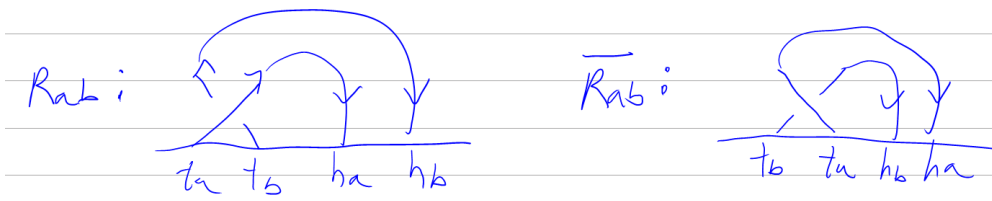Pensieve header: Unitarity for Γ-calculus - finding the ${\mathcal L}_0$ for which $\bar{A}={\mathcal L}_0 A$.

```
In[◦]:=  SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\MetaCalculi"];
         Once[<< KnotTheory`]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

$$In[◦]:=\quad R_{a\_,b\_} := \Gamma@\langle | \varsigma \to \{t_a, t_b, h_a, h_b\}, \omega \to 1, \sigma \to h_a + h_b T_a, \lambda \to \{t_a, t_b\}.\begin{pmatrix} T_a & 1-T_a \\ 0 & 1 \end{pmatrix}.\{h_a, h_b\} |\rangle;$$

$$\overline{R}_{a\_,b\_} := \Gamma@\langle | \varsigma \to \{t_b, t_a, h_b, h_a\}, \omega \to 1, \sigma \to h_a + h_b T_a^{-1}, \lambda \to \{t_a, t_b\}.\begin{pmatrix} T_a^{-1} & 1-T_a^{-1} \\ 0 & 1 \end{pmatrix}.\{h_a, h_b\} |\rangle;$$

$$In[◦]:=\quad \overline{R}_{1,2}$$

Out[◦]=

$$\Gamma\left[\left\langle \left| \varsigma \to \{t_2, t_1, h_2, h_1\}, \omega \to 1, \sigma \to h_1 + \frac{h_2}{T_1}, \lambda \to h_2\left(t_2 + t_1\left(1 - \frac{1}{T_1}\right)\right) + \frac{h_1 t_1}{T_1} \right| \right\rangle \right]$$

```
In[◦]:=  Γ[α_][ϰ_] := α[ϰ];
         Γ[α_][S] := Union@Cases[α[ς], t_a_ :> a, ∞];
         Γ[α_][n] := Length[Γ[α][S]];
         Γ[α_][Σ] := (∂_{h_#} α[σ]) & /@ Γ[α][S];
         Γ[α_][A] := Outer[Factor[∂_{#1,#2} Γ[α][λ]] &, Cases[α[ς], t_], Cases[α[ς], h_]];
```

```
In[◦]:=  Table[{γ@ς, γ@ω, γ@σ, γ@λ, γ@S, γ@Σ, γ@A // MatrixForm}, {γ, {R_{1,2}, R̄_{1,2}}}] // Transpose //
           MatrixForm
```

Out[◦]//MatrixForm=

$$\begin{pmatrix}
\{t_1, t_2, h_1, h_2\} & \{t_2, t_1, h_2, h_1\} \\
1 & 1 \\
h_1 + h_2 T_1 & h_1 + \frac{h_2}{T_1} \\
h_2(t_2 + t_1(1 - T_1)) + h_1 t_1 T_1 & h_2\left(t_2 + t_1\left(1 - \frac{1}{T_1}\right)\right) + \frac{h_1 t_1}{T_1} \\
\{1, 2\} & \{1, 2\} \\
\{1, T_1\} & \left\{1, \frac{1}{T_1}\right\} \\
\begin{pmatrix} T_1 & 1-T_1 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ \frac{-1+T_1}{T_1} & \frac{1}{T_1} \end{pmatrix}
\end{pmatrix}$$

```
In[ ]:= ΓCollect[γ_Γ] := Γ[⟨|ς → γ@ς, ω → Factor@γ@ω, σ → Expand@γ@σ,
          λ → Total[CoefficientRules[γ@λ, γ@ς] /. (ps_ → c_) ⧴ Factor[c] Times @@ ((γ@ς)^ps)]|⟩]
```

```
In[ ]:= ΓCollect[R̄₁,₂]
```

$$\text{Out[ ]= } \Gamma\left[\left\langle\left|\varsigma \to \{t_2,\, t_1,\, h_2,\, h_1\},\, \omega \to 1,\, \sigma \to h_1 + \frac{h_2}{T_1},\, \lambda \to h_2\, t_2 + \frac{h_1\, t_1}{T_1} + \frac{h_2\, t_1\, (-1 + T_1)}{T_1}\right|\right\rangle\right]$$

```
In[ ]:= ConservativeQ[γ_Γ] := Simplify[Expand[γ@λ /. h_ → 1] == Sum[tₐ, {a, γ@S}]]
```

```
In[ ]:= ConservativeQ[R₁,₂]
```

Out[ ]=

True

```
In[ ]:= T* ^:= T⁻¹;
        ⟨c_, d_⟩_ς_ := Expand@Module[{e, f},
            Expand[c (d /. {T → T*, tᵢ_ ⧴ tᵢ*, hᵢ_ ⧴ hᵢ*})] /.
