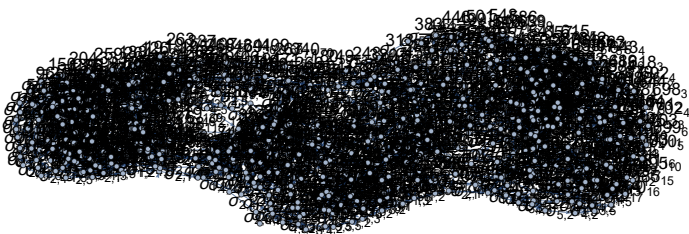
Knot [10, 18] → BR [5, {-1, -1, -1, -2, 1, -2, 3, -2, 3, 4, -3, 4}]



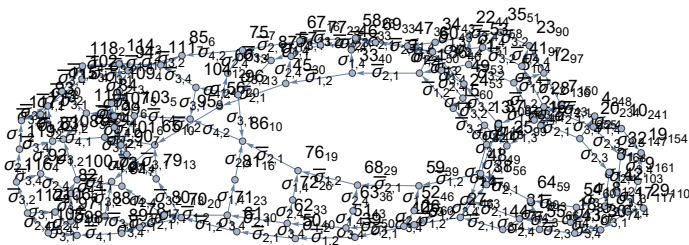
Knot [10, 19] → BR [4, {-1, -1, -1, -1, 2, -1, 2, 2, 3, -2, 3}]



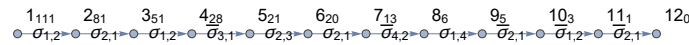
Knot [10, 20] → BR [5, {-1, -1, -1, -1, -2, 1, -2, -3, 2, 4, -3, 4}]



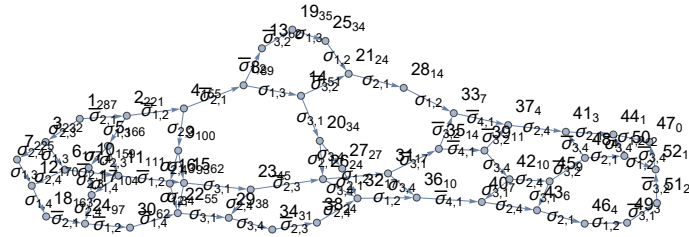
Knot [10, 21] → BR [4, {-1, -1, -2, 1, -2, -2, -2, -2, 3, -2, 3}]



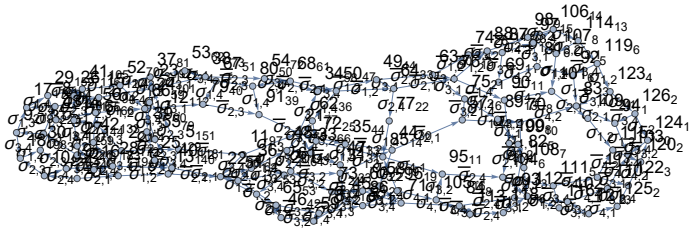
Knot [10, 22] → BR [4, {1, 1, 1, 1, 2, -1, -3, 2, -3, -3, -3}]



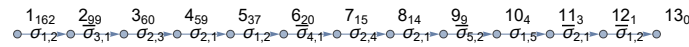
Knot [10, 23] → BR [4, {-1, -1, 2, -1, 2, 2, 2, 2, 3, -2, 3}]



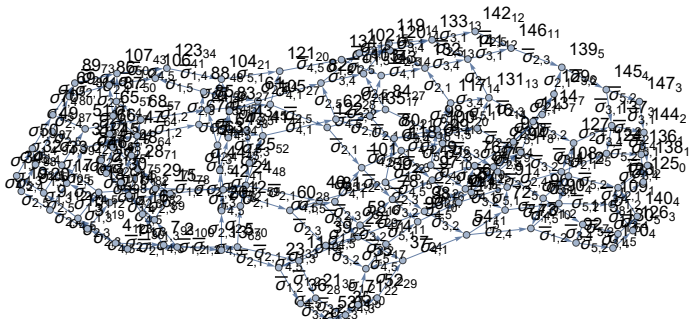
Knot [10, 27] → BR [4, {-1, -1, -1, -1, -2, 1, -2, 3, -2, 3, 3}]



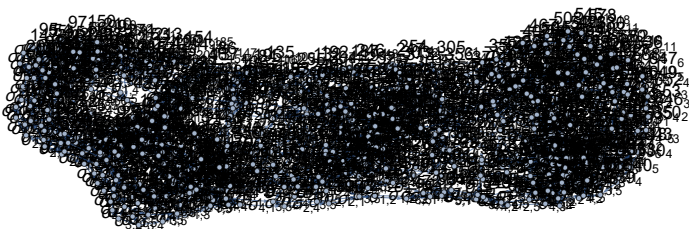
Knot [10, 28] → BR [5, {1, 1, 2, -1, 2, 2, 3, -2, -4, 3, -4, -4}]



Knot [10, 29] → BR [5, {-1, -1, -1, 2, -1, -3, 2, 4, -3, 4}]



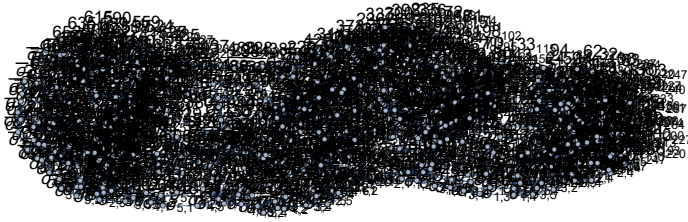
Knot [10, 30] → BR [5, {-1, -1, -2, 1, -2, -2, -3, 2, -3, 4, -3, 4}]



Knot [10, 31] → BR [5, {-1, -1, -1, -2, 1, 3, -2, 3, 3, 4, -3, 4}]



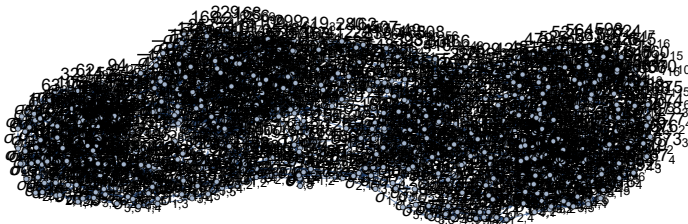
Knot [10, 32] → BR [4, {1, 1, 1, -2, 1, -2, -2, -3, 2, -3, -3}]



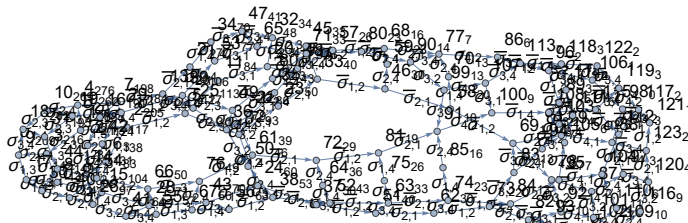
Knot [10, 37] → BR [5, {-1, -1, -1, -2, 1, 3, -2, 3, 4, -3, 4, 4}]



Knot [10, 38] → BR [5, {-1, -1, -1, -2, 1, -2, -2, -3, 2, 4, -3, 4}]



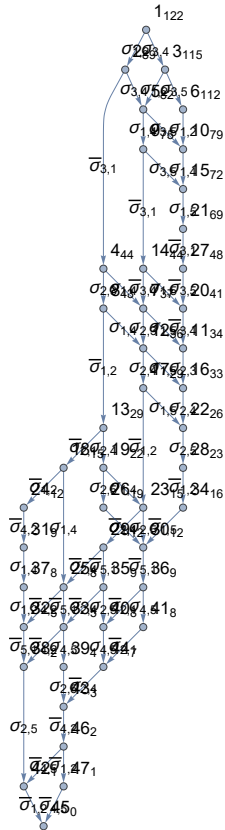
Knot [10, 39] → BR [4, {-1, -1, -1, -2, 1, -2, -2, -2, 3, -2, 3}]



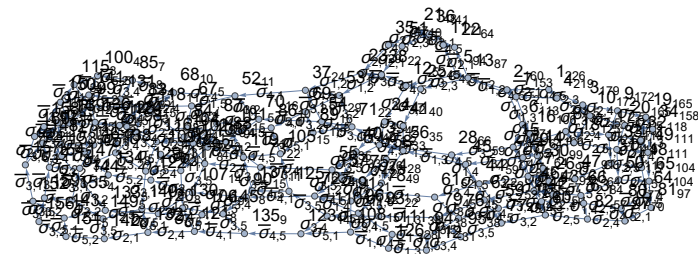
Knot [10, 40] → BR [4, {1, 1, 1, 2, -1, 2, 2, -3, 2, -3, -3}]

1₁₅₉ 2₉₇ 3₆₀ 4₃₅ 5₃₄ 6₂₄ 7₁₄ 8₉ 9₄ 10₃ 11₁ 12₀
 ○-σ_{1,2}-○σ_{2,1}-○σ_{3,2}-○σ_{1,3}-○σ_{1,2}-○σ_{2,1}-○σ_{1,2}-○σ_{4,1}-○σ_{2,4}-○σ_{1,2}-○σ_{2,1}-○

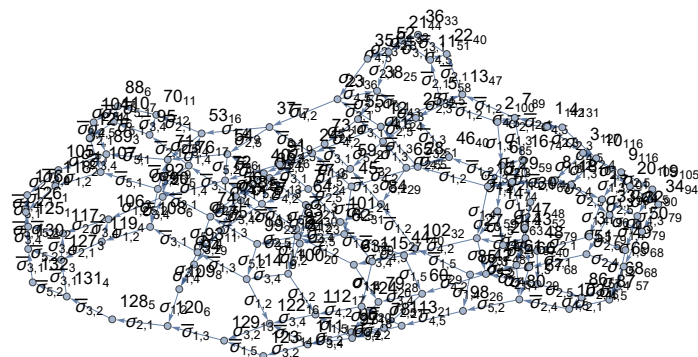
Knot [10, 41] → BR [5, {1, -2, 1, -2, -2, 3, -2, -4, 3, -4}]



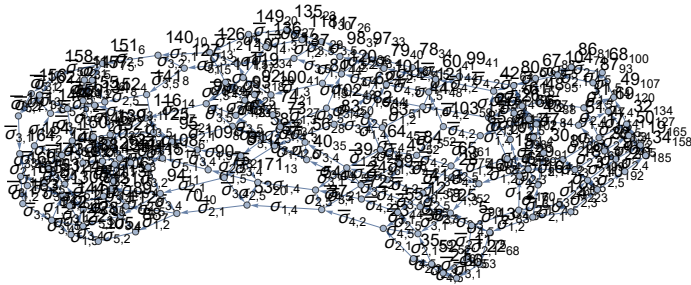
Knot [10, 42] → BR [5, {-1, -1, 2, -1, 2, -3, 2, 4, -3, 4}]



