

RootsOfUnity package

A subpackage for QuantumGroups v2.
Version 2.0, September 18, 2008, Scott Morrison

Introduction

Implementation

```

BeginPackage["QuantumGroups`Steinberg`",
  {"QuantumGroups`", "QuantumGroups`RootsOfUnity`", "QuantumGroups`RootsOfUnity`",
   "QuantumGroups`Representations`", "QuantumGroups`RootSystems`"}];

SteinbergDecomposeRepresentation;

Begin["`Private`"];

SteinbergFold[r_, i_][c_] := Module[{root, innerproduct},
  root = SimpleRoots[r][[i]];
  innerproduct[λ_] := innerproduct[λ] = KillingForm[r][λ + ρ[r], root];
  c /. {Irrep[r][λ_] /; (innerproduct[λ] == 0) => 0,
    Irrep[r][λ_] /; (innerproduct[λ] < 0) => {f[λ];
      -Irrep[r][λ - (2  $\frac{\text{innerproduct}[\lambda]}{\text{innerproduct}[\text{root} - \rho[r]]}$ ) root]}}
]

SteinbergAlcoveFold[r_, L_][c_] := Module[{root, innerproduct, wall},
  root = -AlcoveDefiningRoot[r, L];
  innerproduct[λ_] := innerproduct[λ] = KillingForm[r][λ + ρ[r], root];
  wall = -If[EvenQ[L], L/2, L];
  c /. {Irrep[r][λ_] /; (innerproduct[λ] == wall) => 0, Irrep[r][λ_] /;
    (innerproduct[λ] < wall) => -Irrep[r][λ - (2  $\frac{\text{innerproduct}[\lambda] - \text{wall}}{\text{innerproduct}[\text{root} - \rho[r]]}$ ) root]}}
]

SteinbergFoldAll[r_, L_: 0][c_] :=
  Fold[#2[#1] &, c, If[L > 0, {SteinbergAlcoveFold[r, L]}, {}] ~
  Join ~ Table[SteinbergFold[r, i], {i, 1, Rank[r]}]

```

```
SteinbergDecomposeRepresentation[L_, L_ : 0][Irrep[L_][λ_] ⊗ Irrep[L_][μ_]] :=  
SteinbergDecomposeRepresentation[L, L][Irrep[L][λ] ⊗ Irrep[L][μ]] =  
FixedPoint[SteinbergFoldAll[L, L],  
  Plus @@ (WeightMultiplicities[L, Irrep[L][μ]] /.  
    {κ : {___Integer}, n_Integer} => n Irrep[L][κ + λ] ) /.  
  {n_Integer V : Irrep[L][_] /; n > 0 => DirectSum @@ Table[V, {n}]} /. Plus -> DirectSum  
  
End[];  
  
EndPackage[];
```