Pensieve header: Verifying the full set of KV equations in Γ-calculus.

KV solutions probably coming from pensieve://2012-05/beta5.1/betaExact.nb.

In[∘]:=
```
SetDirectory["C:/drorbn/AcademicPensieve/Projects/MetaCalculi/"];
<< "MetaCalculi.m"
```

```
MetaCalculi` loading...
```

In[∘]:=
```
ΓSimp = Assuming[T1 > 1 ∧ T2 > 1 ∧ T3 > 1, Simplify[PowerExpand[#]]] &;
R[i_, j_] := Γ[Xp[i, j]];
```

In[∘]:=
```
{V0 = Γ[V], V0i = Γ[Vi], V0 ** V0i} // ColumnForm
```

Out[∘]=

$$\begin{pmatrix} \frac{\left(\frac{-1+T_1}{\mathrm{Log}[T_1]}\right)^{1/4}\left(\frac{-1+T_2}{\mathrm{Log}[T_2]}\right)^{1/4}}{\left(\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \frac{\mathrm{Log}[T_1]+\sqrt{\frac{\mathrm{Log}[T_1]\,\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\mathrm{Log}[T_1 T_2]} & \frac{\mathrm{Log}[T_1]-T_2\sqrt{\frac{\mathrm{Log}[T_1]\,\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\mathrm{Log}[T_1 T_2]} \\ s_2 & \frac{\mathrm{Log}[T_2]-\sqrt{\frac{\mathrm{Log}[T_1]\,\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\mathrm{Log}[T_1 T_2]} & \frac{\mathrm{Log}[T_2]+T_2\sqrt{\frac{\mathrm{Log}[T_1]\,\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\mathrm{Log}[T_1 T_2]} \\ \Gamma & 1 & \sqrt{T_1} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\left(\frac{-1+T_1 T_2}{\mathrm{Log}[T_1 T_2]}\right)^{1/4}}{\left(\frac{-1+T_1}{\mathrm{Log}[T_1]}\right)^{1/4}\left(\frac{-1+T_2}{\mathrm{Log}[T_2]}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \frac{(-1+T_1)\,T_2+\frac{\sqrt{\mathrm{Log}[T_2]\,(-1+T_1)\,(-1+T_2)\,(-1+T_1 T_2)}}{\sqrt{\mathrm{Log}[T_1]\,\mathrm{Log}[T_1 T_2]\,T_1}}}{-1+T_1 T_2} & \frac{(-1+T_1)\,T_2-\frac{\sqrt{\mathrm{Log}[T_1]\,(-1+T_1)\,(-1+T_2)\,(-1+T_1 T_2)}}{\sqrt{\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,T_1}}}{-1+T_1 T_2} \\ s_2 & \frac{-1+T_2-\frac{\sqrt{\mathrm{Log}[T_2]\,(-1+T_1)\,(-1+T_2)\,(-1+T_1 T_2)}}{\sqrt{\mathrm{Log}[T_1]\,\mathrm{Log}[T_1 T_2]\,T_1}}}{-1+T_1 T_2} & \frac{-1+T_2+\frac{\sqrt{\mathrm{Log}[T_1]\,(-1+T_1)\,(-1+T_2)\,(-1+T_1 T_2)}}{\sqrt{\mathrm{Log}[T_2]\,\mathrm{Log}[T_1 T_2]\,T_1}}}{-1+T_1 T_2} \\ \Gamma & 1 & \frac{1}{\sqrt{T_1}} \end{pmatrix}$$

$$\begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & 1 & 0 \\ s_2 & 0 & 1 \\ \Gamma & 1 & 1 \end{pmatrix}$$

In[∘]:= **MatrixForm@**

$$\text{Simplify}\left[\begin{pmatrix} \dfrac{\left(\frac{-1+T_1}{\text{Log}[T_1]}\right)^{1/4}\left(\frac{-1+T_2}{\text{Log}[T_2]}\right)^{1/4}}{\left(\frac{-1+T_1 T_2}{\text{Log}[T_1 T_2]}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \dfrac{\text{Log}[T_1]+\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]} & \dfrac{\text{Log}[T_1]-T_2\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]} \\ s_2 & \dfrac{\text{Log}[T_2]-\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]} & \dfrac{\text{Log}[T_2]+T_2\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]} \\ \text{"}\Gamma\text{"} & 1 & \sqrt{T_1} \end{pmatrix} /.\right.$$

$$\left.\text{Log}[x\_] :\to \text{PowerExpand@Log}\left[x\ /.\ T_{i\_}^{p\_.} \to e^{p\,t_i}\right]\right]$$