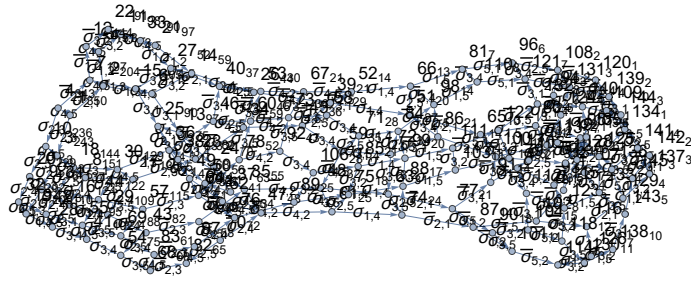
Knot [10, 43] → BR [5, {-1, -1, 2, -1, -3, 2, 4, -3, 4, 4}]



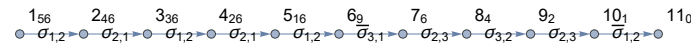
Knot [10, 44] → BR [5, {-1, -1, 2, -1, -3, 2, -3, 4, -3, 4}]



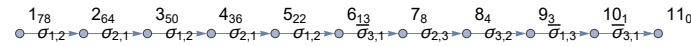
Knot [10, 45] → BR [5, {-1, 2, -1, 2, -3, 2, -3, 4, -3, 4}]



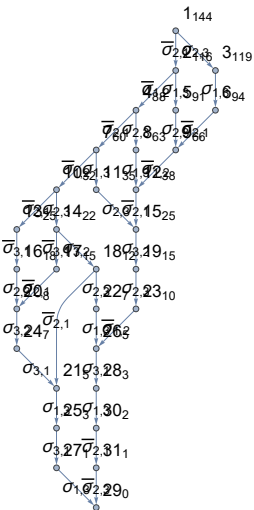
Knot [10, 46] → BR [3, {1, 1, 1, 1, 1, -2, 1, 1, 1, -2}]



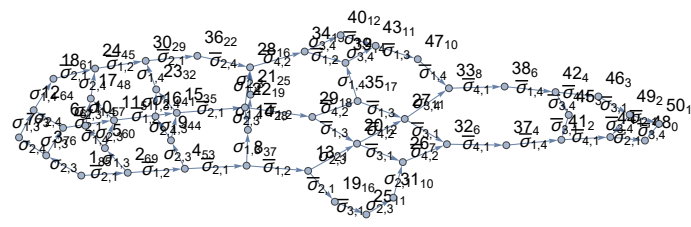
Knot [10, 47] → BR [3, {1, 1, 1, 1, 1, -2, 1, 1, -2, -2}]



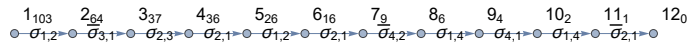
Knot [10, 48] → BR [3, {-1, -1, -1, -1, 2, 2, -1, 2, 2, 2}]



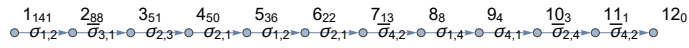
Knot [10, 49] → BR [4, {-1, -1, -1, -1, 2, -1, -3, -2, -2, -3}]



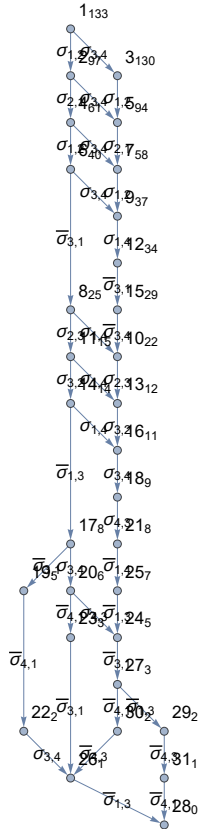
Knot [10, 50] → BR [4, {1, 1, 2, -1, 2, 2, -3, 2, 2, 2, -3}]



Knot [10, 51] → BR [4, {1, 1, 2, -1, 2, 2, -3, 2, 2, -3, -3}]



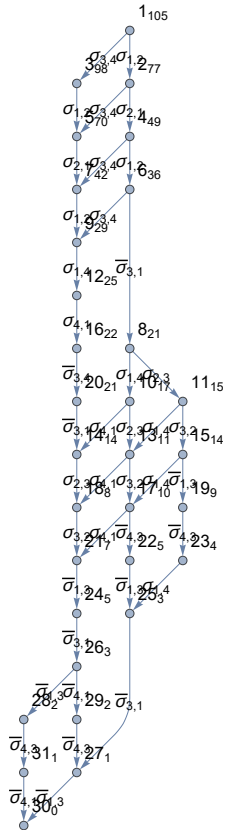
Knot [10, 52] → BR [4, {1, 1, 1, -2, 1, 1, -2, -2, -3, 2, -3}]



Knot [10, 53] → BR [5, {-1, -1, -2, 1, -2, 3, -2, -4, -3, -3, -4}]



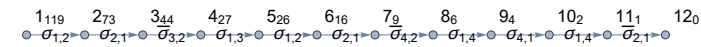
Knot [10, 54] → BR [4, {1, 1, 1, -2, 1, 1, -2, -3, 2, -3, -3}]



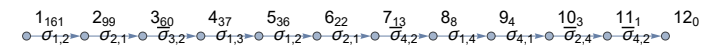
Knot [10, 55] → BR [5, {-1, -1, -1, -2, 1, 3, -2, -4, -3, -3, -3, -4}]



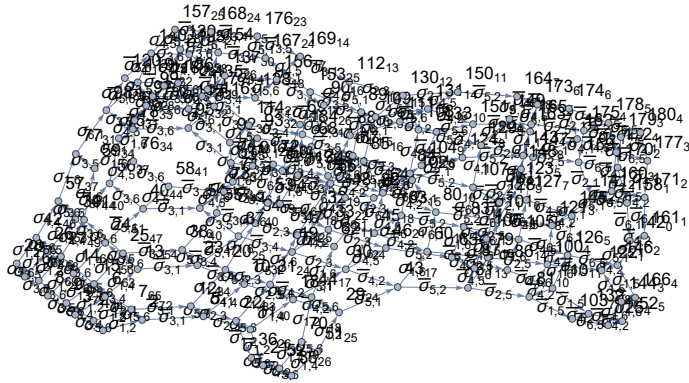
Knot [10, 56] → BR [4, {1, 1, 1, 2, -1, 2, -3, 2, 2, 2, -3}]



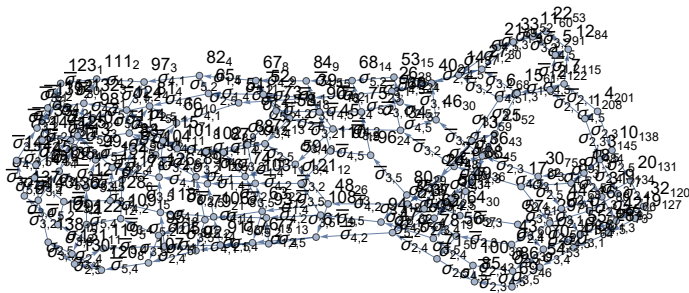
Knot [10, 57] → BR [4, {1, 1, 1, 2, -1, 2, -3, 2, 2, -3, -3}]



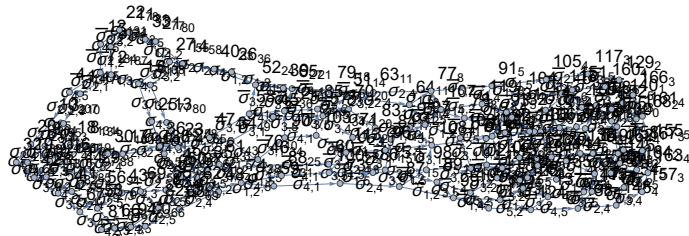
Knot [10, 58] → BR [6, {1, -2, 1, 3, -2, -4, -3, -3, 5, -4, 5}]



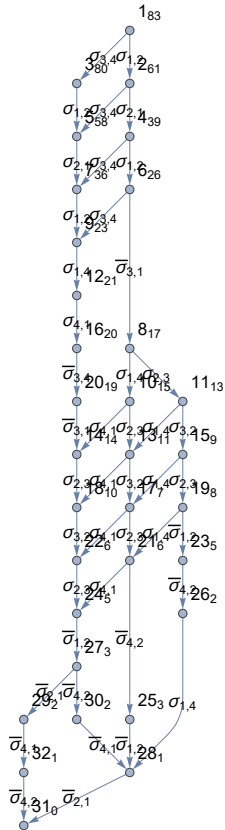
Knot[10, 59] → BR[5, {-1, 2, -1, 2, -3, 2, 2, 4, -3, 4}]



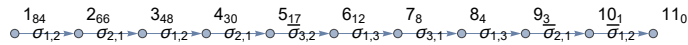
Knot[10, 60] → BR[5, {-1, 2, -1, 2, 2, -3, 2, -3, -2, -4, 3, -4}]



Knot[10, 61] → BR[4, {1, 1, 1, -2, 1, 1, 1, -2, -3, 2, -3}]



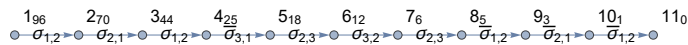
Knot [10, 62] → BR [3, {1, 1, 1, 1, -2, 1, 1, 1, -2, -2}]



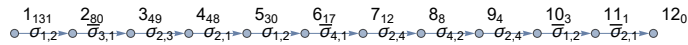
Knot [10, 63] → BR [5, {-1, -1, 2, -1, -3, -2, -2, -2, -3, -4, 3, -4}]



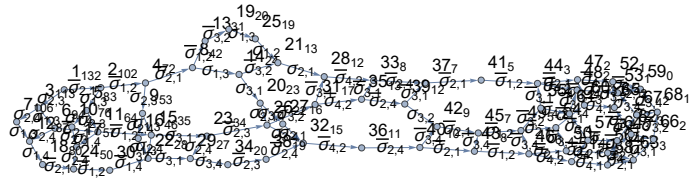
Knot [10, 64] → BR [3, {1, 1, 1, -2, 1, 1, 1, -2, -2, -2}]