            {tᵢ_* tᵢ_ → 0, hᵢ_* hᵢ_ → T - T*, (f : t | h)ⱼ_* (e : t | h)ᵢ_ ⧴
                If[Position[ς, eᵢ][[1, 1]] < Position[ς, fⱼ][[1, 1]], T - 1, 1 - T*]}
          ];
        UnitaryQ[γ_Γ] := Module[{vs},
            vs = Table[-tᵢ + ∂ₜᵢ γ@λ /. T_ → T, {i, γ@S}];
            And @@ Flatten@Table[Simplify[⟨vs[[i]], vs[[j]]⟩_γ@ς == 0], {i, γ@n}, {j, γ@n}]
          ];
```

```
In[ ]:= UnitaryQ /@ {R₁,₂, R̄₁,₂}
```

Out[ ]=

{True, True}

```
In[ ]:= ΓForm[γ_Γ] := Module[{M},
            M = γ[A] // Transpose;
            PrependTo[M, t# & /@ γ[S]];
            M = Join[
              {Prepend[h# & /@ γ[S], γ[ω]]},
              Transpose[M],
              {Prepend[γ[Σ], If[TrueQ[ConservativeQ@γ ∧ UnitaryQ@γ], ■, ■]]}
            ];
            Column[{γ[ς], MatrixForm[M]}]
          ];
        ΓForm[else_] := else /. γ_Γ ⧴ ΓForm[γ];
        Format[γ_Γ, StandardForm] := ΓForm[γ];
```

$In[\circ]:=$ $\{R_{1,2}, \overline{R}_{1,2}\}$

$Out[\circ]=$

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 & 1-T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_2, t_1, h_2, h_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix} \right\}$$

$In[\circ]:=$ $\mathrm{dm}_{i\_j\_\to k\_}[\gamma\_\Gamma] := \mathrm{Module}\Big[\{a, b, c, d, \theta, \epsilon, \phi, \psi, \Xi, \mu\},$

$$\begin{pmatrix} a & b & \theta \\ c & d & \epsilon \\ \phi & \psi & \Xi \end{pmatrix} = \begin{pmatrix} \partial_{t_i,h_i}\gamma@\lambda & \partial_{t_i,h_j}\gamma@\lambda & \partial_{t_i}\gamma@\lambda \\ \partial_{t_j,h_i}\gamma@\lambda & \partial_{t_j,h_j}\gamma@\lambda & \partial_{t_j}\gamma@\lambda \\ \partial_{h_i}\gamma@\lambda & \partial_{h_j}\gamma@\lambda & \gamma@\lambda \end{pmatrix} /. (t \mid h)_{i\mid j} \to 0;$$

$\quad \Gamma \mathrm{Collect}\Big[\Gamma\Big[\lessdot$

$\qquad \varsigma \to \mathrm{DeleteCases}\big[\gamma@\varsigma, h_i \mid t_j\big] /. \{t_i \to t_k, h_j \to h_k\},$

$\qquad \omega \to (\mu = 1 - c) \gamma@\omega,$

$\qquad \sigma \to h_k (\partial_{h_i}\sigma) (\partial_{h_j}\sigma) + (\sigma /. h_{i\mid j} \to 0),$

$\qquad \lambda \to \{t_k, 1\}.\begin{pmatrix} b + a\,d/\mu & \theta + a\,\epsilon/\mu \\ \psi + d\,\phi/\mu & \Xi + \epsilon\,\phi/\mu \end{pmatrix}.\{h_k, 1\}$

$\qquad \lvert\gtrdot\Big] /. \{T_i \to T_k, T_j \to T_k\}\Big]$

$\Big];$

$In[\circ]:=$ $\Gamma /: \mathrm{RotateLeft}[\Gamma[\alpha\_], n\_\_\_] := \Gamma@\mathrm{ReplacePart}[\alpha, \mathrm{Key}@\varsigma \to \mathrm{RotateLeft}[\alpha@\varsigma, n]]$

$In[\circ]:=$ $\{\mathrm{Table}[\mathrm{RotateLeft}[R_{1,2}, k], \{k, 0, 3\}], \mathrm{Table}[\mathrm{RotateLeft}[\overline{R}_{1,2}, k], \{k, 0, 3\}]\}$

$Out[\circ]=$

$$\left\{\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 & 1-T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_2, h_1, h_2, t_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 0 & 1 \\ t_2 & T_1 & 1-T_1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix}, \begin{matrix} \{h_1, h_2, t_1, t_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 & 1-T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix}, \begin{matrix} \{h_2, t_1, t_2, h_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1-T_1 & T_1 \\ t_2 & 1 & 0 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix} \right\},\right.$$

$$\left.\left\{ \begin{matrix} \{t_2, t_1, h_2, h_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix}, \begin{matrix} \{t_1, h_2, h_1, t_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ t_2 & 1 & 0 \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix}, \begin{matrix} \{h_2, h_1, t_2, t_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix}, \begin{matrix} \{h_1, t_2, t_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 0 & 1 \\ t_2 & \frac{1}{T_1} & \frac{-1+T_1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix} \right\}\right\}$$

$In[\circ]:=$
```
Γ /: Insert[γ1_Γ, γ2_Γ, k_] := Γ@<|
    ς → Flatten[Insert[γ1@ς, γ2@ς, k]],
    ω → γ1@ω γ2@ω,
    σ → γ1@σ + γ2@σ,
    λ → γ1@λ + γ2@λ
