

Expected volume for 5_2: 2.8281220883.

```
<< \ c : / drorbn / projects / KAtlas / KnotTheory.m
```

```
Loading KnotTheory`...
```

```
<< \ c : / drorbn / projects / KAtlas / KnotTheoryData.m
```

```
Loading KnotTheoryData.m...
```

```
Lob[t_] := -NIntegrate[Log[Abs[2 Sin[u]]], {u, 0, t}];
```

```
Vol[z_] := Plus @@ (Lob[Arg[#]] & /@ {z, 1 - 1/z, 1 / (1 - z)})
```

```
2 Vol[omega = 1/2 + Sqrt[3]/2 * I]
```

```
2.02988
```

```
Pachner[z_, w_] := {Vol[z], Vol[w], Vol[z] + Vol[w] +  
  Vol[1 / (z * w)] - Vol[z (1 - w) / (1 - z * w)] - Vol[w (1 - z) / (1 - z * w)]}
```

```
Pachner[Random[Real, {-1, 1}] + I * Random[Real, {-1, 1}],
```

```
  Random[Real, {-1, 1}] + I * Random[Real, {-1, 1}]]
```

```
{-0.842242, 0.0317351, -3.99565 × 10-11}
```

```
SetAttributes[p, Orderless]; SetAttributes[{EQs, EQ}, {Orderless, Flat}]
```

```

Rels[K_] := Rels[PD[K]];
Rels[pd_PD] := Module[
  {n, dt, z, e, u, v, j, im4, dm4, w, zz,
   xx, yy, ab, merge, rels, x, y, a, b, c, d, d1, d2, d3, eq},
  im4[a_] := a + 1 /. 5 -> 1; dm4[a_] := a - 1 /. 0 -> 4;
  n = Length[pd];
  dt = EQs @@ Flatten[
    {
      Table[
        t[z[v, j], 3, {1, 2, 4}] ~ e ~ t[z[v, im4@j], 4, {2, 1, 3}],
        {v, n}, {j, 4}
      ],
      Table[e @@ Position[pd, i], {i, 2 n}] /. e[x_, y_] -> {e[x, y], e[y, x]} /. {
        e[{v_, a_}, {u_, b_}] ->
          t[z[v, dm4@a], 2, {1, 3, 4}] ~ e ~ t[z[u, b], 1, {2, 3, 4}] /; OddQ[a + b],
        e[{v_, a_}, {u_, b_}] -> t[z[v, dm4@a], 2, {1, 3, 4}] ~ e ~
          t[z[u, b], 1, {2, 4, 3}] /; EvenQ[a + b]
      }
    ] /. e[t[z_, a_, {c1_, c2_, c3_}], t[w_, b_, {d1_, d2_, d3_}]] -> {
      t[z, a, c1] ~ p ~ t[w, b, d1], t[z, a, c2] ~ p ~ t[w, b, d2], t[z, a, c3] ~ p ~ t[w, b, d3]
    }
  ] /.
  t[z[v_, j_], a_, b_] -> t[z[v, j]] ~ Join ~ (t[a, b] /. {3 -> 4, 4 -> 3}) /; EvenQ[j];
  rels = List @@ Union[
    dt /. p[t[z_, a_, b_], t[w_, c_, d_]] ->
      EQs[t[z, a, b] ~ to ~ t[w, d, c], t[w, c, d] ~ to ~ t[z, b, a]] // . (
        EQs[xx_. * (t[x_, ab_] ~ to ~ t[y_, cd_]), zz_. *
          (t[y_, cd_] ~ to ~ t[z_, ef_])] x
      )
    -> EQs[xx * y[cd] * zz * (t[x, ab] ~ to ~ t[z, ef])]
      ) /. (t[z_, ab_] ~ to ~ t[z_, ab_]) -> z[ab] /.
      (w_z)[ab_] -> Switch[Sort[{ab}],
        {1, 2} | {3, 4}, w[0],
        {1, 3} | {2, 4}, w[1],
        {1, 4} | {2, 3}, w[2]
      ]
  ] /. eq_Times -> (EQ @@ eq);
  merge = Position[Count[#, z[n, _][_] ] & /@ rels, 4];
  rels = Append[Delete[rels, merge], EQ @@ rels[[Join @@ merge]]];
  Append[rels, z1 ~ EQ ~ (EQ @@ Product[
    z[k, 4][0] * z[k, 4][1] * z[k, 3][0] * z[k, 3][2] *
      If[pd[[k, 4]] > pd[[k, 2]] || pd[[k, 2]] - pd[[k, 4]] > 1,
        z[k, 1][1] * z[k, 4][2],
        z[k, 3][1] * z[k, 2][2]
      ],
    {k, n}
  ])] // . {EQ[z_[0], z_[1], z_[2]] -> EQ[-1], EQ[-1, -1] -> EQ[]}
  ] /. z[i_, j_][k_] -> m[ToExpression["z" <> ToString[i] <> ToString[j]], k]
]

```

General::spell1: Possible spelling error: new symbol name "Rels" is similar to existing symbol "Reals".