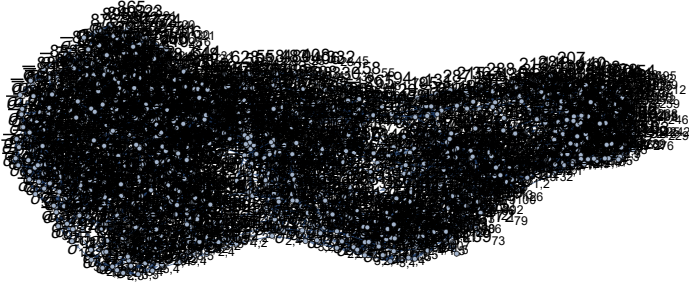
Knot [10, 65] → BR [4, {1, 1, 2, -1, 2, -3, 2, 2, 2, -3, -3}]



Knot [10, 66] → BR [4, {-1, -1, -1, 2, -1, -3, -2, -2, -2, -3, -3}]



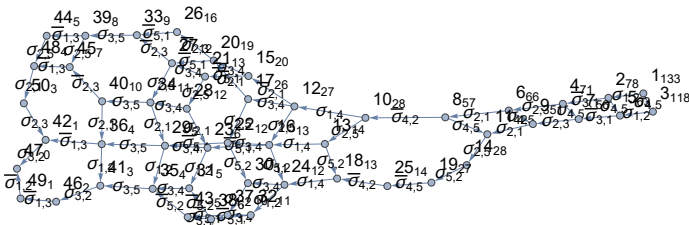
Knot[10, 67] → BR[5, {-1, -1, -1, -2, 1, -2, -3, 2, 2, 4, -3, -2, 4, -3}]



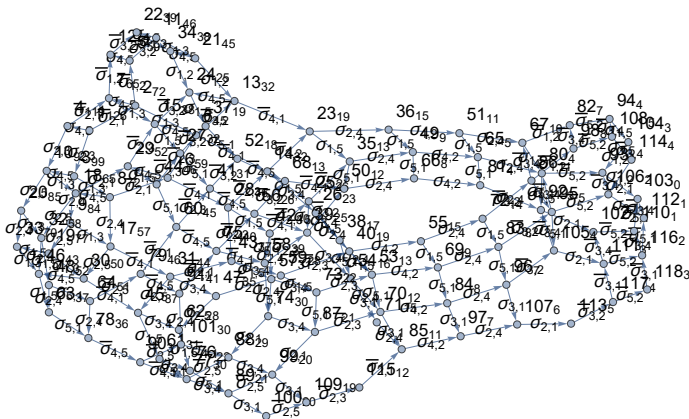
Knot[10, 68] → BR[5, {1, 1, -2, 1, -2, -2, -3, 2, 2, -4, 3, -2, -4, -3}]



Knot[10, 69] → BR[5, {1, 1, 2, -1, -3, 2, 1, 4, -3, 2, -3, 4}]

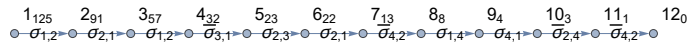


Knot[10, 70] → BR[5, {-1, 2, -1, -3, 2, 2, 2, 4, -3, 4}]



Knot[10, 71] → BR[5, {-1, -1, 2, -1, -3, 2, 2, 4, -3, 4}]

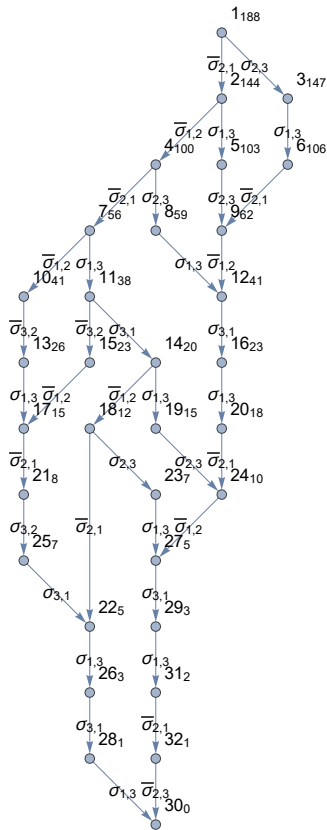
Knot [10, 77] → BR [4, {1, 1, 1, 1, 2, -1, -3, 2, 2, -3, -3}]



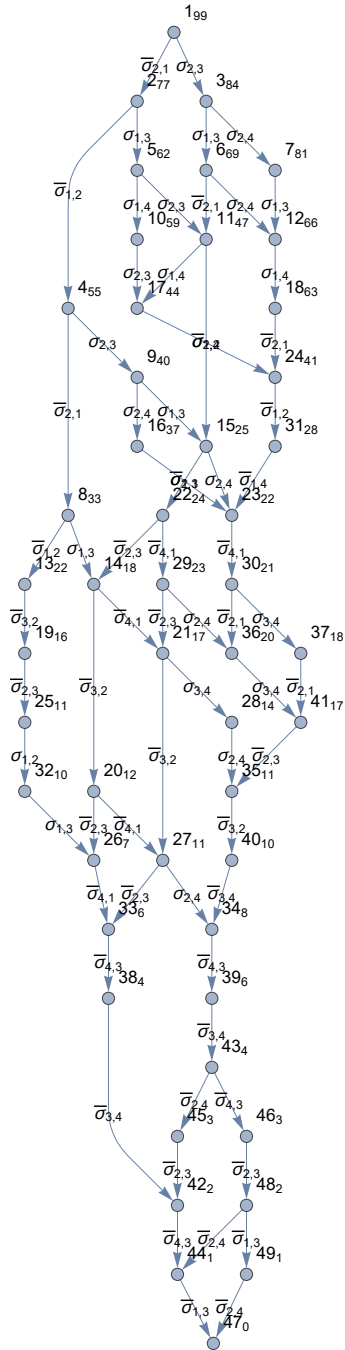
Knot [10, 78] → BR [5, {-1, -1, -2, 1, -2, -1, 3, -2, -4, 3, -4, -4}]



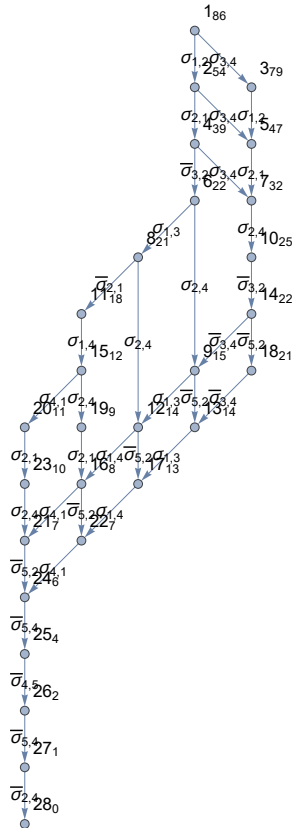
Knot [10, 79] → BR [3, {-1, -1, -1, 2, 2, -1, -1, 2, 2, 2}]



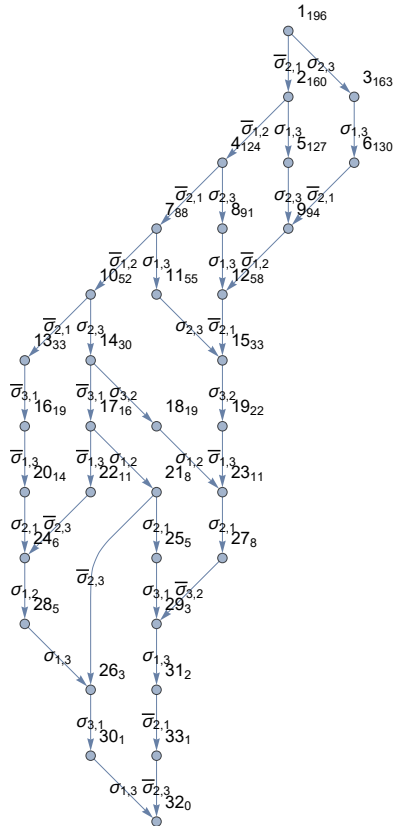
Knot [10, 80] → BR [4, {-1, -1, -1, 2, -1, -1, -3, -2, -2, -2, -3}]



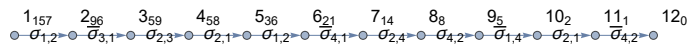
Knot [10, 81] → BR [5, {1, 1, -2, 1, 3, 2, 2, -4, -3, -3, -3, -4}]



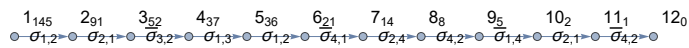
Knot [10, 82] → BR [3, {-1, -1, -1, -1, 2, -1, 2, -1, 2, 2}]



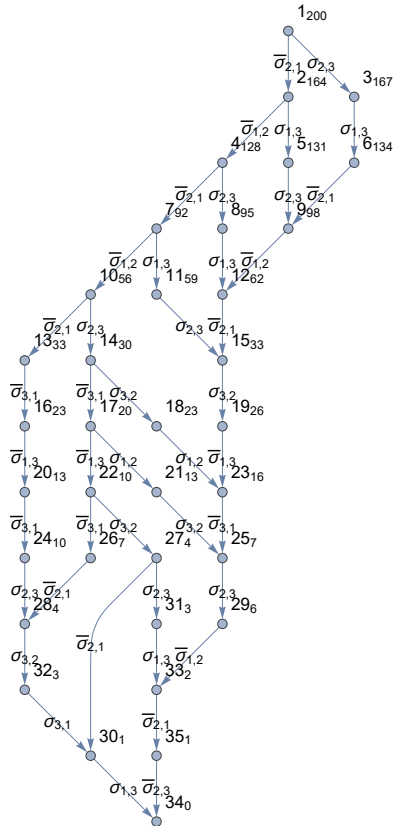
Knot [10, 83] → BR [4, {1, 1, 2, -1, 2, -3, 2, 2, -3, 2, -3}]



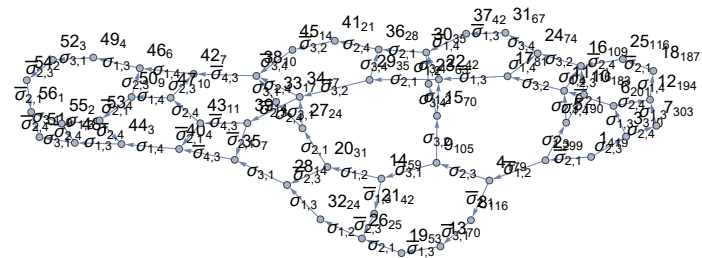
Knot [10, 84] → BR [4, {1, 1, 1, 2, -1, -3, 2, 2, -3, 2, -3}]