    |>
Γ /: Insert[γ2_Γ, k_][γ1_Γ] := Insert[γ1, γ2, k]
```

$In[\circ]:=$
```
ComposeList[{Insert[R_{3,4}, 5], Insert[R_{5,6}, 9], dm_{2,3→2}, dm_{1,4→1}, dm_{1,5→1}, dm_{2,6→2}, dm_{2,1→2}},
    R_{1,2}]
```

Out[●]=

$$\{t_1, t_2, h_1, h_2, t_3, t_4, h_3, h_4\}$$

$$\left\{
\begin{array}{c}
\{t_1, t_2, h_1, h_2\} \\
\begin{pmatrix}
 & 1 & h_1 & h_2 \\
t_1 & T_1 & 1 - T_1 \\
t_2 & 0 & 1 \\
\blacksquare & 1 & T_1
\end{pmatrix}
\end{array}
, \quad
\begin{pmatrix}
 & 1 & h_1 & h_2 & h_3 & h_4 \\
t_1 & T_1 & 1 - T_1 & 0 & 0 \\
t_2 & 0 & 1 & 0 & 0 \\
t_3 & 0 & 0 & T_3 & 1 - T_3 \\
t_4 & 0 & 0 & 0 & 1 \\
\blacksquare & 1 & T_1 & 1 & T_3
\end{pmatrix}
, \right.$$

$$\{t_1, t_2, h_1, h_2, t_3, t_4, h_3, h_4, t_5, t_6, h_5, h_6\}$$

$$\begin{pmatrix}
 & 1 & h_1 & h_2 & h_3 & h_4 & h_5 & h_6 \\
t_1 & T_1 & 1 - T_1 & 0 & 0 & 0 & 0 \\
t_2 & 0 & 1 & 0 & 0 & 0 & 0 \\
t_3 & 0 & 0 & T_3 & 1 - T_3 & 0 & 0 \\
t_4 & 0 & 0 & 0 & 1 & 0 & 0 \\
t_5 & 0 & 0 & 0 & 0 & T_5 & 1 - T_5 \\
t_6 & 0 & 0 & 0 & 0 & 0 & 1 \\
\blacksquare & 1 & T_1 & 1 & T_3 & 1 & T_5
\end{pmatrix}
, $$

$$\{t_1, t_2, h_1, t_4, h_2, h_4, t_5, t_6, h_5, h_6\}$$

$$\begin{pmatrix}
 & 1 & h_1 & h_2 & h_4 & h_5 & h_6 \\
t_1 & T_1 & -((-1 + T_1) T_2) & (-1 + T_1)(-1 + T_2) & 0 & 0 \\
t_2 & 0 & T_2 & 1 - T_2 & 0 & 0 \\
t_4 & 0 & 0 & 1 & 0 & 0 \\
t_5 & 0 & 0 & 0 & T_5 & 1 - T_5 \\
t_6 & 0 & 0 & 0 & 0 & 1 \\
\blacksquare & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
, $$

$$\{t_1, t_2, h_2, h_1, t_5, t_6, h_5, h_6\}$$

$$\begin{pmatrix}
 & 1 & h_1 & h_2 & h_5 & h_6 \\
t_1 & -((-1 + T_1) T_2) & 1 - T_2 + T_1 T_2 & 0 & 0 \\
t_2 & T_2 & 1 - T_2 & 0 & 0 \\
t_5 & 0 & 0 & T_5 & 1 - T_5 \\
t_6 & 0 & 0 & 0 & 1 \\
\blacksquare & 0 & 0 & 0 & 0
\end{pmatrix}
, $$

$$\{t_1, t_2, h_2, t_6, h_1, h_6\}$$

$$\begin{pmatrix}
 & 1 & h_1 & h_2 & h_6 \\
t_1 & -((-1 + T_1) T_2) & T_1(1 - T_2 + T_1 T_2) & -((-1 + T_1)(1 - T_2 + T_1 T_2)) \\
t_2 & T_2 & -T_1(-1 + T_2) & (-1 + T_1)(-1 + T_2) \\
t_6 & 0 & 0 & 1 \\
\blacksquare & 0 & 0 & 0
\end{pmatrix}
, $$

$$\{t_1, t_2, h_1, h_2\}$$

$$\begin{pmatrix}
 & 1 & h_1 & h_2 \\
t_1 & T_1(1 - T_2 + T_1 T_2) & -((-1 + T_1)(1 + T_1 T_2)) \\
t_2 & -T_1(-1 + T_2) & 1 - T_1 + T_1 T_2 \\
\blacksquare & 0 & 0
\end{pmatrix}
, \quad
\begin{array}{c}
\{t_2, h_2\} \\
\begin{pmatrix}
 & & h_2 \\
T_2(1 - T_2 + T_2^2) & h_2 \\
t_2 & 1 \\
\blacksquare & 0
\end{pmatrix}
\end{array}
\right\}$$

In[ ]:= `ς0 = {t₂, h₂, t₆, h₆};`
`S0 = Cases[ς0, t_{i_} :> i]`

Out[ ]=
$\{2, 6\}$

In[ ]:= `Table[-t_i + Sum[a_{i,k} h_k, {k, S0}], {i, S0}]`

Out[ ]=
$\{-t_2 + h_2 a_{2,2} + h_6 a_{2,6}, \ -t_6 + h_2 a_{6,2} + h_6 a_{6,6}\}$

In[ ]:= `us = Flatten@Table[u_{i,j}, {i, S0}, {j, S0}]`

Out[ ]=
$\{u_{2,2}, u_{2,6}, u_{6,2}, u_{6,6}\}$

In[ ]:= `eqns = Flatten@`
`Table[⟨-t_i + Sum[a_{i,k} h_k, {k, S0}], -t_j + Sum[u_{j,k} h_k, {k, S0}]⟩_{ς0} == 0, {i, S0}, {j, S0}]`

Out[ ]=
$$\Big\{-a_{2,2} + \frac{a_{2,2}}{T} - a_{2,6} + \frac{a_{2,6}}{T} + u_{2,2} - T u_{2,2} - \frac{a_{2,2} u_{2,2}}{T} + T a_{2,2} u_{2,2} +$$
$$a_{2,6} u_{2,2} - \frac{a_{2,6} u_{2,2}}{T} + u_{2,6} - T u_{2,6} - a_{2,2} u_{2,6} + T a_{2,2} u_{2,6} - \frac{a_{2,6} u_{2,6}}{T} + T a_{2,6} u_{2,6} == 0,$$
$$-1 + T + a_{2,2} - T a_{2,2} - a_{2,6} + \frac{a_{2,6}}{T} + u_{6,2} - T u_{6,2} - \frac{a_{2,2} u_{6,2}}{T} + T a_{2,2} u_{6,2} + a_{2,6} u_{6,2} -$$
$$\frac{a_{2,6} u_{6,2}}{T} + u_{6,6} - T u_{6,6} - a_{2,2} u_{6,6} + T a_{2,2} u_{6,6} - \frac{a_{2,6} u_{6,6}}{T} + T a_{2,6} u_{6,6} == 0,$$
$$1 - \frac{1}{T} - a_{6,2} + \frac{a_{6,2}}{T} - a_{6,6} + \frac{a_{6,6}}{T} - u_{2,2} + \frac{u_{2,2}}{T} - \frac{a_{6,2} u_{2,2}}{T} + T a_{6,2} u_{2,2} + a_{6,6} u_{2,2} -$$
$$\frac{a_{6,6} u_{2,2}}{T} + u_{2,6} - T u_{2,6} - a_{6,2} u_{2,6} + T a_{6,2} u_{2,6} - \frac{a_{6,6} u_{2,6}}{T} + T a_{6,6} u_{2,6} == 0,$$
$$a_{6,2} - T a_{6,2} - a_{6,6} + \frac{a_{6,6}}{T} - u_{6,2} + \frac{u_{6,2}}{T} - \frac{a_{6,2} u_{6,2}}{T} + T a_{6,2} u_{6,2} + a_{6,6} u_{6,2} - \frac{a_{6,6} u_{6,2}}{T} +$$