Out[∘]//MatrixForm=

$$\begin{pmatrix} \dfrac{\left(\frac{-1+T_1}{t_1}\right)^{1/4}\left(\frac{-1+T_2}{t_2}\right)^{1/4}}{\left(\frac{-1+T_1 T_2}{t_1+t_2}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \dfrac{t_1+\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{t_1+t_2} & \dfrac{t_1-T_2\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{t_1+t_2} \\ s_2 & \dfrac{t_2-\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{t_1+t_2} & \dfrac{t_2+T_2\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{t_1+t_2} \\ \Gamma & 1 & \sqrt{T_1} \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{\left(\frac{T_1-1}{t_1}\right)^{\frac{1}{4}}\left(\frac{T_2-1}{t_2}\right)^{\frac{1}{4}}}{\left(\frac{T_1 T_2-1}{t_1+t_2}\right)^{\frac{1}{4}}} & s_1 & s_2 \\ s_1 & \dfrac{t_1+\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,T_1\,(T_2-1)}{(T_1-1)\,(T_1 T_2-1)}}}{t_1+t_2} & \dfrac{t_1-T_2\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,(T_1-1)\,T_1}{(T_2-1)\,(T_1 T_2-1)}}}{t_1+t_2} \\ s_2 & \dfrac{t_2-\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,T_1\,(T_2-1)}{(T_1-1)\,(T_1 T_2-1)}}}{t_1+t_2} & \dfrac{t_2+T_2\sqrt{\frac{t_1 t_2\,(t_1+t_2)\,(T_1-1)\,T_1}{(T_2-1)\,(T_1 T_2-1)}}}{t_1+t_2} \\ \Gamma & 1 & \sqrt{T_1} \end{pmatrix}$$

Testing dΔ

```
{γ0 = Γ[ω, h_a σ_a + h_z σ_z, {t_a, t_z}.(α θ; ϕ Ξ).{h_a, h_z}],
 t1 = γ0 // A // dΔ[a, x, y] // Γ,
 t2 = γ0 // dΔ[a, x, y],
 t1 == t2 // Simplify}
```

$$\left\{\begin{pmatrix} \omega & s_a & s_z \\ s_a & \alpha & \theta \\ s_z & \phi & \Xi \\ \Gamma & \sigma_a & \sigma_z \end{pmatrix}, \begin{pmatrix} \omega & s_x & s_y & s_z \\ s_x & 1-\phi+\frac{\text{Log}[T_y]\,(-\alpha+\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\text{Log}[T_x]\,(\alpha-\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\theta\,\text{Log}[T_x]}{\text{Log}[T_x]+\text{Log}[T_y]} \\ s_y & \frac{\text{Log}[T_y]\,(\alpha-\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & 1-\phi+\frac{\text{Log}[T_x]\,(-\alpha+\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\theta\,\text{Log}[T_y]}{\text{Log}[T_x]+\text{Log}[T_y]} \\ s_z & \phi & \phi & 1-\theta \\ \Gamma & 1-\alpha-\phi+\sigma_a & 1-\alpha-\phi+\sigma_a & 1-\theta-\Xi+\sigma_z \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \omega & s_x & s_y & s_z \\ s_x & \alpha+\frac{\text{Log}[T_y]\,(-\alpha+\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\text{Log}[T_x]\,(\alpha-\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\theta\,\text{Log}[T_x]}{\text{Log}[T_x]+\text{Log}[T_y]} \\ s_y & \frac{\text{Log}[T_y]\,(\alpha-\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \alpha+\frac{\text{Log}[T_x]\,(-\alpha+\sigma_a)}{\text{Log}[T_x]+\text{Log}[T_y]} & \frac{\theta\,\text{Log}[T_y]}{\text{Log}[T_x]+\text{Log}[T_y]} \\ s_z & \phi & \phi & \Xi \\ \Gamma & \sigma_a & \sigma_a & \sigma_z \end{pmatrix}, \theta+\Xi == 1 \,\&\&\, \alpha+\phi == 1\right\}$$

The Hard R4 Equation

```
(R[2, 3] ** R[1, 3] ** V0) == (V0 ** (R[1, 3] // dΔ[1, 1, 2])) // ΓSimp
```

True

The Twist Equation

```
Θ[1, 2] // Γ
```

$$\begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & \frac{\text{Log}[T_1]+\text{Log}[T_2]\sqrt{T_1 T_2}}{\text{Log}[T_1 T_2]} & -\frac{\text{Log}[T_1]\left(-1+\sqrt{T_1 T_2}\right)}{\text{Log}[T_1 T_2]} \\ s_2 & -\frac{\text{Log}[T_2]\left(-1+\sqrt{T_1 T_2}\right)}{\text{Log}[T_1 T_2]} & \frac{\text{Log}[T_2]+\text{Log}[T_1]\sqrt{T_1 T_2}}{\text{Log}[T_1 T_2]} \\ \Gamma & \sqrt{T_2} & \sqrt{T_1} \end{pmatrix}$$

```
V0 ** (Θ[1, 2] // Γ) == R[1, 2] ** (V0 // dσ[1 → 2, 2 → 1]) // ΓSimp
```

True

The Unitarity Equation

```
(V0 ** (V0 // dA[1, 2]) == (ϵ[1] ϵ[2] // Γ)) // ΓSimp
```

True

The Vertical Flip Equation

```
(V0 ** (V0 // dS[1, 2]) == R[1, 2]) // ΓSimp
True
```

The Cap Equation

Not tested now / not quite meaningful in Γ.