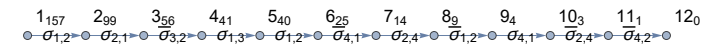
Knot [10, 85] → BR [3, {-1, -1, -1, -1, 2, -1, -1, 2, -1, 2}]



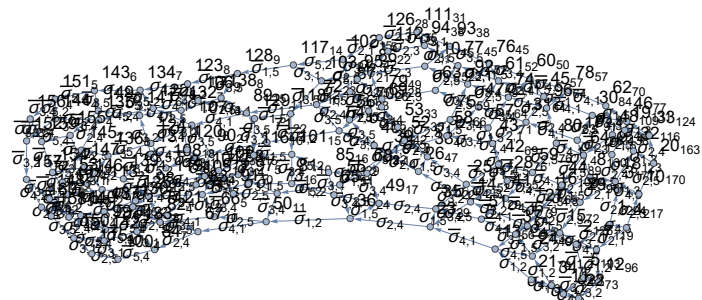
Knot [10, 86] → BR [4, {-1, -1, 2, -1, 2, -1, 2, 2, 3, -2, 3}]



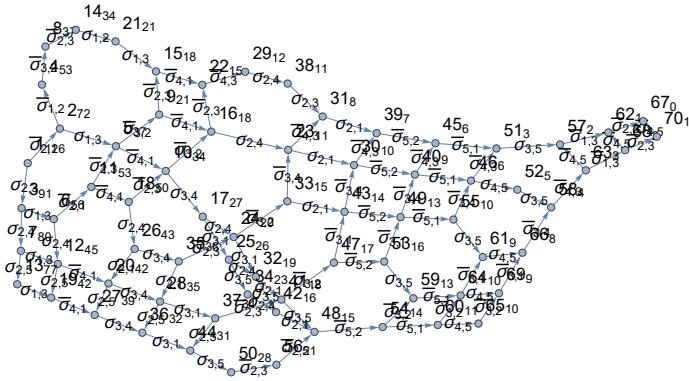
Knot [10, 87] → BR [4, {1, 1, 1, 2, -1, -3, 2, -3, 2, -3, -3}]



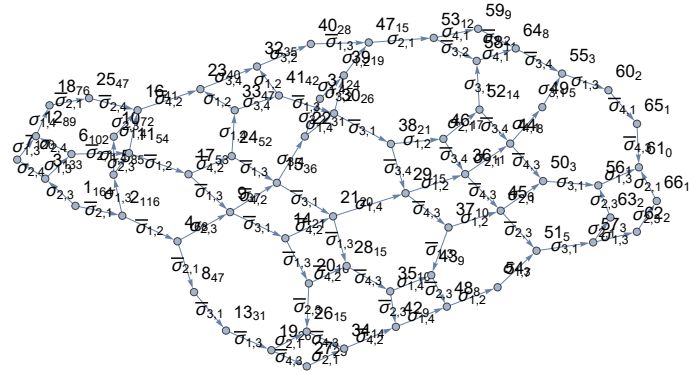
Knot [10, 88] → BR [5, {-1, 2, -1, -3, 2, -3, 2, 4, -3, 4}]



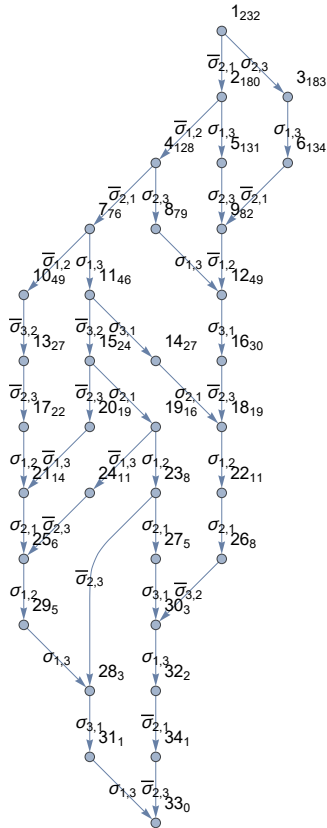
Knot [10, 89] → BR [5, {-1, 2, -1, 2, 3, -2, -1, -4, -3, 2, -3, -4}]



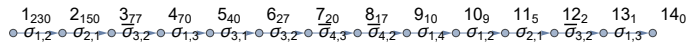
Knot [10, 90] → BR [4, {-1, -1, 2, -1, 2, 3, -2, -1, 3, 2, 2}]



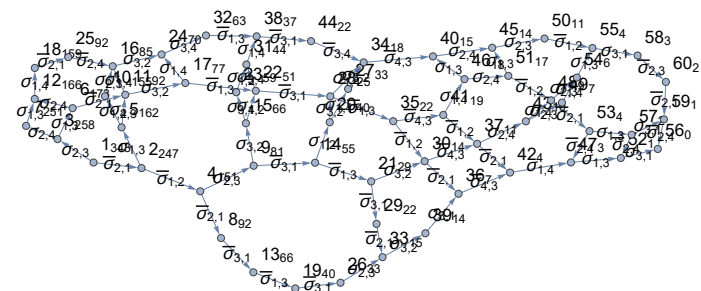
Knot [10, 91] → BR [3, {-1, -1, -1, 2, -1, 2, 2, -1, 2, 2}]



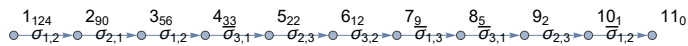
Knot [10, 92] → BR [4, {1, 1, 1, 2, 2, -3, 2, -1, 2, -3, 2}]



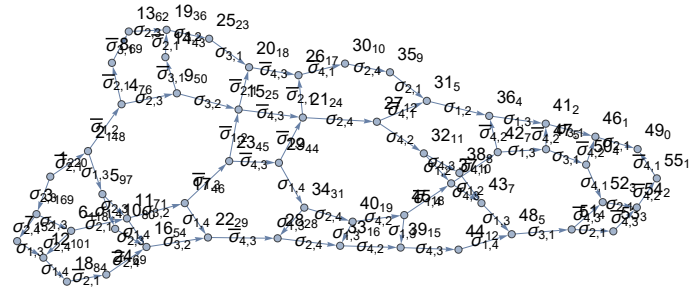
Knot [10, 93] → BR [4, {-1, -1, 2, -1, -1, 2, -1, 2, 3, -2, 3}]



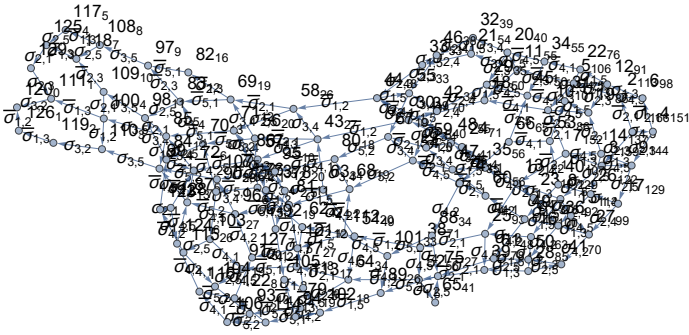
Knot [10, 94] → BR [3, {1, 1, 1, -2, 1, 1, -2, -2, 1, -2}]



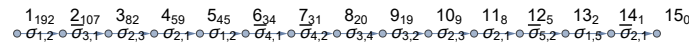
Knot [10, 95] → BR [4, {-1, -1, 2, 2, -3, 2, -1, 2, 3, 3, 2}]



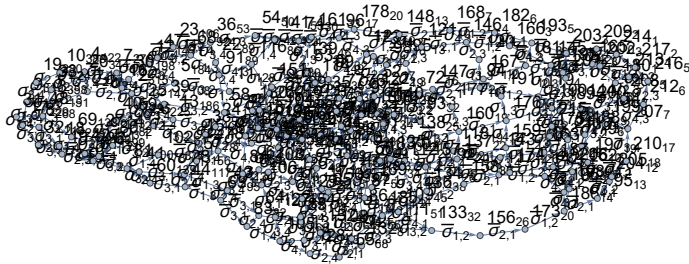
Knot [10, 96] → BR [5, {-1, 2, 1, -3, 2, 1, -3, 4, -3, 2, -3, 4}]



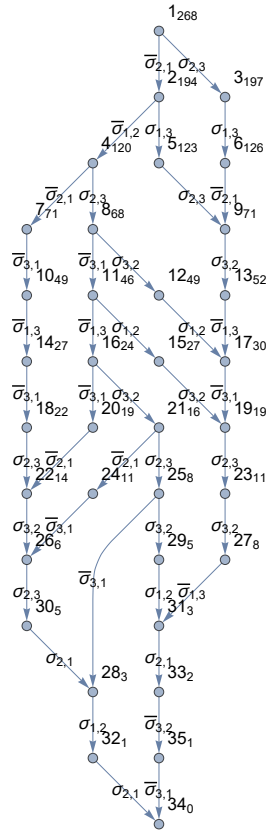
Knot [10, 97] → BR [5, {1, 1, 2, -1, 2, 1, -3, 2, -1, 2, 3, -4, 3, -4}]



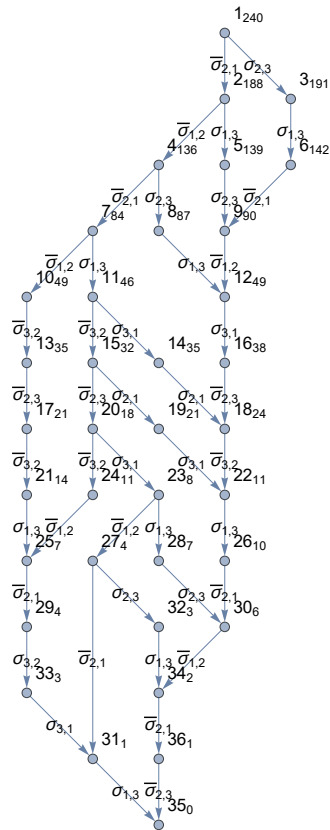
Knot [10, 98] → BR [4, {-1, -1, -2, -2, 3, -2, 1, -2, -2, 3, -2}]



