

Pensieve header: OU matters around the Diamond Lemma.

Old program, from Gamma.nb.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OU"];
```

pdf

```
In[*]:= SetAttributes[VD, Orderless]
```

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```
In[*]:= Tidy[vd_VD] := Module[{ps = Union@@(List@@@vd)},
  Replace[vd, Thread[ps -> Range@Length@ps], {2}]]
```

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```
In[*]:= R12Reduce1[vd_VD] := Tidy@Module[{R2s, R2}, Which[
  Length[R2s = Cases[vd, X_s_[i_, j_] -> X_s_[i + 1, j + 1]] ∩ (List@@vd)] > 0,
  Complement[vd, VD[R2 = First@R2s, R2 /. X_s_[i_, j_] -> X_s_[i - 1, j - 1]]],
  Length[R2s = Cases[vd, X_s_[i_, j_] -> X_s_[i + 1, j - 1]] ∩ (List@@vd)] > 0,
  Complement[vd, VD[R2 = First@R2s, R2 /. X_s_[i_, j_] -> X_s_[i - 1, j + 1]]],
  True, DeleteCases[vd, X_[i_, j_] /; Abs[i - j] == 1]]]
```

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```
In[*]:= R12Reduce[vd_VD] := FixedPoint[R12Reduce1, vd]
```

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```
In[*]:= γ[vd_VD] := Module[{js, s1, i1, j1, s2, i2, j2},
  js = Cases[vd, X_[_, j_] -> j] ∩ Cases[vd, X_[i_, _] -> i - 1];
  If[Length[js] == 0, vd,
  j1 = RandomChoice[js]; i2 = j1 + 1;
  Cases[vd, X_s_[i_, j1] -> {s1 = s; i1 = i}];
  Cases[vd, X_s_[i2, j_] -> {s2 = s; j2 = j}];
  Tidy@Join[Complement[vd, VD[X_s1[i1, j1], X_s2[i2, j2]]],
  VD[X_s2[j1, j2], X_s1[i1, i2], X_s1s2[i1 - s1/3, j2 + s2/3], X_s1s2[i1 + s1/3, j2 - s2/3]]
  ]]
```

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```
In[*]:= Γ[vd_VD] := FixedPoint[γ, vd, 28]
```

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```
In[*]:= Γ[T_] /; Head[T] != VD := Γ[VD[T]]
```

pdf

```
In[*]:= Γ̄[vd_VD] := FixedPoint[γ@*R12Reduce, vd, 28]
```

pdf

```
In[*]:= Γ̄[T_] /; Head[T] != VD := Γ̄[VD[T]]
```

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```
In[ ]:= VPB[n_, {σs___}] := VPB[n, σs];
```

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```
In[ ]:= VD /: vd1_VD ** vd2_VD := Module[{es1, es2, m2},
  es1 = Cases[vd1, EOS[i_] :=> i];
  m2 = Max[es2 = Cases[vd2, EOS[i_] :=> i]];
  Tidy[vd1 ∪ Replace[DeleteCases[vd2, _EOS],
    i_ :=> i/m2 - 1 + es1[[1 + Count[es2, e_ /; i > e]], {2}]]
]
```

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```
In[ ]:= VD[VPB[n_]] := VD@@(EOS /@ Range[n]);
VD[VPB[n_, σi,j]] := Tidy@Append[VD@@(EOS /@ Range[n]), X+1[i - 0.5, j - 0.5]];
VD[VPB[n_, σ̄i,j]] := Tidy@Append[VD@@(EOS /@ Range[n]), X-1[i - 0.5, j - 0.5]];
VD[VPB[n_, σ_, σs___]] := VD[VPB[n, σ]] ** VD[VPB[n, σs]]
```

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```
In[ ]:= VPBGenerators[n_] :=
  VPBGenerators[n] = Flatten@Table[{σi,j, σ̄i,j}, {i, n}, {j, DeleteCases[Range@n, i]}];
```

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```
In[ ]:= ProudFollowers[n_, σi,j] := ProudFollowers[n, σi,j] = Module[{p, q, s},
  Flatten[{σi,j, σj,i, σ̄j,i,
    Table[{σp,q, σq,p, σ̄p,q, σ̄q,p}, {p, {i, j}}, {q, Complement[Range[n], {i, j}]}],
    Table[{σp,q, σ̄p,q},
      {p, Complement[Range[i + 1, n], {j}]}], {q, Complement[Range[n], {i, j, p}]}]
  ]];
ProudFollowers[n_, σ̄i,j] := ProudFollowers[n, σ̄i,j] = ProudFollowers[n, σi,j] /. σi,j → σ̄i,j
```

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```
In[ ]:= ProudVPBs[n_, 0] := {VPB[n]};
ProudVPBs[n_, 1] := VPB[n, #] & /@ VPBGenerators[n];
ProudVPBs[n_, m_] /; m > 1 := Flatten[
  ProudVPBs[n, m - 1] /. VPB[n, σs___, σ_] :=> (VPB[n, σs, σ, #] & /@ ProudFollowers[n, σ])]
```

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```
In[ ]:= CountOUForms[n_, m_] := Module[{k},
  Length@Union@Flatten@Table[Γ@vpb, {k, 0, m}, {vpb, ProudVPBs[n, k]}]]
```