$$u_{6,6} - T u_{6,6} - a_{6,2} u_{6,6} + T a_{6,2} u_{6,6} - \frac{a_{6,6} u_{6,6}}{T} + T a_{6,6} u_{6,6} == 0\Big\}$$

In[ ]:= `{sol} = Solve[eqns, us]`

Out[ ]=
$$\Big\{\Big\{u_{2,2} \to -\Big(\big(T - a_{2,6} - T a_{6,2} + a_{2,6} a_{6,2} - T a_{6,6} - a_{2,2} a_{6,6}\big) \Big/ \big(-T + T^2 - T^2 a_{2,2} + a_{2,6} + T a_{6,2} -$$
$$a_{2,6} a_{6,2} - T a_{2,6} a_{6,2} - T^2 a_{2,6} a_{6,2} - T^2 a_{6,6} + a_{2,2} a_{6,6} + T a_{2,2} a_{6,6} + T^2 a_{2,2} a_{6,6}\big)\Big),$$
$$u_{2,6} \to -\Big(\big(-T + T a_{2,2} + T a_{6,2} + T a_{2,6} a_{6,2} + T a_{6,6} - T a_{2,2} a_{6,6}\big) \Big/ \big(-T + T^2 - T^2 a_{2,2} + a_{2,6} +$$
$$T a_{6,2} - a_{2,6} a_{6,2} - T a_{2,6} a_{6,2} - T^2 a_{2,6} a_{6,2} - T^2 a_{6,6} + a_{2,2} a_{6,6} + T a_{2,2} a_{6,6} + T^2 a_{2,2} a_{6,6}\big)\Big),$$
$$u_{6,2} \to \big(T \, (T - T a_{2,2} - a_{2,6} - T a_{2,6} a_{6,2} - T a_{6,6} + T a_{2,2} a_{6,6})\big) \Big/ \big(-T + T^2 - T^2 a_{2,2} + a_{2,6} +$$
$$T a_{6,2} - a_{2,6} a_{6,2} - T a_{2,6} a_{6,2} - T^2 a_{2,6} a_{6,2} - T^2 a_{6,6} + a_{2,2} a_{6,6} + T a_{2,2} a_{6,6} + T^2 a_{2,2} a_{6,6}\big),$$
$$u_{6,6} \to -\Big(\big(-T + T a_{2,2} + a_{2,6} + T a_{6,2} - a_{2,6} a_{6,2} + a_{2,2} a_{6,6}\big) \Big/ \big(T - T^2 + T^2 a_{2,2} - a_{2,6} - T a_{6,2} +$$
$$a_{2,6} a_{6,2} + T a_{2,6} a_{6,2} + T^2 a_{2,6} a_{6,2} + T^2 a_{6,6} - a_{2,2} a_{6,6} - T a_{2,2} a_{6,6} - T^2 a_{2,2} a_{6,6}\big)\Big)\Big\}\Big\}$$

$In[\circ]:=$ `Flatten@Table[`$u_{i,j}$ `→ Factor[`$u_{i,j}$ `/. sol], {i, S}, {j, S}]`

$Out[\circ]=$

$$\{u_{2,2} \to -\left(\left(-T + a_{2,6} + T\,a_{6,2} - a_{2,6}\,a_{6,2} + T\,a_{6,6} + a_{2,2}\,a_{6,6}\right) \Big/ \left(T - T^2 + T^2\,a_{2,2} - a_{2,6} - T\,a_{6,2} + \right.$$
$$\left.\left. a_{2,6}\,a_{6,2} + T\,a_{2,6}\,a_{6,2} + T^2\,a_{2,6}\,a_{6,2} + T^2\,a_{6,6} - a_{2,2}\,a_{6,6} - T\,a_{2,2}\,a_{6,6} - T^2\,a_{2,2}\,a_{6,6}\right)\right),$$
$$u_{2,6} \to \left(T\,\left(1 - a_{2,2} - a_{6,2} - a_{2,6}\,a_{6,2} - a_{6,6} + a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + T\,a_{6,2} - \right.$$
$$\left. a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right),$$
$$u_{6,2} \to \left(T\,\left(T - T\,a_{2,2} - a_{2,6} - T\,a_{2,6}\,a_{6,2} - T\,a_{6,6} + T\,a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + \right.$$
$$\left. T\,a_{6,2} - a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right),$$
$$u_{6,6} \to \left(-T + T\,a_{2,2} + a_{2,6} + T\,a_{6,2} - a_{2,6}\,a_{6,2} + a_{2,2}\,a_{6,6}\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + T\,a_{6,2} - \right.$$
$$\left. a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right)\}$$

$In[\circ]:=$ `Sum[Factor[`$u_{i,j}$ `/. sol]` $t_i\,h_j$`, {i, S}, {j, S}] // Together`

$Out[\circ]=$

$$\left(T\,h_2\,t_2 - T\,h_6\,t_2 - T^2\,h_2\,t_6 + T\,h_6\,t_6 + T\,h_6\,t_2\,a_{2,2} + T^2\,h_2\,t_6\,a_{2,2} - T\,h_6\,t_6\,a_{2,2} - \right.$$
$$h_2\,t_2\,a_{2,6} + T\,h_2\,t_6\,a_{2,6} - h_6\,t_6\,a_{2,6} - T\,h_2\,t_2\,a_{6,2} + T\,h_6\,t_2\,a_{6,2} - T\,h_6\,t_6\,a_{6,2} + h_2\,t_2\,a_{2,6}\,a_{6,2} +$$
$$T\,h_6\,t_2\,a_{2,6}\,a_{6,2} + T^2\,h_2\,t_6\,a_{2,6}\,a_{6,2} + h_6\,t_6\,a_{2,6}\,a_{6,2} - T\,h_2\,t_2\,a_{6,6} + T\,h_6\,t_2\,a_{6,6} +$$
$$\left. T^2\,h_2\,t_6\,a_{6,6} - h_2\,t_2\,a_{2,2}\,a_{6,6} - T\,h_6\,t_2\,a_{2,2}\,a_{6,6} - T^2\,h_2\,t_6\,a_{2,2}\,a_{6,6} - h_6\,t_6\,a_{2,2}\,a_{6,6}\right) \Big/$$
$$\left(T - T^2 + T^2\,a_{2,2} - a_{2,6} - T\,a_{6,2} + a_{2,6}\,a_{6,2} + T\,a_{2,6}\,a_{6,2} + T^2\,a_{2,6}\,a_{6,2} + \right.$$
$$\left. T^2\,a_{6,6} - a_{2,2}\,a_{6,6} - T\,a_{2,2}\,a_{6,6} - T^2\,a_{2,2}\,a_{6,6}\right)$$

$In[\circ]:=$
```
𝓛0[ς_] := Module[{S, us, eqns, sol},
  S = Cases[ς, t_i_ ⧴ i];
  us = Flatten@Table[u_{i,j}, {i, S}, {j, S}];
  eqns = Flatten@
    Table[⟨-t_i + Sum[a_{i,k} h_k, {k, S}], -t_j + Sum[u_{j,k} h_k, {k, S}]⟩_ς == 0, {i, S}, {j, S}];
  {sol} = Solve[eqns, us];
  Sum[Factor[u_{i,j} /. sol] t_i h_j, {i, S}, {j, S}]
  ]
```

$In[\circ]:=$ `𝓛0[{`t_2, h_2, t_6, h_6`}]`

$Out[\circ]=$