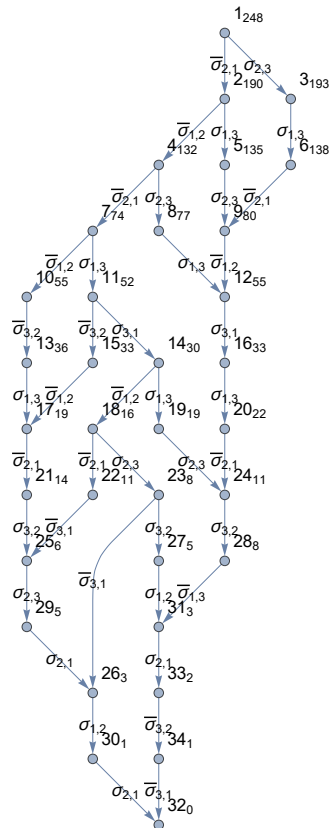
Knot [10, 99] → BR [3, {-1, -1, 2, -1, -1, 2, 2, -1, 2, 2}]



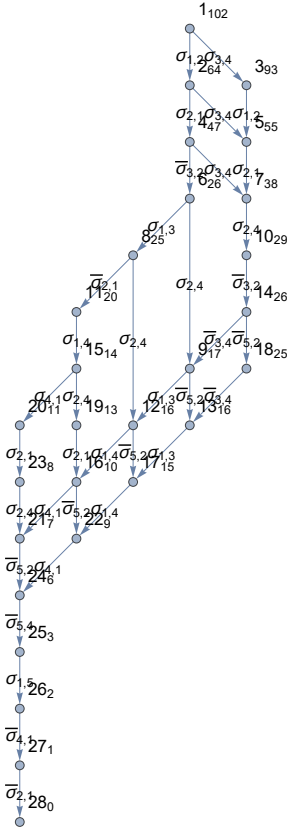
Knot [10, 100] → BR [3, {-1, -1, -1, 2, -1, -1, 2, -1, -1, 2}]



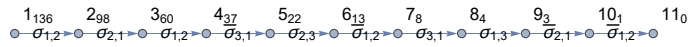
Knot [10, 101] → BR [5, {1, 1, 1, 2, -1, 3, -2, 1, 3, 2, 2, 4, -3, 4}]



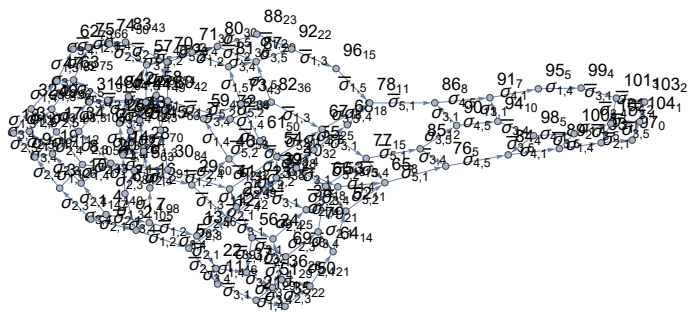
Knot [10, 105] → BR [5, {1, 1, -2, 1, 3, 2, 2, -4, -3, 2, -3, -4}]



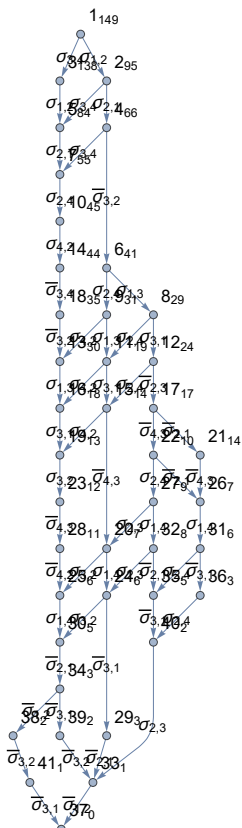
Knot [10, 106] → BR [3, {1, 1, 1, -2, 1, -2, 1, 1, -2, -2}]



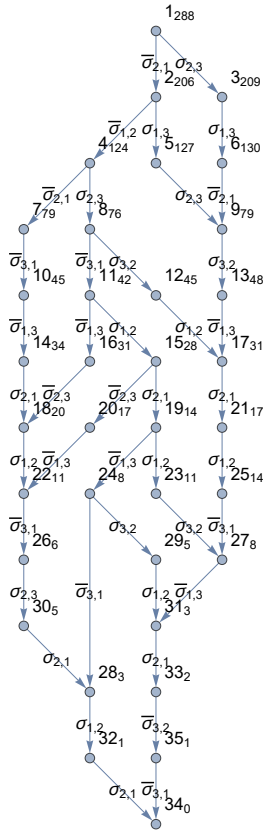
Knot [10, 107] → BR [5, {-1, -1, 2, -1, 3, 2, 2, -4, 3, -2, 3, -4}]



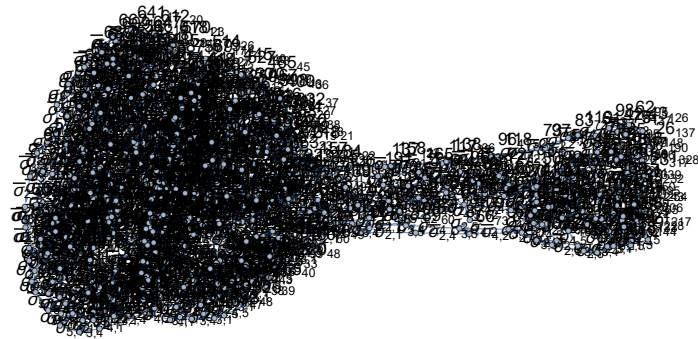
Knot [10, 108] → BR [4, {1, 1, -2, 1, 1, 3, -2, 1, -2, -3, -3}]



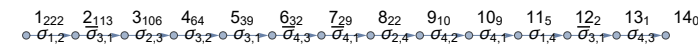
Knot [10, 109] → BR [3, {-1, -1, 2, -1, 2, 2, -1, -1, 2, 2}]



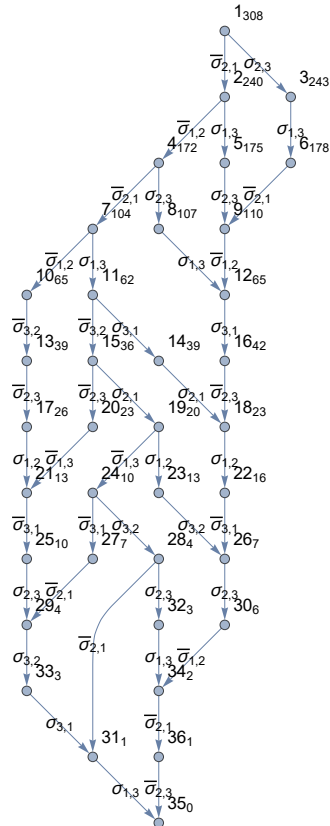
Knot [10, 110] → BR [5, {-1, 2, -1, -3, -2, -2, -2, 4, 3, -2, 3, 4}]



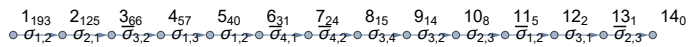
Knot [10, 111] → BR [4, {1, 1, 2, 2, -3, 2, 2, -1, 2, -3, 2}]



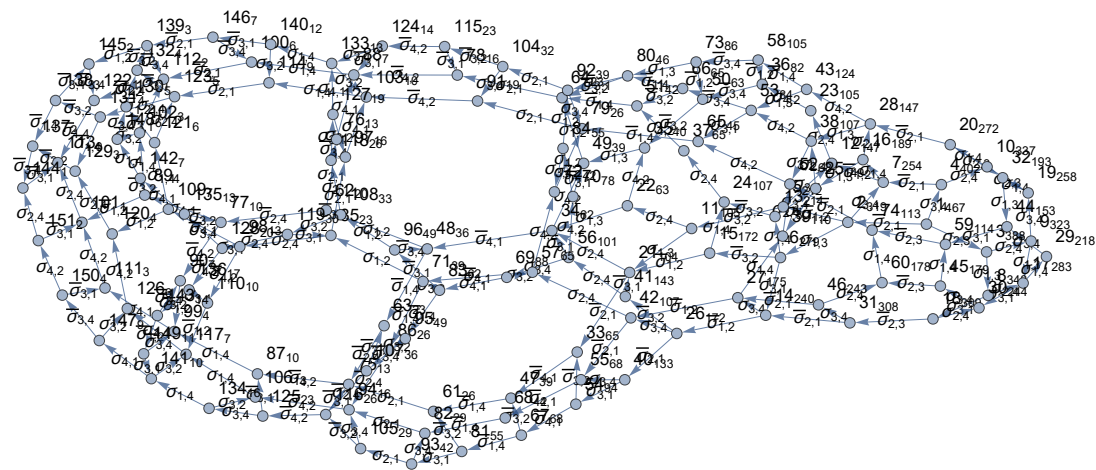
Knot [10, 112] → BR [3, {-1, -1, -1, 2, -1, 2, -1, 2, -1, 2}]



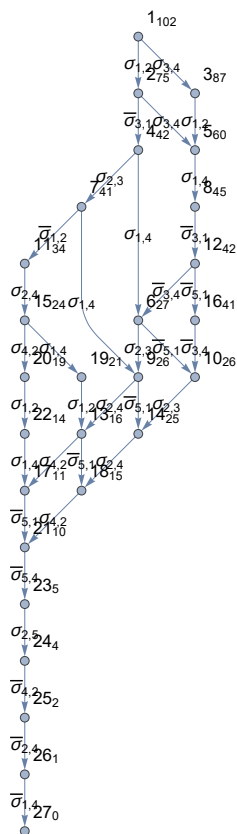
Knot [10, 113] → BR [4, {1, 1, 1, 2, -3, 2, -1, 2, -3, 2, -3}]



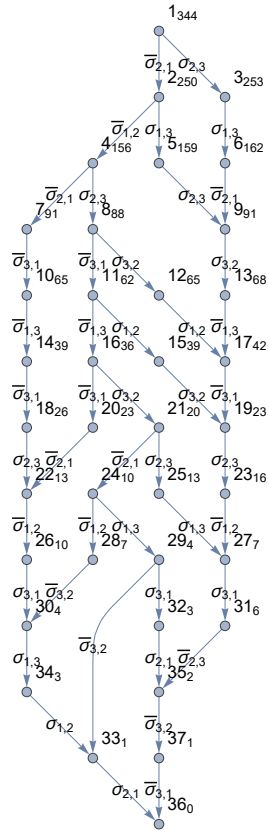
Knot [10, 114] → BR [4, {-1, -1, -2, 1, 3, -2, 3, -2, 3, -2, 3}]