New Code

```
In[ ]:= AllOUs[n_, 0] := {VD@@Flatten@Table[{BT[2 i - 1], EOS[2 i]}, {i, n}]}];
AllOUs[n_, m_] /; m > 0 :=
Sort@Flatten[AllOUs[n, m - 1] /. vd_VD => Module[{BTs, EOSs, k, max0, s},
  BTs = Sort@Cases[vd, BT[i_] => i];
  EOSs = Sort@Cases[vd, EOS[i_] => i];
  max0 = Max[1, Max[Cases[vd, X_[i_, _] => i]]];
  Table[
    Tidy[Append[vd, Xs[p - 0.5, q + 0.5]]],
    {s, {-1, 1}}, {k, Length[BTs]},
    {q, BTs[[k]], EOSs[[k] - 1], {p, Select[BTs, (# >= max0) &]}
  ]]]
```

```
In[ ]:= AllOUs[3, 0]
```

```
Out[ ]:= {VD[BT[1], BT[3], BT[5], EOS[2], EOS[4], EOS[6]]}
```

```
In[ ]:= AllOUs[3, 4] // Length
```

```
Out[ ]:= 86400
```

```
In[ ]:= 4 × 3 × 4 Binomial[4, 2]
```

```
Out[ ]:= 288
```

```
In[ ]:= 23 Binomial[5, 2] 5! / 2
```

```
Out[ ]:= 4800
```

```
In[ ]:= 24 Binomial[6, 2] 6! / 2
```

```
Out[ ]:= 86400
```

```
In[ ]:= 24 Binomial[7, 3] 7! / 3!
```

```
Out[ ]:= 470400
```

```
In[ ]:= AllROUs[n_, m_] :=
  Select[AllOUs[n, m] /. vd_VD => Tidy@DeleteCases[vd, _BT], (# === R12Reduce[#] &)]
```

```
In[ ]:= AllROUs[3, 4] // Length
```

```
Out[ ]:= 41682
```

```
In[ ]:= ξ[vd_VD] := Count[Γ[vd], X[_ , _]]
```



```
In[ ]:= Union[Divisors /@ AllROUs [4, 4]
```

```
Out[ ]:= { {}, {σ1,2}, {σ1,3}, {σ1,4}, {σ2,1}, {σ2,3}, {σ2,4}, {σ3,1}, {σ3,2}, {σ3,4}, {σ4,1}, {σ4,2},
{σ4,3}, {σ̄1,2}, {σ̄1,3}, {σ̄1,4}, {σ̄2,1}, {σ̄2,3}, {σ̄2,4}, {σ̄3,1}, {σ̄3,2}, {σ̄3,4}, {σ̄4,1},
{σ̄4,2}, {σ̄4,3}, {σ1,2, σ2,3}, {σ1,2, σ2,4}, {σ1,2, σ3,1}, {σ1,2, σ3,4}, {σ1,2, σ4,1},
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{σ̄1,4, σ̄2,3}, {σ̄1,4, σ̄3,1}, {σ̄1,4, σ̄3,2}, {σ̄1,4, σ̄4,2}, {σ̄1,4, σ̄4,3}, {σ̄2,1, σ2,3}, {σ̄2,1, σ2,4},
{σ̄2,1, σ3,4}, {σ̄2,1, σ4,3}, {σ̄2,1, σ̄3,2}, {σ̄2,1, σ̄3,4}, {σ̄2,1, σ̄4,2}, {σ̄2,1, σ̄4,3}, {σ̄2,3, σ2,4},
{σ̄2,3, σ4,1}, {σ̄2,3, σ̄3,1}, {σ̄2,3, σ̄3,4}, {σ̄2,3, σ̄4,1}, {σ̄2,3, σ̄4,2}, {σ̄2,4, σ3,1},
{σ̄2,4, σ̄3,1}, {σ̄2,4, σ̄3,2}, {σ̄2,4, σ̄4,1}, {σ̄2,4, σ̄4,3}, {σ̄3,1, σ3,2}, {σ̄3,1, σ3,4},
{σ̄3,1, σ4,2}, {σ̄3,1, σ̄4,2}, {σ̄3,1, σ̄4,3}, {σ̄3,2, σ3,4}, {σ̄3,2, σ4,1}, {σ̄3,2, σ̄4,1},
{σ̄3,2, σ̄4,3}, {σ̄3,4, σ̄4,1}, {σ̄3,4, σ̄4,2}, {σ̄4,1, σ4,2}, {σ̄4,1, σ4,3}, {σ̄4,2, σ4,3}}
```

```
In[ ]:= MinQ[divs_] :=
Sort[Sort[divs /. Thread[Range@4 -> #]] & /@ Permutations[Range@4]] [[1]] === divs
```

```
In[*]:= MinQ /@ { {}, {σ1,2}, {σ1,3}, {σ1,4}, {σ2,1}, {σ2,3}, {σ2,4}, {σ3,1}, {σ3,2}, {σ3,4}, {σ4,1},
  {σ4,2}, {σ4,3}, {σ̄1,2}, {σ̄1,3}, {σ̄1,4}, {σ̄2,1}, {σ̄2,3}, {σ̄2,4}, {σ̄3,1}, {σ̄3,2}, {σ̄3,4},
  {σ̄4,1}, {σ̄4,2}, {σ̄4,3}, {σ1,2 σ2,3}, {σ1,2 σ2,4}, {σ1,2 σ3,1}, {σ1,2 σ3,4}, {σ1,2 σ4,1},
  {σ1,2 σ4,3}, {σ1,2 σ̄1,3}, {σ1,2 σ̄1,4}, {σ1,2 σ̄3,4}, {σ1,2 σ̄4,3}, {σ1,3 σ2,1}, {σ1,3 σ2,4},