$$-\left(\left(h_2\,t_2\,\left(-T + a_{2,6} + T\,a_{6,2} - a_{2,6}\,a_{6,2} + T\,a_{6,6} + a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(T - T^2 + T^2\,a_{2,2} - a_{2,6} - T\,a_{6,2} + \right.\right.$$
$$\left.\left. a_{2,6}\,a_{6,2} + T\,a_{2,6}\,a_{6,2} + T^2\,a_{2,6}\,a_{6,2} + T^2\,a_{6,6} - a_{2,2}\,a_{6,6} - T\,a_{2,2}\,a_{6,6} - T^2\,a_{2,2}\,a_{6,6}\right)\right) +$$
$$\left(h_6\,t_6\,\left(-T + T\,a_{2,2} + a_{2,6} + T\,a_{6,2} - a_{2,6}\,a_{6,2} + a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + T\,a_{6,2} - \right.$$
$$\left. a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right) +$$
$$\left(T\,h_6\,t_2\,\left(1 - a_{2,2} - a_{6,2} - a_{2,6}\,a_{6,2} - a_{6,6} + a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + T\,a_{6,2} - \right.$$
$$\left. a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right) +$$
$$\left(T\,h_2\,t_6\,\left(T - T\,a_{2,2} - a_{2,6} - T\,a_{2,6}\,a_{6,2} - T\,a_{6,6} + T\,a_{2,2}\,a_{6,6}\right)\right) \Big/ \left(-T + T^2 - T^2\,a_{2,2} + a_{2,6} + \right.$$
$$\left. T\,a_{6,2} - a_{2,6}\,a_{6,2} - T\,a_{2,6}\,a_{6,2} - T^2\,a_{2,6}\,a_{6,2} - T^2\,a_{6,6} + a_{2,2}\,a_{6,6} + T\,a_{2,2}\,a_{6,6} + T^2\,a_{2,2}\,a_{6,6}\right)$$

$In[\circ]:=$ $\mathcal{L}0[\{t_2, t_6, h_2, h_6\}]$

$Out[\circ]=$

$$-\left(\left(h_2\, t_2\, (-T + a_{2,6} + T\, a_{6,2} - a_{2,6}\, a_{6,2} + T\, a_{6,6} + a_{2,2}\, a_{6,6})\right) \Big/ \left(T\, a_{2,2} - T^2\, a_{2,6} - T\, a_{6,2} + \right.\right.$$
$$\left.\left. a_{2,6}\, a_{6,2} + T\, a_{2,6}\, a_{6,2} + T^2\, a_{2,6}\, a_{6,2} + T^2\, a_{6,6} - a_{2,2}\, a_{6,6} - T\, a_{2,2}\, a_{6,6} - T^2\, a_{2,2}\, a_{6,6}\right)\right) -$$
$$\left(h_2\, t_6\, \left(-T^2 + T\, a_{2,2} + T\, a_{2,6} - T\, a_{6,2} + T^2\, a_{2,6} + a_{2,6}\, a_{6,2} + T^2\, a_{6,6} - a_{2,2}\, a_{6,6}\right)\right) \Big/$$
$$\left(-T\, a_{2,2} + T^2\, a_{2,6} + T\, a_{6,2} - a_{2,6}\, a_{6,2} - T\, a_{2,6}\, a_{6,2} - \right.$$
$$\left. T^2\, a_{2,6}\, a_{6,2} - T^2\, a_{6,6} + a_{2,2}\, a_{6,6} + T\, a_{2,2}\, a_{6,6} + T^2\, a_{2,2}\, a_{6,6}\right) +$$
$$\left(T\, h_6\, t_6\, (-T + a_{2,2} + a_{2,6} + T\, a_{6,2} - a_{2,6}\, a_{6,2} + a_{2,2}\, a_{6,6})\right) \Big/ \left(-T\, a_{2,2} + T^2\, a_{2,6} + T\, a_{6,2} - \right.$$
$$\left. a_{2,6}\, a_{6,2} - T\, a_{2,6}\, a_{6,2} - T^2\, a_{2,6}\, a_{6,2} - T^2\, a_{6,6} + a_{2,2}\, a_{6,6} + T\, a_{2,2}\, a_{6,6} + T^2\, a_{2,2}\, a_{6,6}\right) +$$
$$\left(h_6\, t_2\, (T - a_{2,2} - a_{2,6} + T\, a_{2,6} - T\, a_{6,2} - T\, a_{2,6}\, a_{6,2} - T\, a_{6,6} + T\, a_{2,2}\, a_{6,6})\right) \Big/ \left(-T\, a_{2,2} + T^2\, a_{2,6} + \right.$$
$$\left.\left. T\, a_{6,2} - a_{2,6}\, a_{6,2} - T\, a_{2,6}\, a_{6,2} - T^2\, a_{2,6}\, a_{6,2} - T^2\, a_{6,6} + a_{2,2}\, a_{6,6} + T\, a_{2,2}\, a_{6,6} + T^2\, a_{2,2}\, a_{6,6}\right)\right.$$

$In[\circ]:=$ $\mathcal{L}0_{\varsigma\_}[\lambda\_] := \text{Factor}\left[\mathcal{L}0[\varsigma]\ /.\ a_{i\_,j\_} \mapsto \partial_{t_i, h_j}\lambda\right]$

$In[\circ]:=$ $\gamma 0 = \overline{R}_{12,1}\ //\ \text{Insert}\left[\text{RotateLeft}\left[\overline{R}_{2,7}, 1\right], 4\right]\ //\ dm_{1,2\to1}\ //\ \text{Insert}[\text{RotateLeft}[R_{6,13}, 1], 7]\ //$
$\qquad dm_{12,13\to12}\ //\ dm_{6,7\to6}\ //\ \text{Insert}\left[\overline{R}_{8,3}, 5\right]\ //\ dm_{1,3\to1}\ //\ dm_{6,8\to6}\ //\ \text{Insert}[R_{14,9}, 6]\ //$
$\qquad dm_{12,14\to12}\ //\ dm_{6,9\to6}\ //\ \text{Insert}\left[\text{RotateLeft}\left[\overline{R}_{4,11}, 1\right], 4\right]\ //\ dm_{1,4\to1}\ //\ dm_{11,12\to11}$

$Out[\circ]=$

$\{t_1, h_1, t_{11}, h_{11}, h_6, t_6\}$

	h_1	h_6
$\frac{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T_{11}+T_1^2T_{11}+T_1T_6T_{11}}{T_1^2T_6T_{11}}$		
t_1	$\frac{(1-T_1+T_1T_6)\,T_{11}}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T_{11}+T_1^2T_{11}+T_1T_6T_{11}}$	$\frac{(-1+T_1)\,(1-T_1+T_1T_6}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T}$
t_6	$\frac{(1-T_1+T_1T_6)\,(-1+T_{11})}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T_{11}+T_1^2T_{11}+T_1T_6T_{11}}$	$\frac{T_1^2T_6T_{11}}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T}$