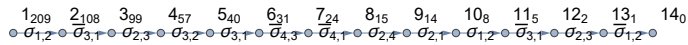
Knot [10, 115] → BR [5, {1, -2, 1, 3, 2, 2, -4, -3, 2, -3, -3, -4}]



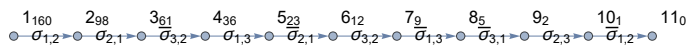
Knot [10, 116] → BR [3, {-1, -1, 2, -1, -1, 2, -1, 2, -1, 2}]



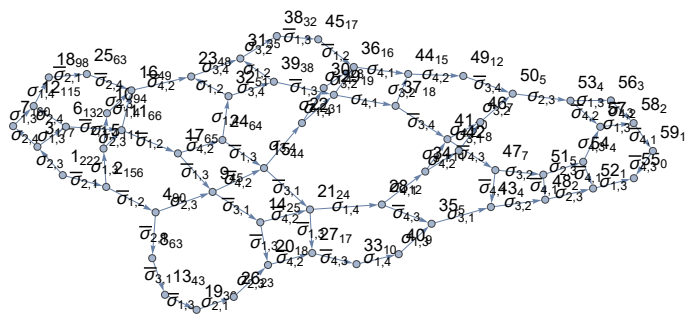
Knot [10, 117] → BR [4, {1, 1, 2, 2, -3, 2, -1, 2, -3, 2, -3}]



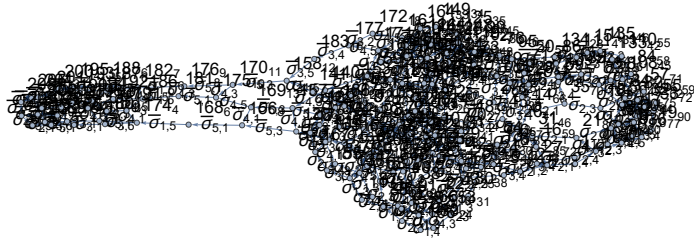
Knot [10, 118] → BR [3, {1, 1, -2, 1, -2, 1, -2, -2, 1, -2}]



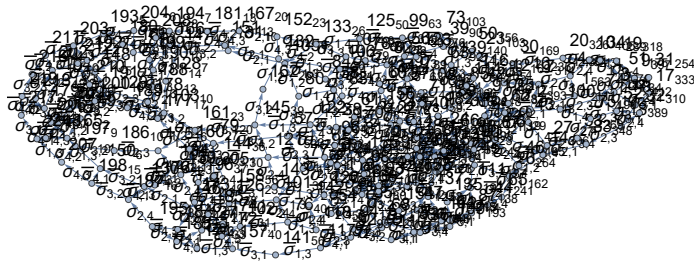
Knot [10, 119] → BR [4, {-1, -1, 2, -1, -3, 2, -1, 2, 3, 3, 2}]



Knot [10, 120] → BR [5, {-1, -1, -2, 1, 3, 2, -1, -4, -3, -2, -2, -3, -3, -4}]



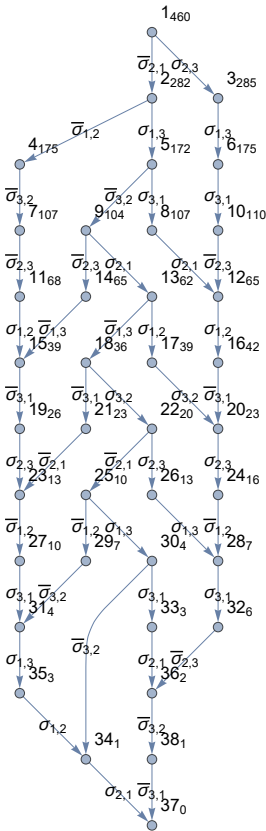
Knot [10, 121] → BR [4, {-1, -1, -2, 3, -2, 1, -2, 3, -2, 3, -2}]



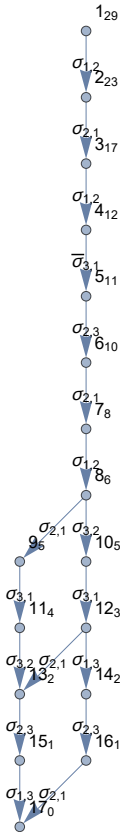
Knot [10, 122] → BR [4, {1, 1, 2, -3, 2, -1, -3, 2, -3, 2, -3}]

1185 296 387 460 551 634 725 824 915 108 115 122 131 140
 ○σ_{1,2}○σ_{3,1}○σ_{2,3}○σ_{2,1}○σ_{4,2}○σ_{4,1}○σ_{3,4}○σ_{3,1}○σ_{2,3}○σ_{1,2}○σ_{3,1}○σ_{2,3}○σ_{1,2}○

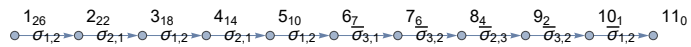
Knot [10, 123] → BR [3, {-1, 2, -1, 2, -1, 2, -1, 2, -1, 2}]



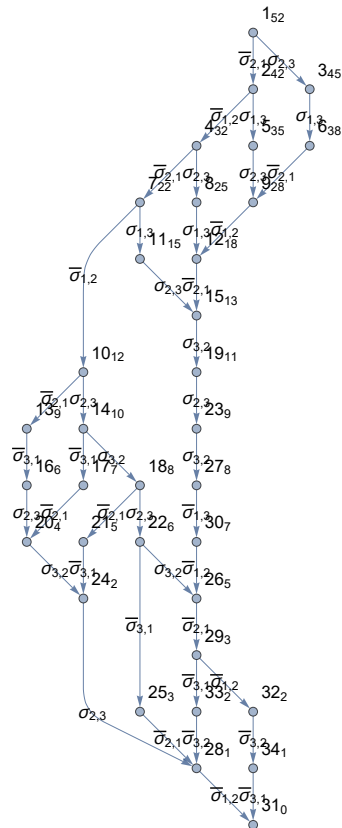
Knot [10, 124] → BR [3, {1, 1, 1, 1, 1, 2, 1, 1, 1, 2}]



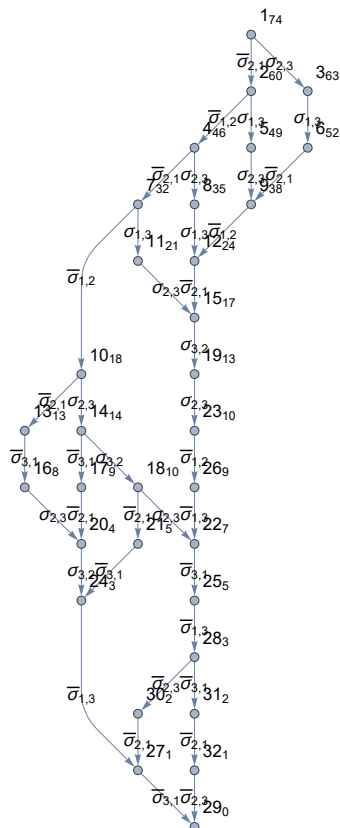
Knot [10, 125] \rightarrow BR [3, {1, 1, 1, 1, 1, -2, -1, -1, -1, -2}]



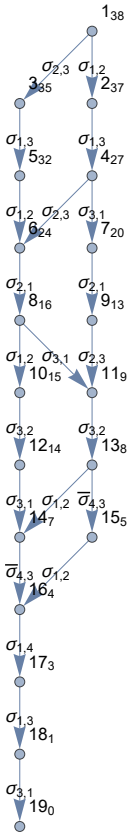
Knot [10, 126] \rightarrow BR [3, {-1, -1, -1, -1, -1, -2, 1, 1, 1, -2}]



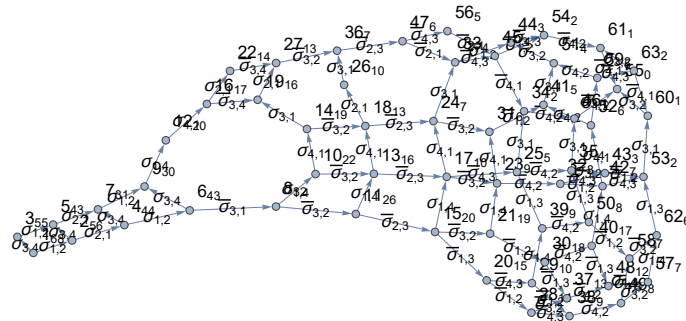
Knot [10, 127] → BR [3, {-1, -1, -1, -1, -1, -2, 1, 1, -2, -2}]



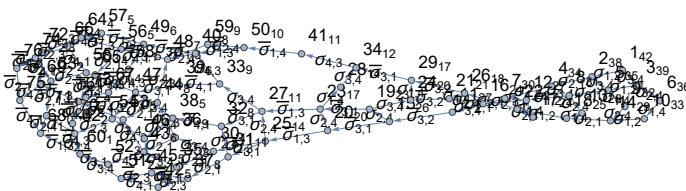
Knot [10, 128] → BR [4, {1, 1, 1, 2, 1, 1, 2, 2, 3, -2, 3}]



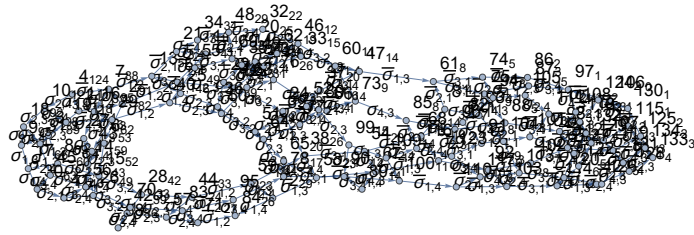
Knot [10, 129] → BR [4, {1, 1, 1, -2, -1, -1, 3, -2, -1, 3, -2}]



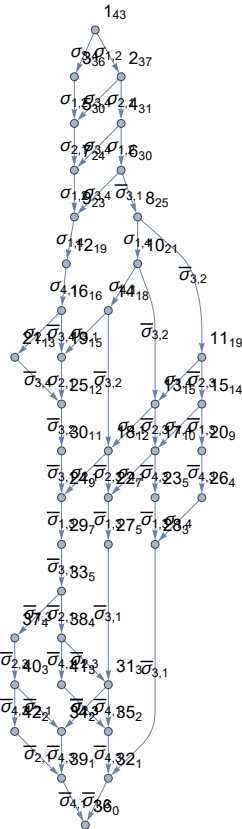
Knot [10, 130] → BR [4, {1, 1, 1, -2, -1, -1, -2, -2, -3, 2, -3}]