  {σ1,3 σ3,2}, {σ1,3 σ3,4}, {σ1,3 σ4,1}, {σ1,3 σ4,2}, {σ1,3 σ̄1,4}, {σ1,3 σ̄2,4}, {σ1,3 σ̄4,2},
  {σ1,4 σ2,1}, {σ1,4 σ2,3}, {σ1,4 σ3,1}, {σ1,4 σ3,2}, {σ1,4 σ4,2}, {σ1,4 σ4,3}, {σ1,4 σ̄2,3},
  {σ1,4 σ̄3,2}, {σ2,1 σ3,2}, {σ2,1 σ3,4}, {σ2,1 σ4,2}, {σ2,1 σ4,3}, {σ2,1 σ̄2,3}, {σ2,1 σ̄2,4},
  {σ2,1 σ̄3,4}, {σ2,1 σ̄4,3}, {σ2,3 σ3,1}, {σ2,3 σ3,4}, {σ2,3 σ4,1}, {σ2,3 σ4,2}, {σ2,3 σ̄2,4},
  {σ2,3 σ̄4,1}, {σ2,4 σ3,1}, {σ2,4 σ3,2}, {σ2,4 σ4,1}, {σ2,4 σ4,3}, {σ2,4 σ̄3,1}, {σ3,1 σ4,2},
  {σ3,1 σ4,3}, {σ3,1 σ̄3,2}, {σ3,1 σ̄3,4}, {σ3,1 σ̄4,2}, {σ3,2 σ4,1}, {σ3,2 σ4,3}, {σ3,2 σ̄3,4},
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  {σ̄1,2 σ̄4,1}, {σ̄1,2 σ̄4,3}, {σ̄1,3 σ1,4}, {σ̄1,3 σ2,4}, {σ̄1,3 σ4,2}, {σ̄1,3 σ̄2,1}, {σ̄1,3 σ̄2,4},
  {σ̄1,3 σ̄3,2}, {σ̄1,3 σ̄3,4}, {σ̄1,3 σ̄4,1}, {σ̄1,3 σ̄4,2}, {σ̄1,4 σ2,3}, {σ̄1,4 σ3,2}, {σ̄1,4 σ̄2,1},
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  {σ̄2,3 σ4,1}, {σ̄2,3 σ̄3,1}, {σ̄2,3 σ̄3,4}, {σ̄2,3 σ̄4,1}, {σ̄2,3 σ̄4,2}, {σ̄2,4 σ3,1},
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  {σ̄3,2 σ̄4,3}, {σ̄3,4 σ̄4,1}, {σ̄3,4 σ̄4,2}, {σ̄4,1 σ4,2}, {σ̄4,1 σ4,3}, {σ̄4,2 σ4,3}}
```

```
Out[*]:= {True, True, False, False, False, False, False, False, False, False, False, False, False,
  True, False, False, False, False, False, False, False, False, False, False, False,
  True, False, False, True, False, False, True, False, True, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, True, False, False, True, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False}
```

```
In[*]:= Select[{{}, {σ1,2}, {σ1,3}, {σ1,4}, {σ2,1}, {σ2,3}, {σ2,4}, {σ3,1}, {σ3,2}, {σ3,4}, {σ4,1},
  {σ4,2}, {σ4,3}, {σ̄1,2}, {σ̄1,3}, {σ̄1,4}, {σ̄2,1}, {σ̄2,3}, {σ̄2,4}, {σ̄3,1}, {σ̄3,2}, {σ̄3,4},
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  {σ1,2 σ4,3}, {σ1,2 σ̄1,3}, {σ1,2 σ̄1,4}, {σ1,2 σ̄3,4}, {σ1,2 σ̄4,3}, {σ1,3 σ2,1}, {σ1,3 σ2,4},
  {σ1,3 σ3,2}, {σ1,3 σ3,4}, {σ1,3 σ4,1}, {σ1,3 σ4,2}, {σ1,3 σ̄1,4}, {σ1,3 σ̄2,4}, {σ1,3 σ̄4,2},
  {σ1,4 σ2,1}, {σ1,4 σ2,3}, {σ1,4 σ3,1}, {σ1,4 σ3,2}, {σ1,4 σ4,2}, {σ1,4 σ4,3}, {σ1,4 σ̄2,3},
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  {σ2,1 σ̄3,4}, {σ2,1 σ̄4,3}, {σ2,3 σ3,1}, {σ2,3 σ3,4}, {σ2,3 σ4,1}, {σ2,3 σ4,2}, {σ2,3 σ̄2,4},
  {σ2,3 σ̄4,1}, {σ2,4 σ3,1}, {σ2,4 σ3,2}, {σ2,4 σ4,1}, {σ2,4 σ4,3}, {σ2,4 σ̄3,1}, {σ3,1 σ4,2},
  {σ3,1 σ4,3}, {σ3,1 σ̄3,2}, {σ3,1 σ̄3,4}, {σ3,1 σ̄4,2}, {σ3,2 σ4,1}, {σ3,2 σ4,3}, {σ3,2 σ̄3,4},
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  {σ̄1,2 σ1,4}, {σ̄1,2 σ3,4}, {σ̄1,2 σ4,3}, {σ̄1,2 σ̄2,3}, {σ̄1,2 σ̄2,4}, {σ̄1,2 σ̄3,1}, {σ̄1,2 σ̄3,4},
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```

