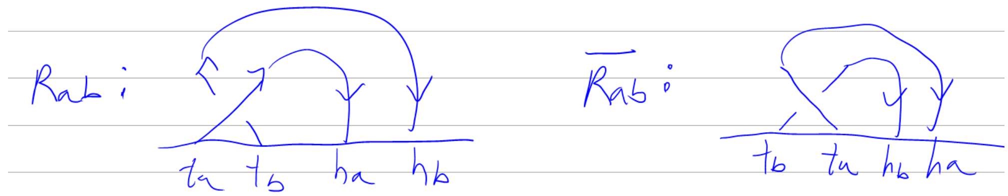
Pensieve header: Unitarity for Γ-calculus.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\MetaCalculi"];
Once[<< KnotTheory`]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```



In[∘]:=

$$R_{a\_,b\_} := \Gamma\left[\{t_a, t_b, h_a, h_b\}, 1, h_a + h_b T_a, \{t_a, t_b\}.\begin{pmatrix}1 & 1-T_a\\ 0 & T_a\end{pmatrix}.\{h_a, h_b\}\right];$$

$$\overline{R}_{a\_,b\_} := \Gamma\left[\{t_b, t_a, h_b, h_a\}, 1, h_a + h_b T_a^{-1}, \{t_a, t_b\}.\begin{pmatrix}1 & 1-T_a^{-1}\\ 0 & T_a^{-1}\end{pmatrix}.\{h_a, h_b\}\right];$$

In[∘]:= $\overline{R}_{1,2}$

Out[∘]=

$$\Gamma\left[\{t_2, t_1, h_2, h_1\}, 1, h_1 + \frac{h_2}{T_1}, h_1 t_1 + h_2 \left(t_1 \left(1 - \frac{1}{T_1}\right) + \frac{t_2}{T_1}\right)\right]$$

In[∘]:=
```
ΓCollect[Γ[ς_, ω_, σ_, λ_]] := Γ[ς, Factor@ω, Expand@σ,
   Total[CoefficientRules[λ, ς] /. (ps_ → c_) ⧴ Factor[c] Times @@ (ς^ps)]]
```

In[∘]:= `ΓCollect[R̄1,2]`

Out[∘]=

$$\Gamma\left[\{t_2, t_1, h_2, h_1\}, 1, h_1 + \frac{h_2}{T_1}, h_1 t_1 + \frac{h_2 t_2}{T_1} + \frac{h_2 t_1 (-1 + T_1)}{T_1}\right]$$

In[∘]:=
```
γ_Γ[ς] := γ⟦1⟧; γ_Γ[ω] := γ⟦2⟧; γ_Γ[σ] := γ⟦3⟧; γ_Γ[λ] := γ⟦4⟧;
γ_Γ[S] := Union@Cases[γ[ς], t_a_ ⧴ a, ∞];
γ_Γ[Σ] := (∂_h_# γ[σ]) & /@ γ[S];
γ_Γ[A] := Outer[Factor[∂_t_#1 h_#2 γ[λ]] &, γ[S], γ[S]];
```

In[∘]:=
```
{R̄_{1,2}[ς], R̄_{1,2}[ω], R̄_{1,2}[σ], R̄_{1,2}[λ], R̄_{1,2}[S], R̄_{1,2}[Σ], R̄_{1,2}[A] // MatrixForm} // Column
```

Out[∘]=

$\{t_2, t_1, h_2, h_1\}$

1

$h_1 + \frac{h_2}{T_1}$

$h_1 t_1 + h_2 \left(t_1 \left(1 - \frac{1}{T_1}\right) + \frac{t_2}{T_1}\right)$

$\{1, 2\}$

$\left\{1, \frac{1}{T_1}\right\}$

$\begin{pmatrix} 1 & \frac{-1+T_1}{T_1} \\ 0 & \frac{1}{T_1} \end{pmatrix}$

In[∘]:=
```
ΓForm[γ_Γ] := Module[{M},
   M = γ[A] // Transpose;
   PrependTo[M, t_# & /@ γ[S]];
   M = Join[
     {Prepend[h_# & /@ γ[S], γ[ω]]},
     Transpose[M],
     {Prepend[γ[Σ], "Γ"]}
    ];
   Column[{γ[ς], MatrixForm[M]}]
  ];
ΓForm[else_] := else /. γ_Γ ⧴ ΓForm[γ];
Format[γ_Γ, StandardForm] := ΓForm[γ];
```

In[∘]:=
```
{R_{1,2}, R̄_{1,2}}
```

Out[∘]=

$$\left\{ \begin{array}{c} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 1 - T_1 \\ t_2 & 0 & T_1 \\ \Gamma & 1 & T_1 \end{pmatrix} \end{array}, \begin{array}{c} \{t_2, t_1, h_2, h_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & \frac{-1+T_1}{T_1} \\ t_2 & 0 & \frac{1}{T_1} \\ \Gamma & 1 & \frac{1}{T_1} \end{pmatrix} \end{array} \right\}$$