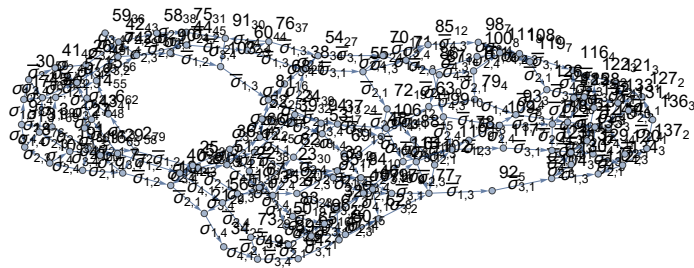
Knot [10, 131] → BR [4, {-1, -1, -1, -2, 1, 1, -2, -2, -3, 2, -3}]



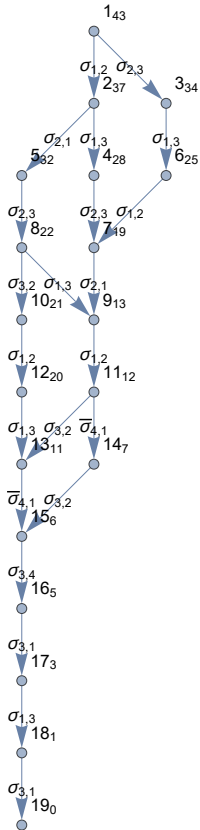
Knot [10, 132] → BR [4, {1, 1, 1, -2, -1, -1, -2, -3, 2, -3, -3}]



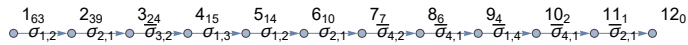
Knot [10, 133] → BR [4, {-1, -1, -1, -2, 1, 1, -2, -3, 2, -3, -3}]



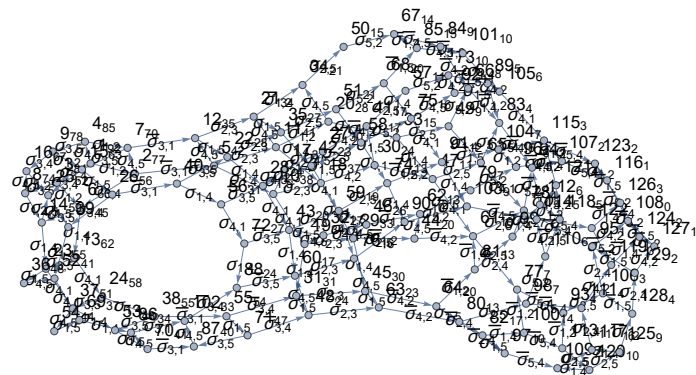
Knot [10, 134] → BR [4, {1, 1, 1, 2, 1, 1, 2, 3, -2, 3, 3}]



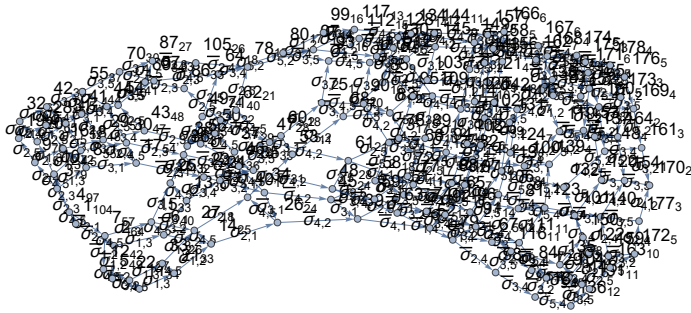
Knot [10, 135] → BR [4, {1, 1, 1, 2, -1, 2, -3, -2, -2, -2, -3}]



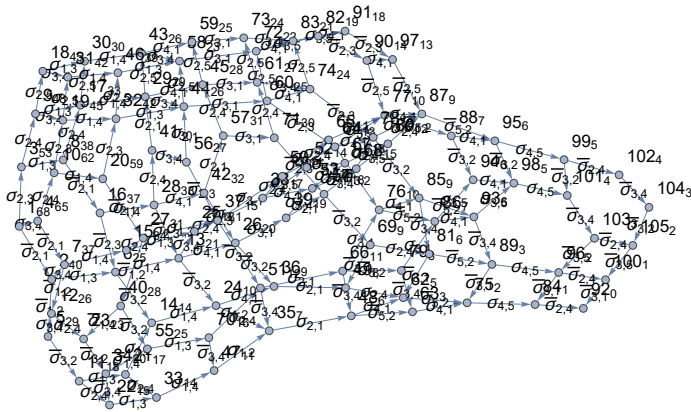
Knot [10, 136] → BR [5, {1, -2, 1, -2, -3, 2, 2, 4, -3, 4}]



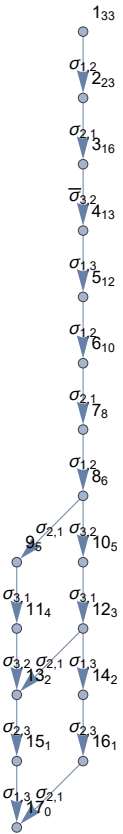
Knot [10, 137] → BR [5, {-1, 2, -1, 2, -3, -2, -2, 4, -3, 4}]



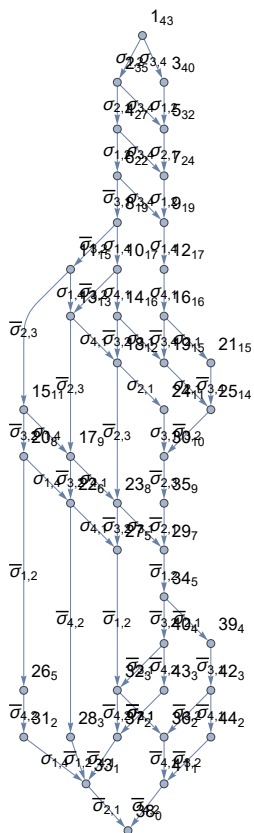
Knot [10, 138] → BR [5, {-1, 2, -1, 2, 3, 2, 2, -4, 3, -4}]



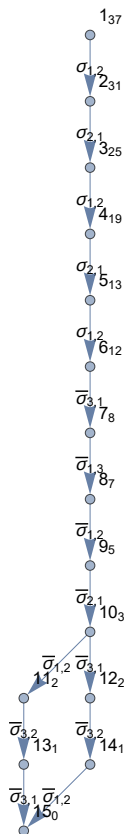
Knot [10, 139] → BR [3, {1, 1, 1, 1, 2, 1, 1, 1, 2, 2}]



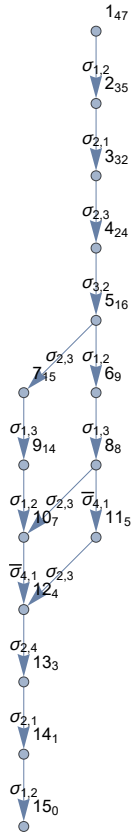
Knot [10, 140] → BR [4, {1, 1, 1, -2, -1, -1, -1, -2, -3, 2, -3}]



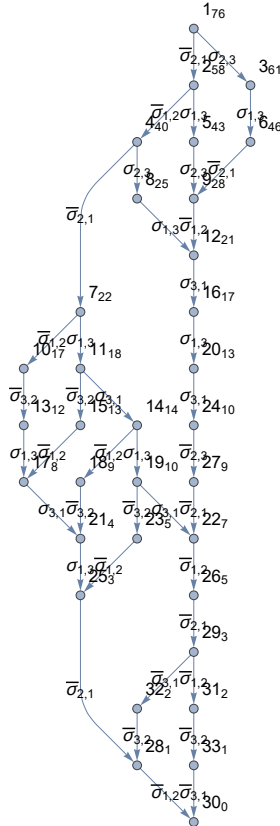
Knot [10, 141] → BR [3, {1, 1, 1, 1, -2, -1, -1, -1, -2, -2}]



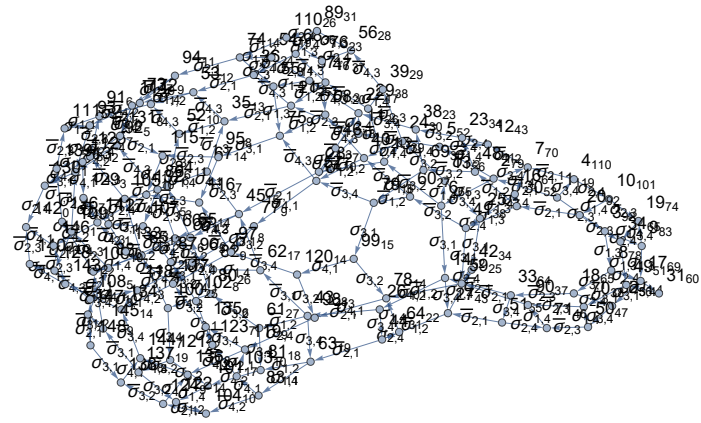
Knot [10, 142] \rightarrow BR [4, {1, 1, 1, 2, 1, 1, 1, 2, 3, -2, 3}]



Knot [10, 143] \rightarrow BR [3, {-1, -1, -1, -1, -2, 1, 1, 1, -2, -2}]



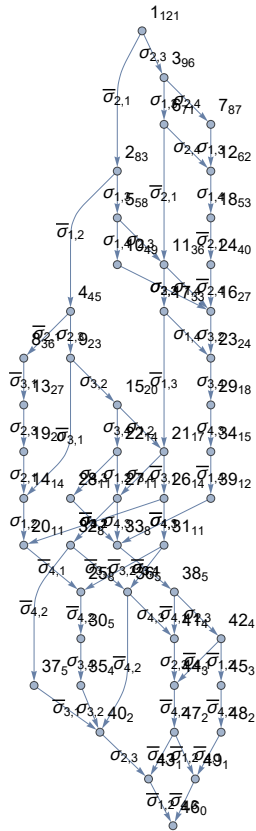
$\text{Knot}[10, 144] \rightarrow \text{BR}[4, \{-1, -1, -2, 1, -2, -1, 3, -2, -1, 3, 2\}]$