```
Out[*]:= {{}, {σ1,2}, {σ̄1,2}, {σ1,2 σ2,3}, {σ1,2 σ3,4},
  {σ1,2 σ̄1,3}, {σ1,2 σ̄3,4}, {σ̄1,2 σ̄2,3}, {σ̄1,2 σ̄3,4}}
```

```
In[*]:= AllROUS[2, 2]
```

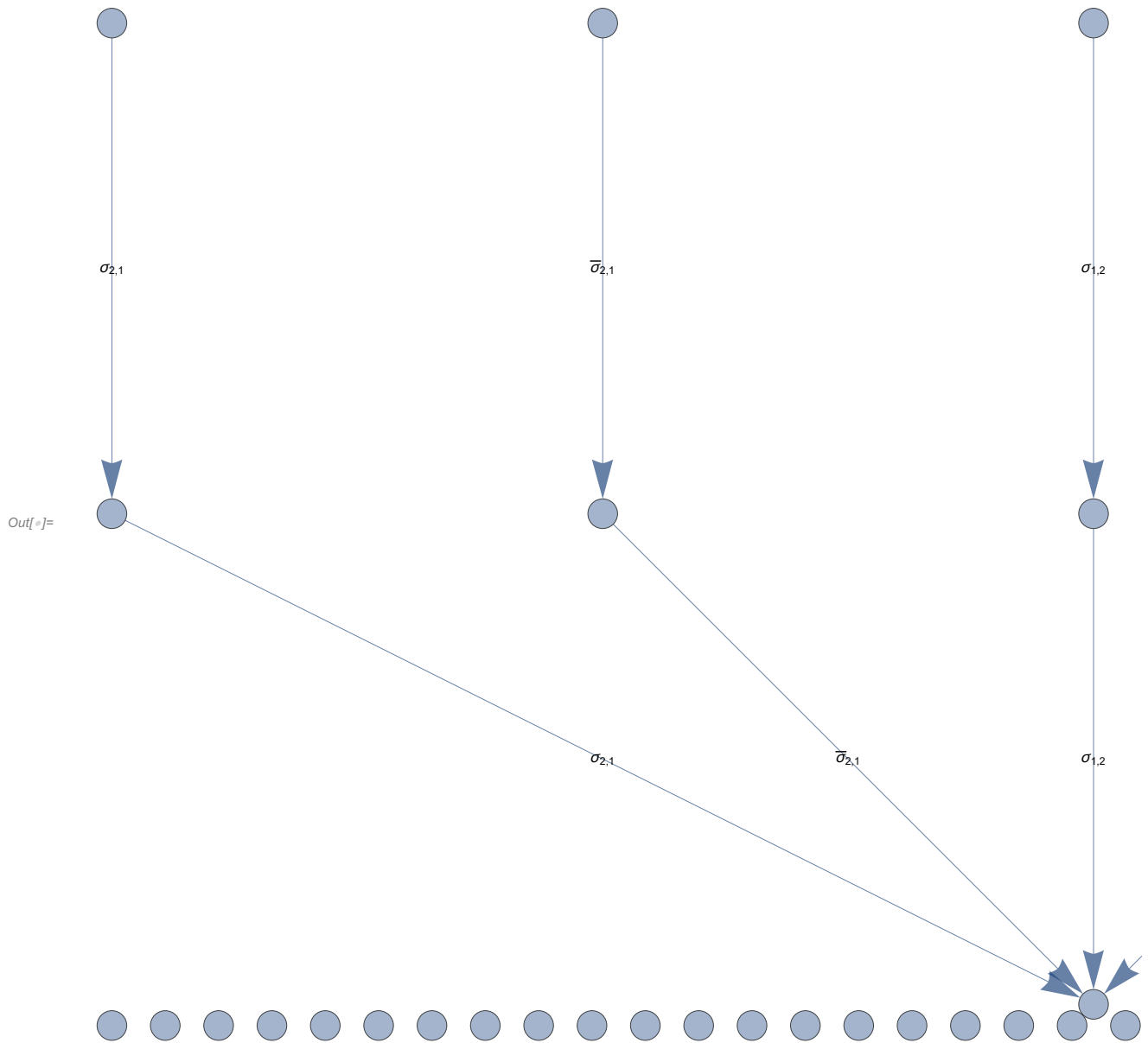
```
Out[*]:= {VD[EOS[1], EOS[6], X-1[2, 4], X-1[3, 5]], VD[EOS[1], EOS[6], X1[2, 4], X1[3, 5]],
  VD[EOS[2], EOS[6], X-1[3, 5], X-1[4, 1]], VD[EOS[2], EOS[6], X-1[3, 5], X1[4, 1]],
  VD[EOS[2], EOS[6], X-1[4, 1], X1[3, 5]], VD[EOS[2], EOS[6], X1[3, 5], X1[4, 1]],
  VD[EOS[3], EOS[6], X-1[4, 1], X-1[5, 2]], VD[EOS[3], EOS[6], X-1[4, 2], X-1[5, 1]],
  VD[EOS[3], EOS[6], X1[4, 1], X1[5, 2]], VD[EOS[3], EOS[6], X1[4, 2], X1[5, 1]],
  VD[EOS[2], EOS[6], X-1[1, 4], X-1[3, 5]], VD[EOS[2], EOS[6], X-1[1, 4], X1[3, 5]],
  VD[EOS[2], EOS[6], X-1[3, 5], X1[1, 4]], VD[EOS[2], EOS[6], X1[1, 4], X1[3, 5]],
  VD[EOS[3], EOS[6], X-1[1, 5], X-1[4, 2]], VD[EOS[3], EOS[6], X-1[1, 5], X1[4, 2]],
  VD[EOS[3], EOS[6], X-1[4, 2], X1[1, 5]], VD[EOS[3], EOS[6], X1[1, 5], X1[4, 2]],
  VD[EOS[4], EOS[6], X-1[1, 3], X-1[5, 2]], VD[EOS[4], EOS[6], X-1[1, 3], X1[5, 2]],
  VD[EOS[4], EOS[6], X-1[5, 2], X1[1, 3]], VD[EOS[4], EOS[6], X1[1, 3], X1[5, 2]],
  VD[EOS[3], EOS[6], X-1[1, 4], X-1[2, 5]], VD[EOS[3], EOS[6], X-1[1, 5], X-1[2, 4]],
  VD[EOS[3], EOS[6], X1[1, 4], X1[2, 5]], VD[EOS[3], EOS[6], X1[1, 5], X1[2, 4]],
  VD[EOS[4], EOS[6], X-1[1, 3], X-1[2, 5]], VD[EOS[4], EOS[6], X-1[1, 3], X1[2, 5]],
  VD[EOS[4], EOS[6], X-1[2, 5], X1[1, 3]], VD[EOS[4], EOS[6], X1[1, 3], X1[2, 5]],
  VD[EOS[5], EOS[6], X-1[1, 3], X-1[2, 4]], VD[EOS[5], EOS[6], X1[1, 3], X1[2, 4]]}
```