t_{11}	$\frac{T_1\,(-1+T_6)\,T_6T_{11}}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1T_{11}+T_1^2T_{11}+T_1T_6T_{11}}$	$-\frac{(-1+T_6)\,T_{11}\,(-1+2T_1-T_1^2+T_{11}-2T_1}{-1+2T_1-T_1^2-T_1T_6+T_1^2T_6+T_{11}-2T_1}$
🟩	0	0

$In[\circ]:=$ $\text{lhs} = \mathcal{L}0_{\gamma 0[\varsigma]}\left[\gamma 0[\lambda]\ /.\ T\_ \to T\right]$

$Out[\circ]=$

$$\frac{1}{T^2\left(-3+4T-3T^2+T^3\right)}$$
$$\left(-T^2 h_1 t_1 + T^3 h_1 t_1 - T^4 h_1 t_1 + T h_6 t_1 - 4 T^2 h_6 t_1 + 5 T^3 h_6 t_1 - 3 T^4 h_6 t_1 + T^5 h_6 t_1 - T h_{11} t_1 + \right.$$
$$2 T^2 h_{11} t_1 - 2 T^3 h_{11} t_1 + T^4 h_{11} t_1 - T h_1 t_6 + T^2 h_1 t_6 - 2 h_6 t_6 + 6 T h_6 t_6 - 10 T^2 h_6 t_6 + 8 T^3 h_6 t_6 -$$
$$4 T^4 h_6 t_6 + T^5 h_6 t_6 + 2 h_{11} t_6 - 5 T h_{11} t_6 + 6 T^2 h_{11} t_6 - 4 T^3 h_{11} t_6 + T^4 h_{11} t_6 - T^2 h_1 t_{11} +$$
$$\left. 2 T^3 h_1 t_{11} - 2 T^4 h_1 t_{11} + T^5 h_1 t_{11} + T h_6 t_{11} - 2 T^2 h_6 t_{11} + 2 T^3 h_6 t_{11} - T^4 h_6 t_{11} - T h_{11} t_{11}\right)$$

$In[\circ]:=$ **rhs = Factor$\left[\gamma0[\lambda] \text{ /. } T_{\_} \to T^{-1}\right]$**

$Out[\circ]=$

$$\frac{1}{T^2 \left(-3 + 4T - 3T^2 + T^3\right)}$$

$\left(-T^2 h_1 t_1 + T^3 h_1 t_1 - T^4 h_1 t_1 + T h_6 t_1 - 4 T^2 h_6 t_1 + 5 T^3 h_6 t_1 - 3 T^4 h_6 t_1 + T^5 h_6 t_1 - T h_{11} t_1 +$
$2 T^2 h_{11} t_1 - 2 T^3 h_{11} t_1 + T^4 h_{11} t_1 - T h_1 t_6 + T^2 h_1 t_6 - 2 h_6 t_6 + 6 T h_6 t_6 - 10 T^2 h_6 t_6 + 8 T^3 h_6 t_6 -$
$4 T^4 h_6 t_6 + T^5 h_6 t_6 + 2 h_{11} t_6 - 5 T h_{11} t_6 + 6 T^2 h_{11} t_6 - 4 T^3 h_{11} t_6 + T^4 h_{11} t_6 - T^2 h_1 t_{11} +$
$2 T^3 h_1 t_{11} - 2 T^4 h_1 t_{11} + T^5 h_1 t_{11} + T h_6 t_{11} - 2 T^2 h_6 t_{11} + 2 T^3 h_6 t_{11} - T^4 h_6 t_{11} - T h_{11} t_{11}\right)$

$In[\circ]:=$ **lhs == rhs**

$Out[\circ]=$

True

$In[\circ]:=$ **Together$[\mathcal{L}0@\{t_1, h_1, t_{11}, h_{11}, h_6, t_6\}]$**

$Out[\circ]=$

$\big(T h_6 t_1 - T^2 h_6 t_1 - T h_{11} t_1 + T^2 h_{11} t_1 - T h_6 t_6 + T^2 h_6 t_6 + T h_{11} t_6 - T^2 h_{11} t_6 - T h_6 t_1 a_{1,1} + T^2 h_6 t_1 a_{1,1} +$
$T h_{11} t_1 a_{1,1} - T^2 h_{11} t_1 a_{1,1} - T^2 h_6 t_6 a_{1,1} + T^2 h_{11} t_6 a_{1,1} + T h_6 t_{11} a_{1,1} - T h_{11} t_{11} a_{1,1} + T h_1 t_1 a_{1,6} -$
$T^2 h_1 t_1 a_{1,6} + T^2 h_1 t_6 a_{1,6} + T h_6 t_6 a_{1,6} - T^2 h_6 t_6 a_{1,6} - T h_{11} t_6 a_{1,6} - T^2 h_1 t_{11} a_{1,6} + T h_{11} t_{11} a_{1,6} -$
$h_1 t_1 a_{1,11} + T h_1 t_1 a_{1,11} - T h_1 t_6 a_{1,11} + T h_6 t_6 a_{1,11} + T h_1 t_{11} a_{1,11} - T h_6 t_{11} a_{1,11} - h_{11} t_{11} a_{1,11} +$
$T h_{11} t_{11} a_{1,11} + T^2 h_6 t_1 a_{6,1} - T^2 h_{11} t_1 a_{6,1} - T^2 h_6 t_{11} a_{6,1} + T^2 h_{11} t_{11} a_{6,1} - T h_6 t_1 a_{1,6} a_{6,1} +$
$T^2 h_6 t_1 a_{1,6} a_{6,1} + T h_{11} t_1 a_{1,6} a_{6,1} - T^2 h_6 t_6 a_{1,6} a_{6,1} + T^3 h_6 t_6 a_{1,6} a_{6,1} + T^2 h_{11} t_6 a_{1,6} a_{6,1} +$
$T^3 h_1 t_{11} a_{1,6} a_{6,1} + T h_6 t_{11} a_{1,6} a_{6,1} - T h_{11} t_{11} a_{1,6} a_{6,1} - T^2 h_{11} t_{11} a_{1,6} a_{6,1} - T h_6 t_1 a_{1,11} a_{6,1} -$
$T^2 h_6 t_6 a_{1,11} a_{6,1} - T^2 h_1 t_{11} a_{1,11} a_{6,1} + T h_6 t_{11} a_{1,11} a_{6,1} + T^2 h_6 t_{11} a_{1,11} a_{6,1} - T^2 h_{11} t_{11} a_{1,11} a_{6,1} -$
$T^2 h_1 t_1 a_{6,6} - T h_6 t_1 a_{6,6} + T^2 h_6 t_1 a_{6,6} + T h_{11} t_1 a_{6,6} + T h_6 t_6 a_{6,6} - 2 T^2 h_6 t_6 a_{6,6} + T^3 h_6 t_6 a_{6,6} -$
$T h_{11} t_6 a_{6,6} + T^2 h_{11} t_6 a_{6,6} + T^3 h_1 t_{11} a_{6,6} - T^2 h_{11} t_{11} a_{6,6} + T h_6 t_1 a_{1,1} a_{6,6} - T^2 h_6 t_1 a_{1,1} a_{6,6} -$
$T h_{11} t_1 a_{1,1} a_{6,6} + T^2 h_6 t_6 a_{1,1} a_{6,6} - T^3 h_6 t_6 a_{1,1} a_{6,6} - T^2 h_{11} t_6 a_{1,1} a_{6,6} - T^3 h_1 t_{11} a_{1,1} a_{6,6} -$
$T h_6 t_{11} a_{1,1} a_{6,6} + T h_{11} t_{11} a_{1,1} a_{6,6} + T^2 h_{11} t_{11} a_{1,1} a_{6,6} + h_1 t_1 a_{1,11} a_{6,6} + T h_1 t_6 a_{1,11} a_{6,6} -$
$T h_1 t_{11} a_{1,11} a_{6,6} - T^2 h_1 t_{11} a_{1,11} a_{6,6} + T h_6 t_{11} a_{1,11} a_{6,6} + h_{11} t_{11} a_{1,11} a_{6,6} + T h_1 t_1 a_{6,11} -$