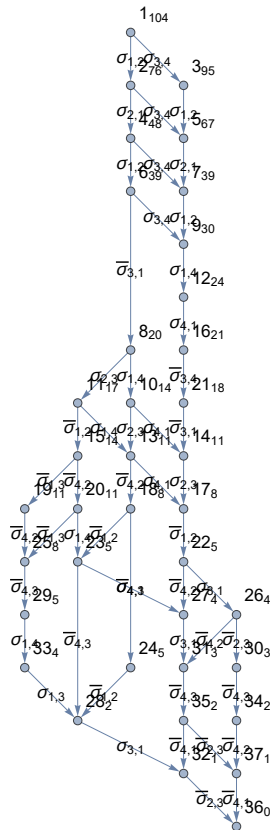
$\text{Knot}[10, 145] \rightarrow \text{BR}[4, \{-1, -1, -2, 1, -2, -1, -3, -2, 1, -2, -3\}]$



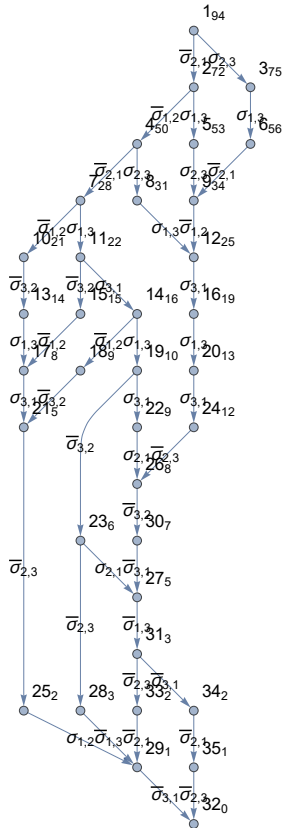
Knot [10, 146] → BR [4, {-1, -1, 2, -1, 2, 1, -3, 2, -1, 2, -3}]



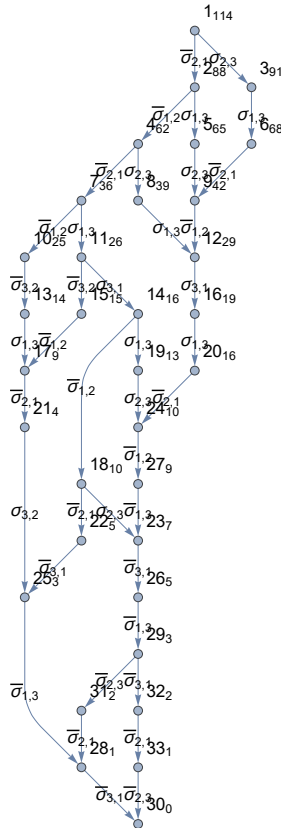
Knot [10, 147] → BR [4, {1, 1, 1, -2, 1, -2, -3, 2, -1, 2, -3}]



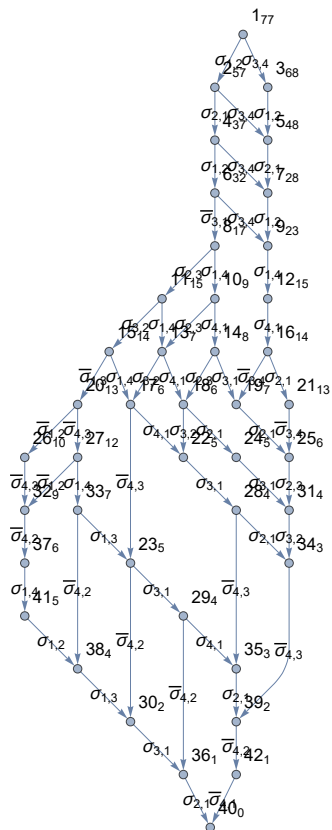
Knot [10, 148] → BR [3, {-1, -1, -1, -1, -2, 1, 1, -2, 1, -2}]



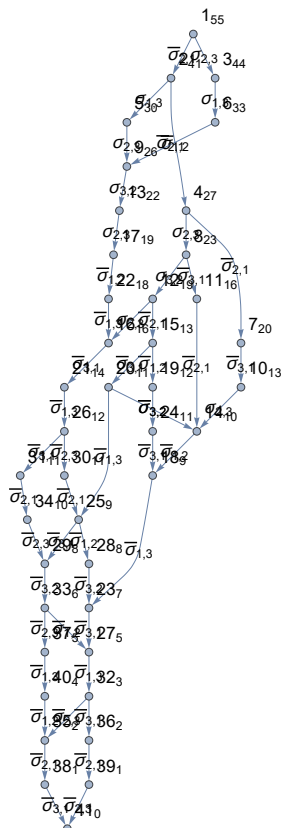
$\text{Knot}[10, 149] \rightarrow \text{BR}[3, \{-1, -1, -1, -1, -2, 1, -2, 1, -2, -2\}]$



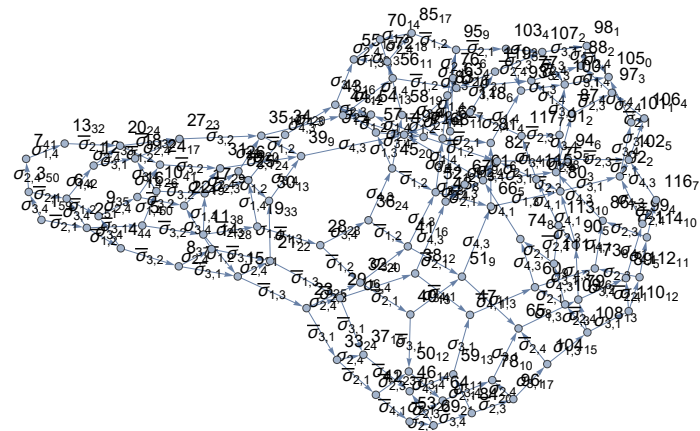
Knot [10, 150] → BR [4, {1, 1, 1, -2, 1, 1, 3, -2, -1, 3, 2}]



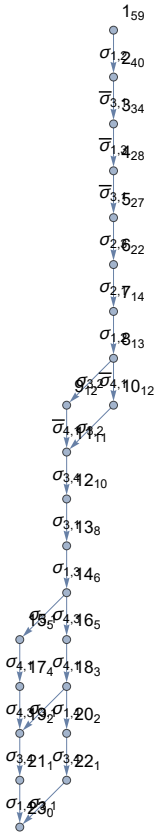
Knot [10, 151] → BR [4, {1, 1, 1, 2, -1, -1, 3, -2, 1, 3, -2}]



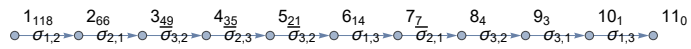
Knot [10, 153] → BR [4, { -1, -1, -1, -2, -1, -1, 3, 2, 2, 2, 3 }]



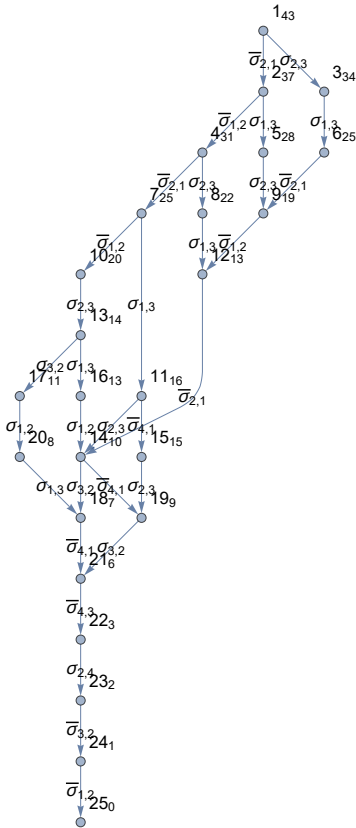
Knot [10, 154] → BR [4, { 1, 1, 2, -1, 2, 1, 3, 2, 2, 2, 3 }]



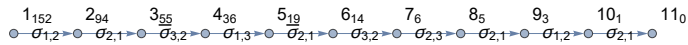
Knot [10, 155] → BR [3, {1, 1, 1, 2, -1, -1, 2, -1, -1, 2}]



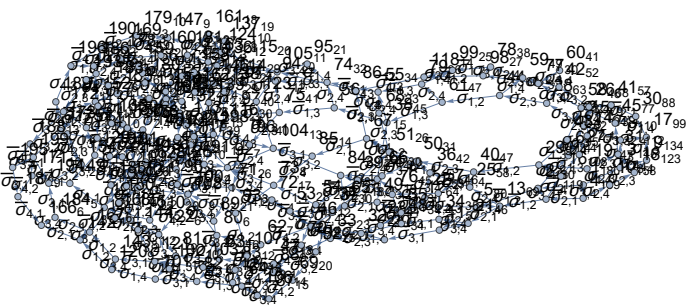
Knot [10, 156] → BR [4, {-1, -1, -1, 2, 1, 1, -3, -2, 1, -2, -3}]



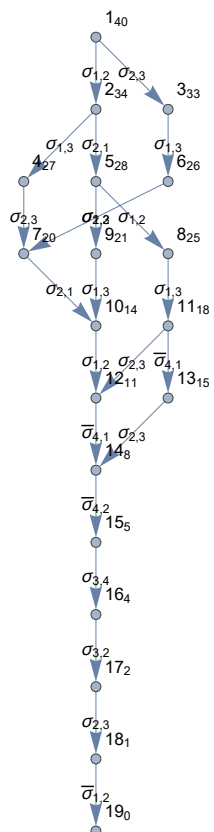
Knot [10, 157] \rightarrow BR [3, {1, 1, 1, 2, 2, -1, 2, -1, 2, 2}]



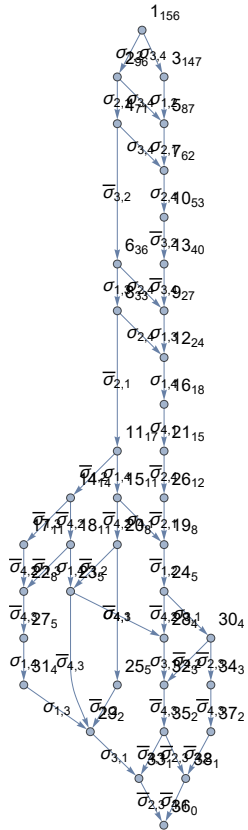
Knot [10, 158] \rightarrow BR [4, {-1, -1, -1, -2, 1, 1, 3, 2, -1, 2, 3}]



Knot [10, 159] \rightarrow BR [3, {-1, -1, -1, -2, 1, -2, 1, 1, -2, -2}]



Knot [10, 161] \rightarrow BR [3, {-1, -1, -1, -2, 1, -2, -1, -1, -2, -2}]



Knot [10, 165] → BR [4, {1, 1, 2, -1, -3, 2, -1, 2, 3, 3, 2}]

