

## KnotTheory`QuantumKnotInvariants` package

A subpackage for KnotTheory`, providing a simple interface to the QuantumGroups` package.

June 23, 2006, Scott Morrison

```
BeginPackage["KnotTheory`QuantumKnotInvariants`",
  {"KnotTheory`", "QuantumGroups`", "QuantumGroups`Braiding`",
   "QuantumGroups`Representations`", "QuantumGroups`Utilities`DataPackage`"}];
```

```
QuantumKnotInvariant::about =
  "Quantum knot invariants are calculated using the mathematica
  package QuantumGroups`, written by Scott Morrison 2003-2008.";
```

```
QuantumKnotInvariant::usage =
  "QuantumKnotInvariant[ $\Gamma$ , V][K][q] calculates the quantum knot
  invariant of the knot K in the representation V of the quantum
  group  $\Gamma$ . This relies on the QuantumGroups` package, and you
  should look there for details of how  $\Gamma$  and V may be specified.
```

Examples:

```
QuantumKnotInvariant[Subscript[A,2], Irrep[Subscript[A,2]][{1, 0}]] [Knot[5, 2]][q]
QuantumKnotInvariant[Subscript[G,2], Irrep[Subscript[G,2]][{1,
0}] $\oplus$ Irrep[Subscript[G,2]][{0, 1}]] [Knot[5, 2]][q]"
```

```
PackageQuantumKnotInvariants::usage =
  "PackageQuantumKnotInvariants[ $\Gamma$ ] saves all calculated quantum knot invariants for
  the quantum group K into a data file in the QuantumGroupsDataDirectory[]."

```

```
Begin["`Private`"]
```

```
q = Global`q;
```

```
Wants[x_] :=
  (Off[Get::noopen];
  Off[Needs::nocont];
  Needs[x];
  On[Needs::nocont];
  On[Get::noopen];)
```

```
ExtractMatrices[indices : {__Integer}, matrices_] :=
  Extract[matrices, indices /. {n_ /; n < 0 => {2, -n}, n_ /; n > 0 => {1, n}}]
```

```
TogetherDot[x_] := x
TogetherDot[x_, y_] := x.y
TogetherDot[x_, y_, z_] := TogetherDot[Together[x.y], z]
```

```

QuantumKnotInvariant[ $\mathcal{I}_{-n}$ ,  $V_{-}$ ][ $K_{-}$ ] :=
QuantumKnotInvariant[ $\mathcal{I}_n$ ,  $V$ ][ $K$ ] = Module[{br = BR[ $K$ ], k, data},
  CreditMessage[QuantumKnotInvariant::about];
  If[br === BR[1, {}],
    Return[Function[{Global`q $\theta$ }, Evaluate[qDimension[ $\mathcal{I}_n$ ][ $V$ ] /. q  $\rightarrow$  Global`q $\theta$ ]]];
  Wants["QuantumGroups`Data`" <> ToString[ $\mathcal{I}$ ] <> ToString[ $n$ ] <> "` BraidingData`"];
  k = br[[1]];
  data = BraidingData[ $\mathcal{I}_n$ ][ $V$ , k];
  Function[{Global`q $\theta$ }, Evaluate[
    Expand[Together[Plus @@ (#[[1]] Tr[TogetherDot @@ ExtractMatrices[br[[2]], #[[2]]] &) /@
      data]]] /. q  $\rightarrow$  Global`q $\theta$ ]]
]

```

```

PackageQuantumKnotInvariants[ $\mathcal{I}_{-n}$ ] := PackageData[
  {{QuantumKnotInvariant, HoldPattern[QuantumKnotInvariant[ $\mathcal{I}_n$ , _][_Knot]]}},
  {ToString[ $\mathcal{I}$ ] <> ToString[ $n$ ], "QuantumKnotInvariants"},
  "Needs"  $\rightarrow$  {"QuantumGroups`", "KnotTheory`", "KnotTheory`QuantumKnotInvariants`"},
  "ExtraPrivateCode"  $\rightarrow$  "q $\theta$ =Global`q $\theta$ ";, "UseGzip"  $\rightarrow$  False
]

```

```
End[]
```

```
EndPackage[]
```