General::spell1: Possible spelling error: new symbol name "rels" is similar to existing symbol "Rels".

rels = Rels[Knot[7, 2]]

```
{EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]],
EQ[m[z21, 0], m[z22, 0], m[z23, 0], m[z24, 0]],
EQ[m[z14, 0], m[z34, 0]], EQ[m[z31, 0], m[z32, 0], m[z33, 0], m[z34, 0]],
EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]],
EQ[m[z41, 0], m[z42, 0], m[z43, 0], m[z44, 0]],
EQ[m[z42, 0], m[z52, 0]], EQ[m[z51, 0], m[z52, 0], m[z53, 0], m[z54, 0]],
EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z42, 1], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2]],
EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]],
EQ[m[z32, 0], m[z62, 0]],
EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34, 2], m[z61, 2], m[z62, 1]],
EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34, 2], m[z62, 1], m[z63, 2]],
EQ[m[z44, 0], m[z64, 0]], EQ[m[z61, 0], m[z62, 0], m[z63, 0], m[z64, 0]],
EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z52, 1], m[z53, 2], m[z61, 2], m[z64, 1]],
EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z51, 2], m[z52, 1], m[z63, 2], m[z64, 1]],
EQ[m[z32, 1], m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z62, 2], m[z63, 1], m[z64, 2]],
EQ[m[z31, 2], m[z32, 1], m[z43, 2], m[z44, 1], m[z61, 1], m[z62, 2], m[z63, 1], m[z64, 2]],
EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]],
EQ[m[z12, 0], m[z22, 0], m[z72, 0]],
EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]],
EQ[m[z21, 0], m[z73, 0]], EQ[m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2],
m[z51, 2], m[z54, 1], m[z72, 1], m[z73, 2]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]],
EQ[m[z42, 1], m[z43, 2], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]],
EQ[m[z11, 2], m[z12, 1], m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z73, 2], m[z74, 1]],
EQ[m[z12, 1], m[z13, 2], m[z21, 2], m[z21, 2], m[z22, 1], m[z24, 1],
m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2],
m[z72, 2], m[z73, 0], m[z73, 1], m[z73, 1], m[z74, 0], m[z74, 2], m[z74, 2]],
EQ[-1, z1, m[z11, 1], m[z13, 0], m[z13, 2], m[z21, 1], m[z23, 0], m[z23, 2], m[z31, 1],
m[z33, 0], m[z33, 2], m[z41, 1], m[z43, 0], m[z43, 2], m[z51, 1], m[z53, 0],
m[z53, 2], m[z61, 1], m[z63, 0], m[z63, 2], m[z71, 1], m[z73, 0], m[z73, 2]]}
```

```

rels = Delete[rels, Position[rels, z1][[1, 1]]]
{EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]],
 EQ[m[z21, 0], m[z22, 0], m[z23, 0], m[z24, 0]],
 EQ[m[z14, 0], m[z34, 0]], EQ[m[z31, 0], m[z32, 0], m[z33, 0], m[z34, 0]],
 EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]],
 EQ[m[z41, 0], m[z42, 0], m[z43, 0], m[z44, 0]],
 EQ[m[z42, 0], m[z52, 0]], EQ[m[z51, 0], m[z52, 0], m[z53, 0], m[z54, 0]],
 EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z42, 1], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2]],
 EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]],
 EQ[m[z32, 0], m[z62, 0]],
 EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34, 2], m[z61, 2], m[z62, 1]],
 EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34, 2], m[z62, 1], m[z63, 2]],
 EQ[m[z44, 0], m[z64, 0]], EQ[m[z61, 0], m[z62, 0], m[z63, 0], m[z64, 0]],
 EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z52, 1], m[z53, 2], m[z61, 2], m[z64, 1]],
 EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z51, 2], m[z52, 1], m[z63, 2], m[z64, 1]],
 EQ[m[z32, 1], m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z62, 2], m[z63, 1], m[z64, 2]],
 EQ[m[z31, 2], m[z32, 1], m[z43, 2], m[z44, 1], m[z61, 1], m[z62, 2], m[z63, 1], m[z64, 2]],
 EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]],
 EQ[m[z12, 0], m[z22, 0], m[z72, 0]],
 EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]],
 EQ[m[z21, 0], m[z73, 0]], EQ[m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2],
 m[z51, 2], m[z54, 1], m[z72, 1], m[z73, 2]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]],
 EQ[m[z42, 1], m[z43, 2], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]],
 EQ[m[z11, 2], m[z12, 1], m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z73, 2], m[z74, 1]],
 EQ[m[z12, 1], m[z13, 2], m[z21, 2], m[z21, 2], m[z22, 1], m[z24, 1],
 m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2],
 m[z72, 2], m[z73, 0], m[z73, 1], m[z73, 1], m[z74, 0], m[z74, 2], m[z74, 2]]}

```

Length /@ rels

```
{4, 4, 2, 4, 8, 4, 2, 8, 8, 4, 8, 4, 3, 8, 2, 8, 3, 8, 8, 20}
```

Plus @@ len /@ Length /@ rels

```
2 len[2] + 4 len[3] + 3 len[4] + 6 len[8] + len[20]
```

RandomPermutation[L_List] := Last /@ Sort[{Random[], #} & /@ L];