$T h_6 t_1 a_{6,11} + T h_6 t_6 a_{6,11} - T^2 h_6 t_6 a_{6,11} - T^2 h_1 t_{11} a_{6,11} + T^2 h_6 t_{11} a_{6,11} + T h_{11} t_{11} a_{6,11} -$
$T^2 h_{11} t_{11} a_{6,11} + T h_6 t_1 a_{1,1} a_{6,11} + T^2 h_6 t_6 a_{1,1} a_{6,11} + T^2 h_1 t_{11} a_{1,1} a_{6,11} - T h_6 t_{11} a_{1,1} a_{6,11} -$
$T^2 h_6 t_{11} a_{1,1} a_{6,11} + T^2 h_{11} t_{11} a_{1,1} a_{6,11} - h_1 t_1 a_{1,6} a_{6,11} - T h_1 t_6 a_{1,6} a_{6,11} + T h_1 t_{11} a_{1,6} a_{6,11} +$
$T^2 h_1 t_{11} a_{1,6} a_{6,11} - T h_6 t_{11} a_{1,6} a_{6,11} - h_{11} t_{11} a_{1,6} a_{6,11} - T h_6 t_1 a_{11,1} + T h_{11} t_1 a_{11,1} + T h_6 t_6 a_{11,1} -$
$T h_{11} t_6 a_{11,1} - T h_1 t_1 a_{1,6} a_{11,1} + T^2 h_1 t_1 a_{1,6} a_{11,1} - T^2 h_{11} t_1 a_{1,6} a_{11,1} - T^2 h_1 t_6 a_{1,6} a_{11,1} -$
$T h_6 t_6 a_{1,6} a_{11,1} + T h_{11} t_6 a_{1,6} a_{11,1} + T^2 h_{11} t_6 a_{1,6} a_{11,1} - T h_{11} t_{11} a_{1,6} a_{11,1} + h_1 t_1 a_{1,11} a_{11,1} -$
$T h_1 t_1 a_{1,11} a_{11,1} + T^2 h_6 t_1 a_{1,11} a_{11,1} + T h_{11} t_1 a_{1,11} a_{11,1} - T^2 h_{11} t_1 a_{1,11} a_{11,1} + T h_1 t_6 a_{1,11} a_{11,1} -$
$T h_6 t_6 a_{1,11} a_{11,1} - T^2 h_6 t_6 a_{1,11} a_{11,1} + T^2 h_{11} t_6 a_{1,11} a_{11,1} + T h_6 t_{11} a_{1,11} a_{11,1} + h_{11} t_{11} a_{1,11} a_{11,1} -$
$T h_{11} t_{11} a_{1,11} a_{11,1} + T^2 h_1 t_1 a_{6,6} a_{11,1} + T h_6 t_1 a_{6,6} a_{11,1} - T h_{11} t_1 a_{6,6} a_{11,1} - T^2 h_{11} t_1 a_{6,6} a_{11,1} -$
$T h_6 t_6 a_{6,6} a_{11,1} + T^2 h_6 t_6 a_{6,6} a_{11,1} + T h_{11} t_6 a_{6,6} a_{11,1} + T^2 h_{11} t_{11} a_{6,6} a_{11,1} - h_1 t_1 a_{1,11} a_{6,6} a_{11,1} -$
$T^2 h_6 t_1 a_{1,11} a_{6,6} a_{11,1} - T h_{11} t_1 a_{1,11} a_{6,6} a_{11,1} - T h_1 t_6 a_{1,11} a_{6,6} a_{11,1} - T^3 h_6 t_6 a_{1,11} a_{6,6} a_{11,1} -$
$T^2 h_{11} t_6 a_{1,11} a_{6,6} a_{11,1} - T^3 h_1 t_{11} a_{1,11} a_{6,6} a_{11,1} - T h_6 t_{11} a_{1,11} a_{6,6} a_{11,1} - h_{11} t_{11} a_{1,11} a_{6,6} a_{11,1} -$
$T h_1 t_1 a_{6,11} a_{11,1} + T h_6 t_1 a_{6,11} a_{11,1} + T^2 h_6 t_1 a_{6,11} a_{11,1} - T^2 h_{11} t_1 a_{6,11} a_{11,1} - T h_6 t_6 a_{6,11} a_{11,1} -$
$T^2 h_6 t_{11} a_{6,11} a_{11,1} - T h_{11} t_{11} a_{6,11} a_{11,1} + T^2 h_{11} t_{11} a_{6,11} a_{11,1} + h_1 t_1 a_{1,6} a_{6,11} a_{11,1} +$
$T^2 h_6 t_1 a_{1,6} a_{6,11} a_{11,1} + T h_{11} t_1 a_{1,6} a_{6,11} a_{11,1} + T h_1 t_6 a_{1,6} a_{6,11} a_{11,1} + T^3 h_6 t_6 a_{1,6} a_{6,11} a_{11,1} +$
$T^2 h_{11} t_6 a_{1,6} a_{6,11} a_{11,1} + T^3 h_1 t_{11} a_{1,6} a_{6,11} a_{11,1} + T h_6 t_{11} a_{1,6} a_{6,11} a_{11,1} + h_{11} t_{11} a_{1,6} a_{6,11} a_{11,1} +$

$$T^2 h_1 t_1 a_{11,6} - T^2 h_{11} t_1 a_{11,6} - T^2 h_1 t_6 a_{11,6} + T^2 h_{11} t_6 a_{11,6} + T h_1 t_1 a_{1,1} a_{11,6} - T^2 h_1 t_1 a_{1,1} a_{11,6} +$$
$$T^2 h_{11} t_1 a_{1,1} a_{11,6} + T^2 h_1 t_6 a_{1,1} a_{11,6} + T h_6 t_6 a_{1,1} a_{11,6} - T h_{11} t_6 a_{1,1} a_{11,6} - T^2 h_{11} t_6 a_{1,1} a_{11,6} +$$
$$T h_{11} t_{11} a_{1,1} a_{11,6} - T^2 h_1 t_1 a_{1,11} a_{11,6} + T h_1 t_6 a_{1,11} a_{11,6} + T^2 h_1 t_6 a_{1,11} a_{11,6} - T^2 h_6 t_6 a_{1,11} a_{11,6} -$$
$$T h_{11} t_6 a_{1,11} a_{11,6} - T^2 h_1 t_{11} a_{1,11} a_{11,6} - T^2 h_1 t_1 a_{6,1} a_{11,6} - T h_6 t_1 a_{6,1} a_{11,6} + T h_{11} t_1 a_{6,1} a_{11,6} +$$
$$T^2 h_{11} t_1 a_{6,1} a_{11,6} + T h_6 t_6 a_{6,1} a_{11,6} - T^2 h_6 t_6 a_{6,1} a_{11,6} - T h_{11} t_6 a_{6,1} a_{11,6} - T^2 h_{11} t_{11} a_{6,1} a_{11,6} +$$
$$h_1 t_1 a_{1,11} a_{6,1} a_{11,6} + T^2 h_6 t_1 a_{1,11} a_{6,1} a_{11,6} + T h_{11} t_1 a_{1,11} a_{6,1} a_{11,6} + T h_1 t_6 a_{1,11} a_{6,1} a_{11,6} +$$
$$T^3 h_6 t_6 a_{1,11} a_{6,1} a_{11,6} + T^2 h_{11} t_6 a_{1,11} a_{6,1} a_{11,6} + T^3 h_1 t_{11} a_{1,11} a_{6,1} a_{11,6} + T h_6 t_{11} a_{1,11} a_{6,1} a_{11,6} +$$
$$h_{11} t_{11} a_{1,11} a_{6,1} a_{11,6} - T h_1 t_1 a_{6,11} a_{11,6} - T^2 h_1 t_1 a_{6,11} a_{11,6} + T^2 h_6 t_1 a_{6,11} a_{11,6} +$$
