

Pensieve header: Counting virtual pure braids.

(Alt) In[*]:=

```
CountVPB[n_, m_] := Module[{gens, diags, s, rels, r = 0, ij, ijk, ijk1, i, j, k, l, mat},
  gens = Flatten@Table[{σi,j, σ̄i,j}, {i, n}, {j, DeleteCases[Range@n, i]}];
  diags = Flatten[Table[VPB[n, #] & /@ Tuples[gens, s], {s, 0, m}]];
  rels = Flatten[{
    Table[{i, j} = ij; {
      T[VPB[n, Join[p, {σi,j, σ̄i,j}, q]], VPB[n, Join[p, q]]],
      T[VPB[n, Join[p, {σ̄i,j, σi,j}, q]], VPB[n, Join[p, q]]]
    },
    {s, 0, m - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
    {ijk, Join@@(Permutations /@ Subsets[Range[n], {2}])}
  ],
  Table[{i, j, k} = ijk; {
    T[VPB[n, Join[p, {σi,j, σi,k, σj,k}, q]], VPB[n, Join[p, {σj,k, σi,k, σi,j}, q]]],
    T[VPB[n, Join[p, {σ̄j,i, σi,k, σj,k}, q]], VPB[n, Join[p, {σj,k, σi,k, σ̄j,i}, q]]],
    T[VPB[n, Join[p, {σi,j, σi,k, σ̄k,j}, q]], VPB[n, Join[p, {σ̄k,j, σi,k, σi,j}, q]]],
    T[VPB[n, Join[p, {σi,j, σ̄k,i, σ̄k,j}, q]], VPB[n, Join[p, {σ̄k,j, σ̄k,i, σi,j}, q]]],
    T[VPB[n, Join[p, {σ̄j,i, σ̄k,i, σj,k}, q]], VPB[n, Join[p, {σj,k, σ̄k,i, σ̄j,i}, q]]],
    T[VPB[n, Join[p, {σ̄j,i, σ̄k,i, σ̄k,j}, q]], VPB[n, Join[p, {σ̄k,j, σ̄k,i, σ̄j,i}, q]]]
  },
  {s, 0, m - 3}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
  {ijk, Join@@(Permutations /@ Subsets[Range[n], {3}])}
  ],
  mat = SparseArray[
    Flatten@Cases[rels, T[b1_, b2_] =>
      {++r, Position[diags, b1][[1, 1]]} -> 1, {r, Position[diags, b2][[1, 1]]} -> -1}],
    {r, Length@diags}
  ];
  Length@diags - If[r == 0, 0, MatrixRank@mat]
]
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(Alt) In[*]:= **CountVPB[2, 1]**

(Alt) Out[*]= 5

(Alt) In[*]:= **CountVPB[2, 2]**

(Alt) Out[*]= 17

(Alt) In[*]:= **CountVPB[2, 3]**

(Alt) Out[*]= 53

(Alt) In[*]:= **CountVPB[2, 4]**

(Alt) Out[*]= 161

(Alt) In[*]:= **CountVPB[3, 1]**

(Alt) Out[*]= 13

(Alt) In[*]:= **CountVPB[3, 2]**

(Alt) Out[*]= 145

(Alt) In[*]:= **CountVPB[3, 3]**

(Alt) Out[*]= 1561

(Alt) In[*]:= **CountVPB[3, 4]**

(Alt) Out[*]= 16 741

(Alt) In[*]:= **CountVPB[3, 5]**

(Alt) Out[*]= \$Aborted