

Pensieve header: α and Γ extracts from MetaCalculi.nb at pensieve://Projects/MetaCalculi/.

```
tm[x_, y_, z_][ $\beta$ _A] :=  $\alpha$ Collect[ $\beta$  /. {t[x]  $\rightarrow$  t[z], t[y]  $\rightarrow$  t[z], c_x  $\rightarrow$  c_z, c_y  $\rightarrow$  c_z}];
```

```
hm[x_, y_, z_][A[ $\omega$ _,  $\sigma$ _,  $\mu$ _]] := Module[
  { $\alpha$  = D[ $\mu$ , h[x]],  $\beta$  = D[ $\mu$ , h[y]],  $\Xi$  =  $\mu$  /. h[x] | h[y]  $\rightarrow$   $\emptyset$ },
  A[ $\omega$ ,
    h[z] ( $\partial_{h[x]} \sigma$ ) ( $\partial_{h[y]} \sigma$ ) + ( $\sigma$  /. h[x] | y  $\rightarrow$   $\emptyset$ ),
     $\Xi$  + h[z] ( $\alpha$  + ( $\partial_{h[x]} \sigma$ )  $\beta$ )
  ] //  $\alpha$ Collect];
```

```
tha[u_, x_][A[ $\omega$ _,  $\sigma$ _,  $\mu$ _]] := Module[{ $\alpha$ ,  $\theta$ ,  $\phi$ ,  $\Xi$ ,  $\nu$ },
   $\alpha$  = Coefficient[ $\mu$ , h[x] t[u]];  $\theta$  = D[ $\mu$ , t[u]] /. h[x]  $\rightarrow$   $\emptyset$ ;
   $\phi$  = D[ $\mu$ , h[x]] /. t[u]  $\rightarrow$   $\emptyset$ ;  $\Xi$  =  $\mu$  /. h[x] | t[u]  $\rightarrow$   $\emptyset$ ;
   $\nu$  = 1 + c_u  $\alpha$ ;
  A[ $\omega$  *  $\nu$ ,  $\sigma$ , Plus[
     $\alpha$  ( $\partial_{h[x]} \sigma$ ) /  $\nu$  * h[x] t[u],  $\theta$  ( $\partial_{h[x]} \sigma$ ) /  $\nu$  * t[u],
     $\phi$  /  $\nu$  * h[x],  $\Xi$  - c_u  $\phi$  *  $\theta$  /  $\nu$ 
  ]] //  $\alpha$ Collect];
```

```
dmi_j  $\rightarrow$  k[ $\Gamma$ [ $\omega$ _,  $\sigma$ _,  $\lambda$ _]] := Module[{ $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\theta$ ,  $\epsilon$ ,  $\phi$ ,  $\psi$ ,  $\Xi$ ,  $\mu$ },
  ( $\begin{matrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi \end{matrix}$ ) = ( $\begin{matrix} \partial_{t_i, h_i} \lambda & \partial_{t_i, h_j} \lambda & \partial_{t_i} \lambda \\ \partial_{t_j, h_i} \lambda & \partial_{t_j, h_j} \lambda & \partial_{t_j} \lambda \\ \partial_{h_i} \lambda & \partial_{h_j} \lambda & \lambda \end{matrix}$ ) /. (t | h)i|j  $\rightarrow$   $\emptyset$ ;
   $\Gamma$ Collect[ $\Gamma$ [( $\mu$  = 1 -  $\beta$ )  $\omega$ ,
    h_k ( $\partial_{h_i} \sigma$ ) ( $\partial_{h_j} \sigma$ ) + ( $\sigma$  /. hi|j  $\rightarrow$   $\emptyset$ ),
    {t_k, 1} . ( $\gamma$  +  $\alpha$   $\delta$  /  $\mu$   $\epsilon$  +  $\delta$   $\theta$  /  $\mu$ ) . {h_k, 1}
     $\phi$  +  $\alpha$   $\psi$  /  $\mu$   $\Xi$  +  $\theta$   $\psi$  /  $\mu$ 
  ]] /. {T_i  $\rightarrow$  T_k, T_j  $\rightarrow$  T_k, b_i  $\rightarrow$  b_k, b_j  $\rightarrow$  b_k} //  $\Gamma$ Collect];
```

```
A[ $\omega$ _,  $\sigma$ _,  $\mu$ _] //  $\Gamma$  := Module[{S}, S = dL[{ $\omega$ ,  $\mu$ }];
   $\Gamma$ [ $\omega$ ,
    Total[h# & /@ S] +  $\mu$  /. {h[a_]  $\Rightarrow$  h_a, t[a_]  $\Rightarrow$  c_a},
    Total[(t# h#) & /@ S] + ( $\mu$  /. t[a_]  $\Rightarrow$  c_a /. h[a_]  $\Rightarrow$  h_a t_a) -  $\mu$  /.
    {h[a_]  $\Rightarrow$  h_a, t[a_]  $\Rightarrow$  c_a t_a}
  ] /. c_a  $\Rightarrow$  Log[T_a] //  $\Gamma$ Collect[FullSimplify[# /. Sinh[x_]  $\Rightarrow$   $\frac{e^x - e^{-x}}{2}$ ] &]];
```

```
 $\Gamma$ [ $\omega$ _,  $\sigma$ _,  $\lambda$ _] // A := Module[{S,  $\mu$ }, S = dL[ $\Gamma$ [ $\omega$ ,  $\sigma$ ,  $\lambda$ ]];
   $\mu$  = Total[( $\partial_{h\#} \sigma$ ) t# h#] & /@ S] -  $\lambda$ ;
  A[ $\omega$ ,
     $\sigma$  /. h_a  $\Rightarrow$  h[a],  $\mu$  /. {h_a  $\Rightarrow$  h[a], t_a  $\Rightarrow$  t[a] / c_a}
  ] /. T_a  $\Rightarrow$  ec_a //  $\alpha$ Collect];
```