```

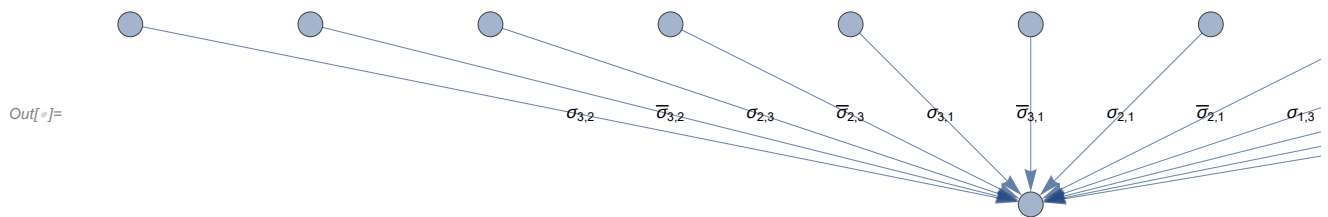
In[ ]:= OUGraph[n_, m_] := Module[{gens, OUs, k, d, g, q, m1, m2},
  gens = VPBGenerators[n];
  OUs = Flatten@Table[AllROUs[n, k], {k, 0, m}];
  OURule = Dispatch@Thread[OUs → Range@Length@OUs];
  Graph[
    Range@Length@OUs,
    Union@Flatten@Table[