$$T h_{11} t_1 a_{6,11} a_{11,6} + T h_1 t_6 a_{6,11} a_{11,6} - T^2 h_6 t_6 a_{6,11} a_{11,6} + T^3 h_6 t_6 a_{6,11} a_{11,6} - T h_{11} t_6 a_{6,11} a_{11,6} +$$
$$T^2 h_{11} t_6 a_{6,11} a_{11,6} + T^3 h_1 t_{11} a_{6,11} a_{11,6} - h_1 t_1 a_{1,1} a_{6,11} a_{11,6} - T^2 h_6 t_1 a_{1,1} a_{6,11} a_{11,6} -$$
$$T h_{11} t_1 a_{1,1} a_{6,11} a_{11,6} - T h_1 t_6 a_{1,1} a_{6,11} a_{11,6} - T^3 h_6 t_6 a_{1,1} a_{6,11} a_{11,6} - T^2 h_{11} t_6 a_{1,1} a_{6,11} a_{11,6} -$$
$$T^3 h_1 t_{11} a_{1,1} a_{6,11} a_{11,6} - T h_6 t_{11} a_{1,1} a_{6,11} a_{11,6} - h_{11} t_{11} a_{1,1} a_{6,11} a_{11,6} - T h_1 t_1 a_{11,11} +$$
$$T^2 h_6 t_1 a_{11,11} + T h_{11} t_1 a_{11,11} - T^2 h_{11} t_1 a_{11,11} + T h_1 t_6 a_{11,11} - T^2 h_6 t_6 a_{11,11} - T h_{11} t_6 a_{11,11} +$$
$$T^2 h_{11} t_6 a_{11,11} - h_1 t_1 a_{1,1} a_{11,11} + T h_1 t_1 a_{1,1} a_{11,11} - T^2 h_6 t_1 a_{1,1} a_{11,11} - T h_{11} t_1 a_{1,1} a_{11,11} +$$
$$T^2 h_{11} t_1 a_{1,1} a_{11,11} - T h_1 t_6 a_{1,1} a_{11,11} + T h_6 t_6 a_{1,1} a_{11,11} + T^2 h_6 t_6 a_{1,1} a_{11,11} - T^2 h_{11} t_6 a_{1,1} a_{11,11} -$$
$$T h_6 t_{11} a_{1,1} a_{11,11} - h_{11} t_{11} a_{1,1} a_{11,11} + T h_{11} t_{11} a_{1,1} a_{11,11} + T^2 h_1 t_1 a_{1,6} a_{11,11} - T h_1 t_6 a_{1,6} a_{11,11} -$$
$$T^2 h_1 t_6 a_{1,6} a_{11,11} + T^2 h_6 t_6 a_{1,6} a_{11,11} + T h_{11} t_6 a_{1,6} a_{11,11} + T^2 h_1 t_{11} a_{1,6} a_{11,11} + T h_1 t_1 a_{6,1} a_{11,11} -$$
$$T h_6 t_1 a_{6,1} a_{11,11} - T^2 h_6 t_1 a_{6,1} a_{11,11} + T^2 h_{11} t_1 a_{6,1} a_{11,11} + T h_6 t_6 a_{6,1} a_{11,11} + T^2 h_6 t_{11} a_{6,1} a_{11,11} +$$
$$T h_{11} t_{11} a_{6,1} a_{11,11} - T^2 h_{11} t_{11} a_{6,1} a_{11,11} - h_1 t_1 a_{1,6} a_{6,1} a_{11,11} - T^2 h_6 t_1 a_{1,6} a_{6,1} a_{11,11} -$$
$$T h_{11} t_1 a_{1,6} a_{6,1} a_{11,11} - T h_1 t_6 a_{1,6} a_{6,1} a_{11,11} - T^3 h_6 t_6 a_{1,6} a_{6,1} a_{11,11} - T^2 h_{11} t_6 a_{1,6} a_{6,1} a_{11,11} -$$
$$T^3 h_1 t_{11} a_{1,6} a_{6,1} a_{11,11} - T h_6 t_{11} a_{1,6} a_{6,1} a_{11,11} - h_{11} t_{11} a_{1,6} a_{6,1} a_{11,11} + T h_1 t_1 a_{6,6} a_{11,11} +$$
$$T^2 h_1 t_1 a_{6,6} a_{11,11} - T^2 h_6 t_1 a_{6,6} a_{11,11} - T h_{11} t_1 a_{6,6} a_{11,11} - T h_1 t_6 a_{6,6} a_{11,11} + T^2 h_6 t_6 a_{6,6} a_{11,11} -$$
$$T^3 h_6 t_6 a_{6,6} a_{11,11} + T h_{11} t_6 a_{6,6} a_{11,11} - T^2 h_{11} t_6 a_{6,6} a_{11,11} - T^3 h_1 t_{11} a_{6,6} a_{11,11} + h_1 t_1 a_{1,1} a_{6,6} a_{11,11} +$$
$$T^2 h_6 t_1 a_{1,1} a_{6,6} a_{11,11} + T h_{11} t_1 a_{1,1} a_{6,6} a_{11,11} + T h_1 t_6 a_{1,1} a_{6,6} a_{11,11} + T^3 h_6 t_6 a_{1,1} a_{6,6} a_{11,11} +$$
$$\left. T^2 h_{11} t_6 a_{1,1} a_{6,6} a_{11,11} + T^3 h_1 t_{11} a_{1,1} a_{6,6} a_{11,11} + T h_6 t_{11} a_{1,1} a_{6,6} a_{11,11} + h_{11} t_{11} a_{1,1} a_{6,6} a_{11,11} \right) \Big/$$
$$\left(T a_{1,6} - T^2 a_{1,6} - a_{1,11} + T a_{1,11} + T^3 a_{1,6} a_{6,1} - T^2 a_{1,11} a_{6,1} - T^2 a_{6,6} + T^3 a_{6,6} - T^3 a_{1,1} a_{6,6} + \right.$$
$$a_{1,11} a_{6,6} + T a_{6,11} - T^2 a_{6,11} + T^2 a_{1,1} a_{6,11} - a_{1,6} a_{6,11} - T a_{1,6} a_{11,1} + a_{1,11} a_{11,1} + T^2 a_{6,6} a_{11,1} -$$
$$a_{1,11} a_{6,6} a_{11,1} - T a_{1,11} a_{6,6} a_{11,1} - T^2 a_{1,11} a_{6,6} a_{11,1} - T^3 a_{1,11} a_{6,6} a_{11,1} - T a_{6,11} a_{11,1} + a_{1,6} a_{6,11} a_{11,1} +$$
$$T a_{1,6} a_{6,11} a_{11,1} + T^2 a_{1,6} a_{6,11} a_{11,1} + T^3 a_{1,6} a_{6,11} a_{11,1} + T a_{1,1} a_{11,6} - T^2 a_{1,11} a_{11,6} - T^2 a_{6,1} a_{11,6} +$$
$$a_{1,11} a_{6,1} a_{11,6} + T a_{1,11} a_{6,1} a_{11,6} + T^2 a_{1,11} a_{6,1} a_{11,6} + T^3 a_{1,11} a_{6,1} a_{11,6} + T^3 a_{6,11} a_{11,6} -$$
$$a_{1,1} a_{6,11} a_{11,6} - T a_{1,1} a_{6,11} a_{11,6} - T^2 a_{1,1} a_{6,11} a_{11,6} - T^3 a_{1,1} a_{6,11} a_{11,6} - a_{1,1} a_{11,11} + T^2 a_{1,6} a_{11,11} +$$
$$T a_{6,1} a_{11,11} - a_{1,6} a_{6,1} a_{11,11} - T a_{1,6} a_{6,1} a_{11,11} - T^2 a_{1,6} a_{6,1} a_{11,11} - T^3 a_{1,6} a_{6,1} a_{11,11} -$$
$$\left. T^3 a_{6,6} a_{11,11} + a_{1,1} a_{6,6} a_{11,11} + T a_{1,1} a_{6,6} a_{11,11} + T^2 a_{1,1} a_{6,6} a_{11,11} + T^3 a_{1,1} a_{6,6} a_{11,11} \right)$$