QuantumRoots package

A subpackage for QuantumGroups v2.
Version 2.0, June 11, 2005, Scott Morrison

Introduction

This package calculates the quantum roots for a quantum group.

Implementation

Start of package

Specify package dependencies:

```
BeginPackage["QuantumGroups`QuantumRoots`",
  {"QuantumGroups`", "QuantumGroups`Utilities`Debugging`",
   "QuantumGroups`MatrixPresentations`", "QuantumGroups`Algebra`",
   "QuantumGroups`RootSystems`", "QuantumGroups`Representations`",
   "QuantumGroups`BraidAction`", "QuantumGroups`WeylGroups`"}];
```

Usage messages

```
ExpandQuantumRoot::usage = "";
```

```
QuantumPositiveRoots::usage = "";
```

```
QuantumNegativeRoots::usage = "";
```

```
QuantumRootHeight::usage = "";
```

Internals

```
Begin["`Private`"];
```

```
q = Global`q;
```

```
LoadQuantumRoots[Γ_n_] := Module[{},
  Off[Get::noopen, Needs::nocont];
  Needs["QuantumGroups`Data`" <> SymbolName[Γ] <> ToString[n] <> "`QuantumRoots`"];
  On[Get::noopen, Needs::nocont];
  LoadQuantumRoots[Γn] = False;
  True
 ]
```

```
ExpandQuantumRoot[Γ_][SuperPlus[X_Γ,r_]] /;
  MemberQ[SimpleRoots[Γ], PositiveRoots[Γ][[r]]] :=
 SuperPlus[X_Position[SimpleRoots[Γ],PositiveRoots[Γ][[r]]][[1,1]]]
ExpandQuantumRoot[Γ_][SuperPlus[X_Γ,r_]] := ExpandQuantumRoot[Γ][SuperPlus[XΓ,r]] =
  If[LoadQuantumRoots[Γ], ExpandQuantumRoot[Γ][SuperPlus[XΓ,r]],
   With[{i = LongestWordDecomposition[Γ]},
    DebugPrintHeld["Calculating ", ExpandQuantumRoot[Γ][SuperPlus[XΓ,r]]];
    BraidAction[Γ][T# & /@ Take[i, r - 1], SuperPlus[Xi[[r]]]]
   ]
  ]
```

```
ExpandQuantumRoot[Γ_][SuperMinus[X_Γ,r_]] :=
 ExpandQuantumRoot[Γ][SuperPlus[XΓ,r]] /. q → q^-1 /. {SuperPlus[Xi_] :> SuperMinus[Xi]} /.
  Z_NonCommutativeMultiply :> Reverse[Z]
```

```
Δ[A : (SuperPlus[X_Γ,r_] | SuperMinus[X_Γ,r_])] := Δ[ExpandQuantumRoot[Γ][A]]
```

```
OperatorWeight[Γ_][SuperPlus[X_Γ,r_]] := PositiveRoots[Γ][[r]]
OperatorWeight[Γ_][SuperMinus[X_Γ,r_]] := -PositiveRoots[Γ][[r]]
```

```
QuantumPositiveRoots[Γ_] :=
 Table[SuperPlus[XΓ,i], {i, 1, Length[LongestWordDecomposition[Γ]]}]
QuantumNegativeRoots[Γ_] :=
 Table[SuperMinus[XΓ,i], {i, 1, Length[LongestWordDecomposition[Γ]]}]
```

```
MatrixPresentation[Γ_][A : (SuperPlus[X_Γ,r_] | SuperMinus[X_Γ,r_])][V_, β_, λ_] :=
 MatrixPresentation[Γ][A][V, β, λ] =
  MatrixPresentation[Γ][ExpandQuantumRoot[Γ][A]][V, β, λ]
```

```
QuantumRootHeight[Γ_][SuperPlus[X_Γ,r_] | SuperMinus[X_Γ,r_]] :=
 Plus @@ (PositiveRoots[Γ][[r]].Inverse[SimpleRoots[Γ]])
```

```
End[];
```

End of package