      m1 = Count[d, X[_[_], _]];
      m2 = Count[q =  $\overline{\Gamma}$ [VD[VPB[n, g]] ** d], X[_[_], _]];
      If[m2 < m1, Labeled[(d ↔ q) /. OURule, g], Nothing],
      {d, OUs}, {g, gens}
    ]
  ]
]

```


In[]:= **OUGraph[2, 2]**

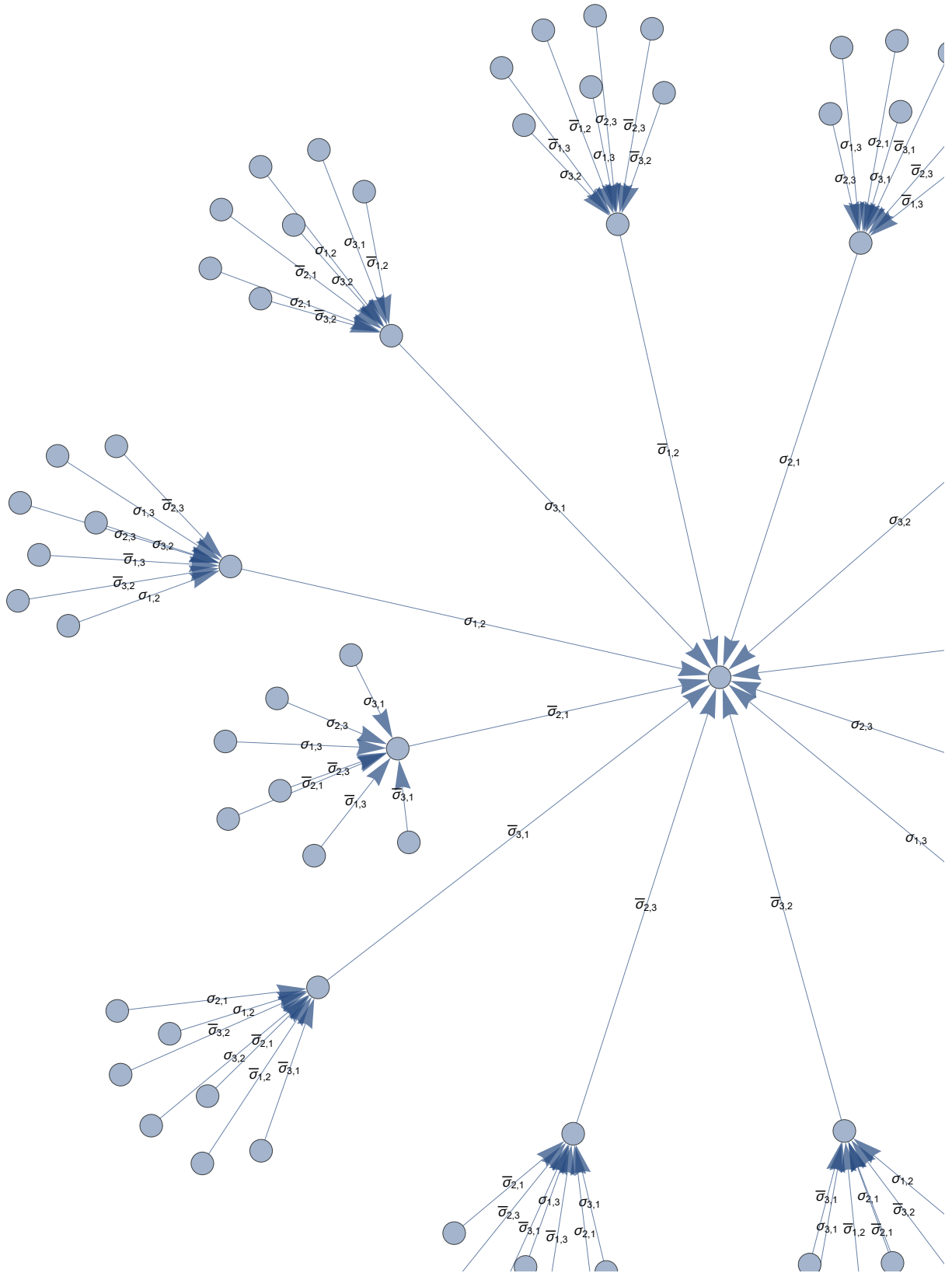


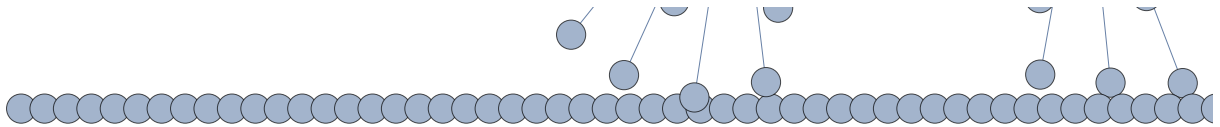
In[]:= **OUGraph[3, 1]**



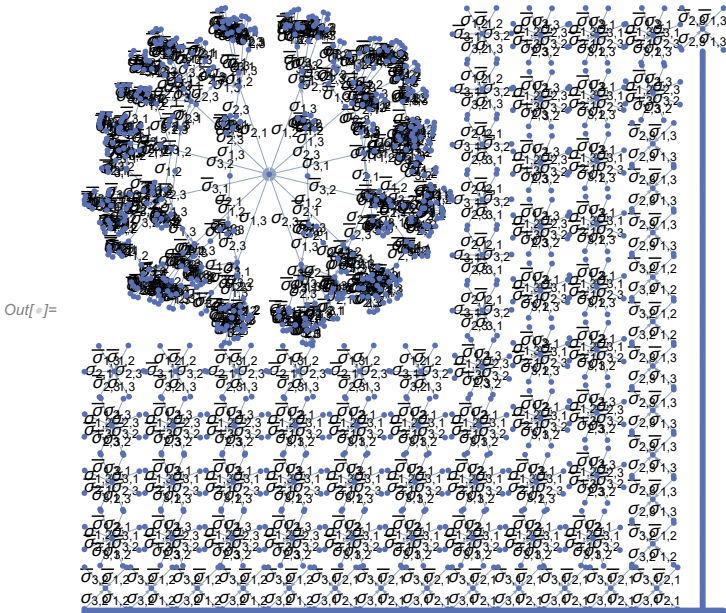
In[]:= **OUGraph**[3, 2]

Out[]:=



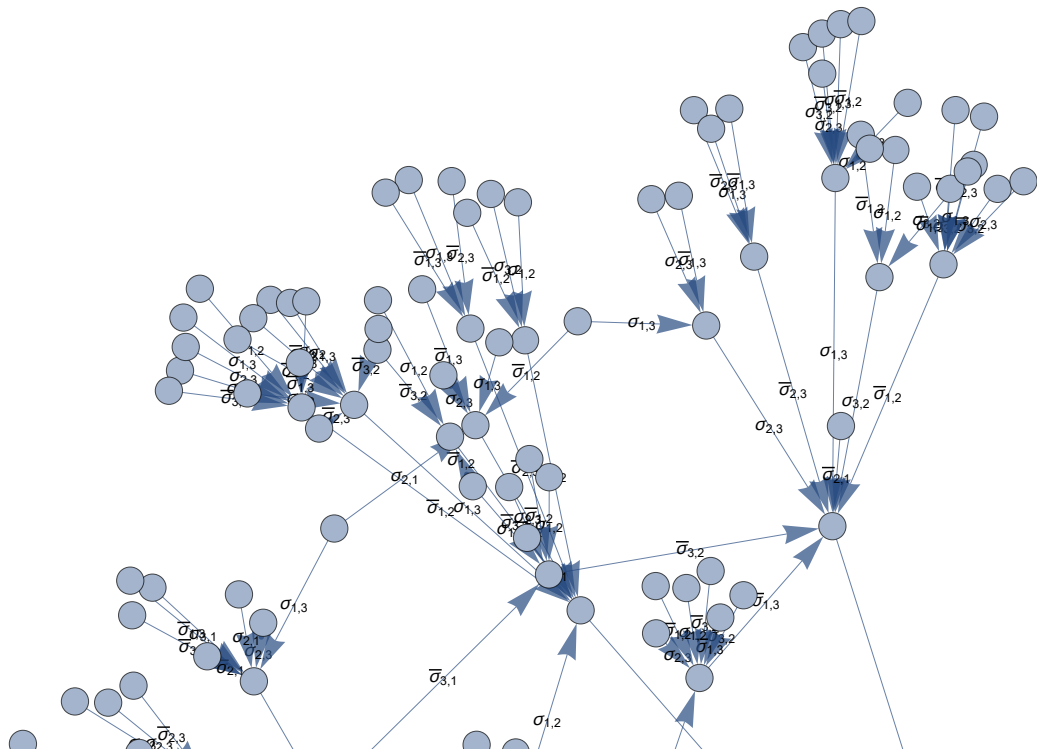


In[]:= **g = OUGraph [3, 3]**

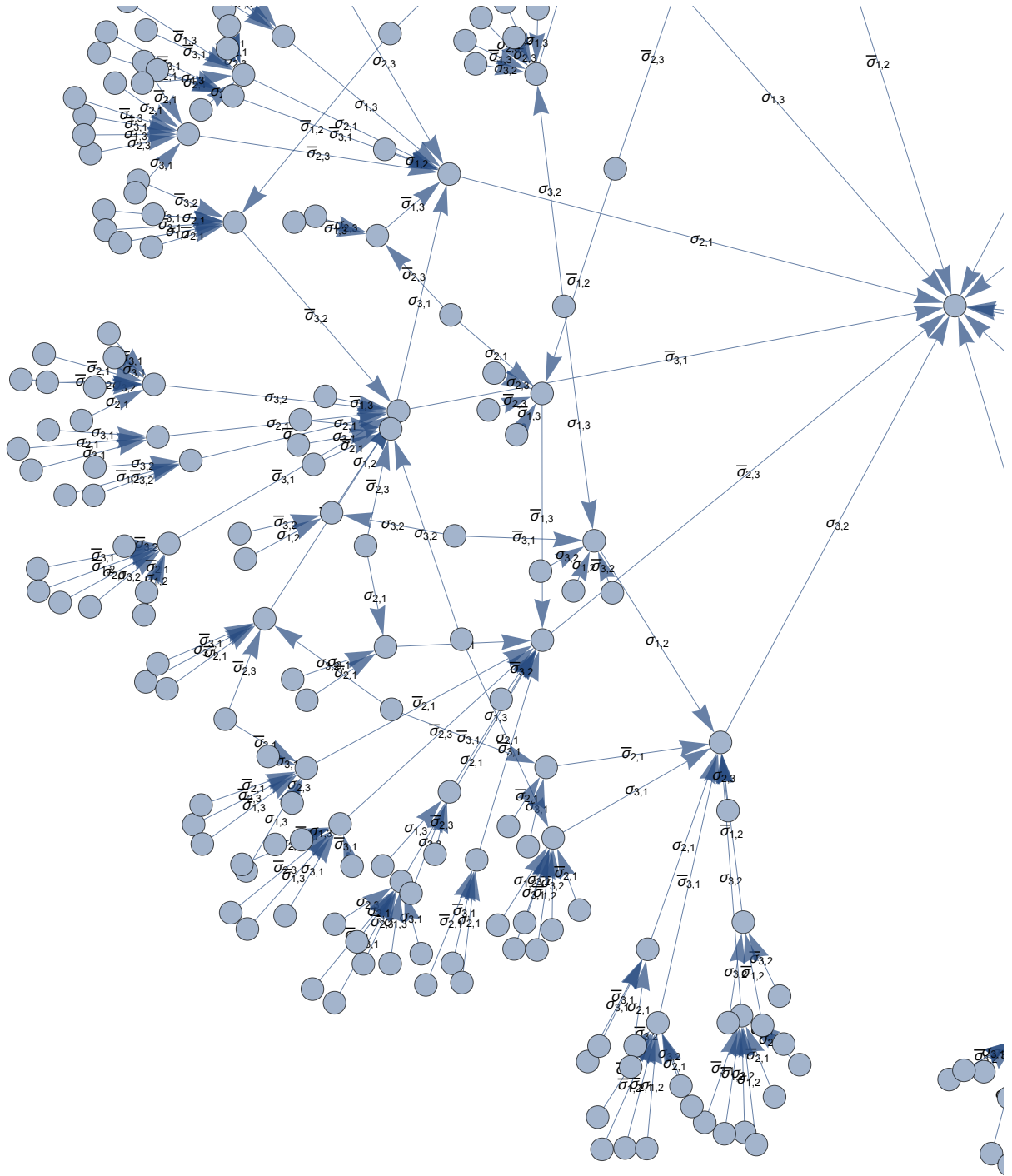


Out[]:=

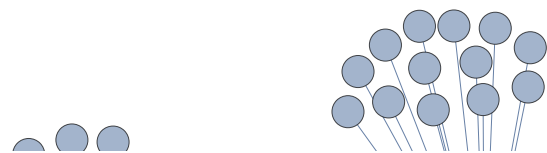
In[]:= **First@WeaklyConnectedGraphComponents [g]**



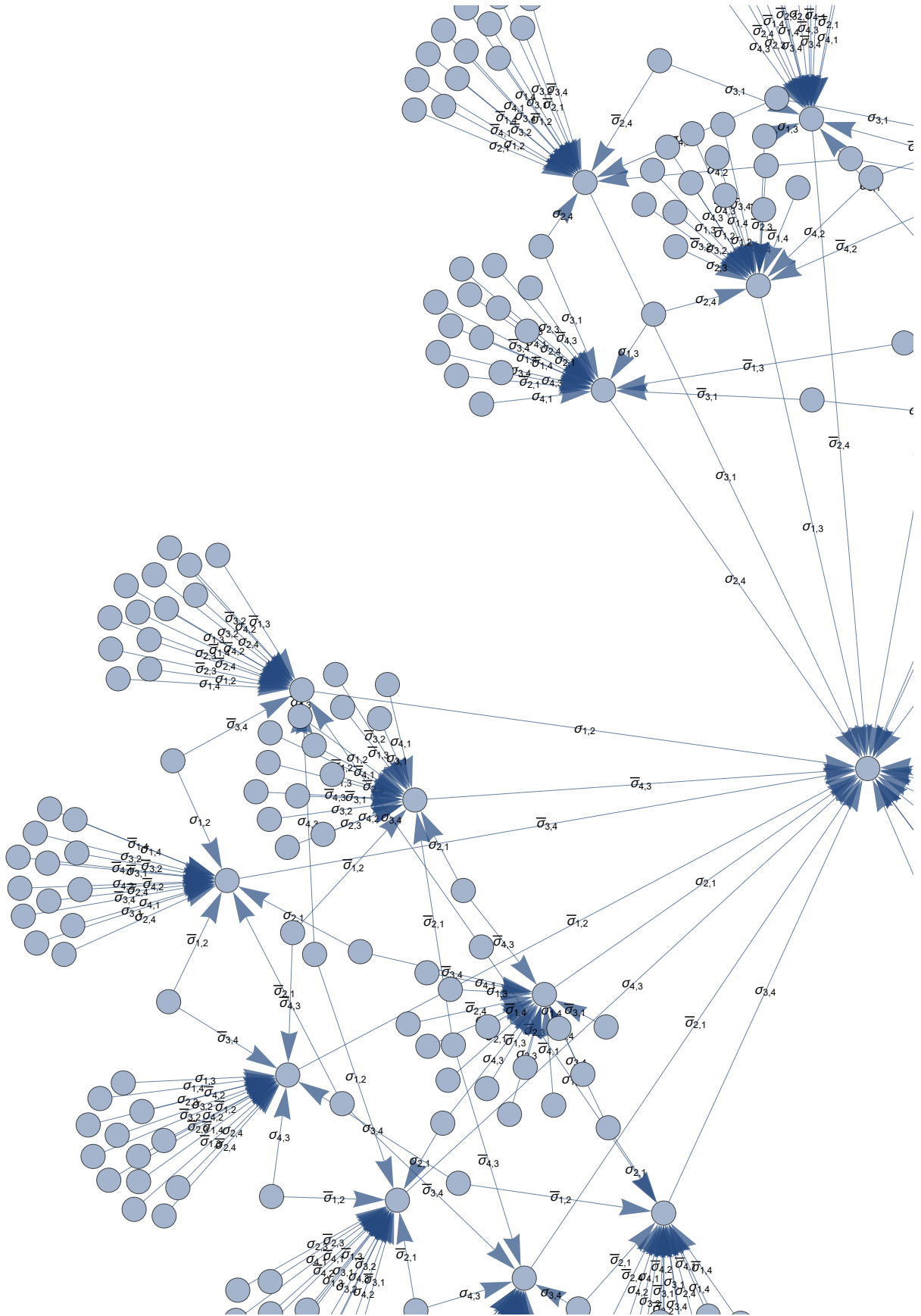
Out[]:=

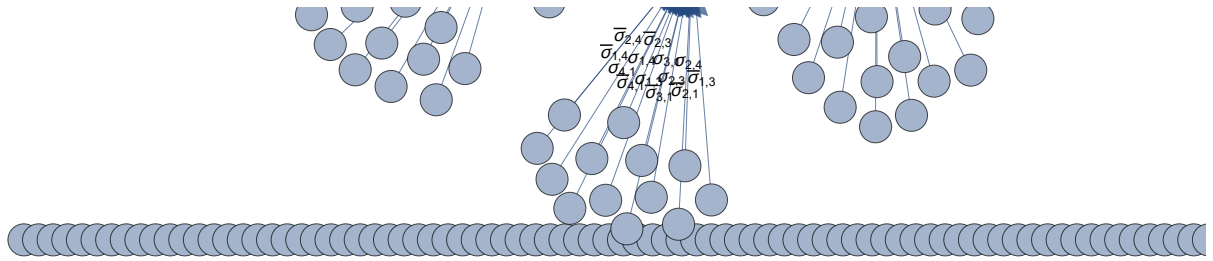


In[]:= **g = OUGraph[4, 2]**

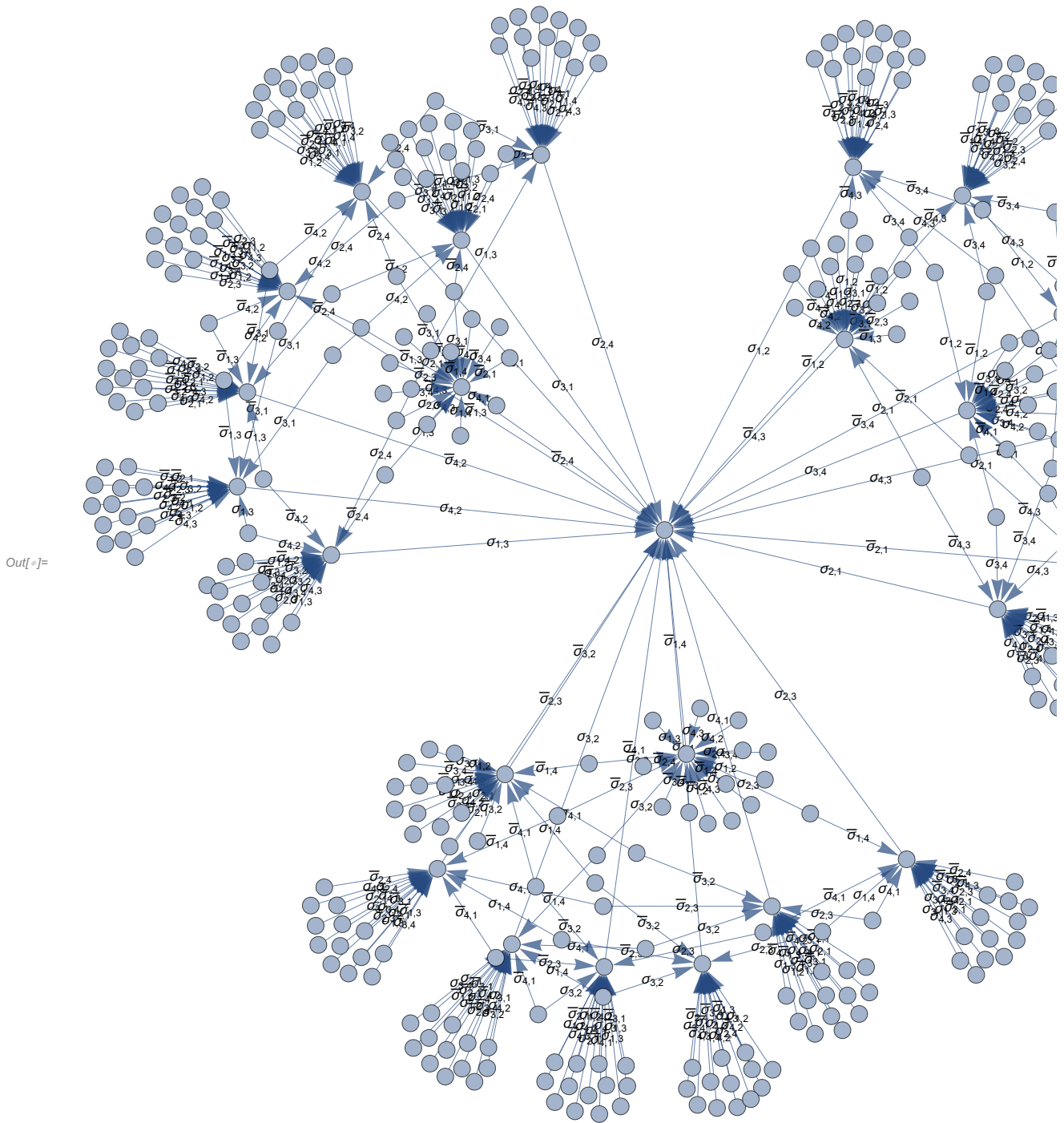


Out[]:=





```
In[ ]:= First@WeaklyConnectedGraphComponents [g]
```



Out[]:=