\$Log = True;

m[z_, k1_][k2_] := m[z, (k1 + k2) ~Mod~ 3];

RemoveTriangle[s_, k_Integer] :=

```

(Replace[s[[k]], EQ[m[z1_, p1_], m[z2_, p2_], m[z3_, p3_]] => (
  $Log && Print[StringForm["Doing triangle: k=`; s[[k]]=`; s=`", k, s[[k]], s]];
  z1 = m[z1, p1]; z2 = m[z2, p2]; z3 = m[z3, p3];
  {w1, w2} = m[#, 0] & /@ Unique[{w, w}];
  t = Delete[s, k] //. {
    z3 -> EQ[w1, w2], z2 -> EQ[w1[2], w2[1]], z1 -> EQ[w1[1], w2[2]],
    EQ[z3[1], z3[2]] -> EQ[-1, w1[2], w2[1], w1[1], w2[2]],
    EQ[z2[1], z2[2]] -> EQ[-1, w1, w2, w1[1], w2[2]],
    EQ[z1[1], z1[2]] -> EQ[-1, w1, w2, w1[2], w2[1]],
    EQ[z1[2], z2[1]] -> w1, EQ[z1[1], z2[2]] -> w2, EQ[z2[2], z3[1]] -> w1[1],
    EQ[z3[1], z1[2]] -> w2[1],
    EQ[z3[2], z1[1]] -> w1[2], EQ[z2[1], z3[2]] -> w2[2],

```

```

    EQ[m[z_, 0], m[z_, 1], m[z_, 2]] → EQ[-1], EQ[-1, -1] → EQ[]
  };
  If[FreeQ[t, z123 = First[z1] | First[z2] | First[z3]],
    Return[t],
    $Log && Print[StringForm[
      "Failed triangle; t=`; problems at `", t, Select[t, !FreeQ[#, z123] &]]
    ]
  ]];
  Return[Failed]
);
ContractDomino[s_, k1_Integer, k2_Integer] := (
  Print[StringForm[
    "{k1, k2} = {`, `}; s[{{k1,k2}}]={`, `}", k1, k2, s[{{k1}}, s[{{k2}}]]];
  Replace[s[{{k1, k2}}],
    {EQ[_ , _ , m[w1_, p1_], m[w2_, p2_]], EQ[m[w1_, pa_], m[w2_, pb_], _ , _]} ⇒ (
    $Log && Print[StringForm["Found domino at k1=` , k2=`", k1, k2]];
    w1 = m[w1, p1]; w2 = m[w2, p2];
    {z1, z2, z3} = m[#, 0] & /@ Unique[{z, z, z}];
    t = Append[
      s /. {
        EQ[w1, w2] → z3, EQ[w1[2], w2[1]] → z2, EQ[w1[1], w2[2]] → z1,
        w1 → EQ[z1[2], z2[1]], w2 → EQ[z1[1], z2[2]], w1[1] → EQ[z2[2], z3[1]],
        w2[1] → EQ[z3[1], z1[2]],
        w1[2] → EQ[z3[2], z1[1]], w2[2] → EQ[z2[1], z3[2]]
      },
      EQ[z1, z2, z3]
    ];
    t = RemoveTriangle[RemoveTriangle[t, k2], k1];
    $Log && Print["t=", t];
    Return[t]
  ) /; {p1, p2} == Mod[{pa + 1, pb + 2}, 3]
];
Return[Failed]
);
Red[rels_List] := Module[
  {s = rels, ks, k, t, at, at1, at2, k1, k2},
  Label[Start]; s = RandomPermutation[DeleteCases[s, EQ[]]];
  For[k = 1, k ≤ Length[s], ++k, Replace[s[[k]], EQ[m[z1_, p1_], m[z2_, p2_]] ⇒ (
    $Log && Print[StringForm["Doing bigon: k=`; s[[k]]=`; s=`", k, s[[k]], s]];
    z1 = m[z1, p1]; z2 = m[z2, p2];
    t =
      s /. {EQ[z1, z2] → EQ[], EQ[z1[1], z2[2]] → EQ[], EQ[z1[2], z2[1]] → EQ[]};
    at = Position[t, # // First] & /@ {z1, z2};
    If[(Length /@ at) === {0, 0}, s = t; Goto[Start]];
    If[(Length /@ at) != {1, 1},
      $Log && Print[StringForm["Bad bigon: at=`; t=`", at, t]],
      at1 = at[[1, 1, 1]]; at2 = at[[2, 1, 1]];
      s = If[at1 == at2,
        $Log && Print["Equal ats ", t[[at1]]]; t /. EQ[z1, z2] → EQ[],
        Append[Delete[t, {{at1}, {at2}}],

```

```

EQ[t[[at1]], t[[at2]]] /. EQ[z1, z2] → EQ[]
];
Goto[Start]
]
)]];
Do[If[Failed != (t = RemoveTriangle[s, k]), s = t; Goto[Start]], {k, Length[s]}];
$Log && Print["Contracting dominos; s=", s];
Do[
  If[{4, 4} == (Length /@ s[{k1, k2}])] &&
    Failed != (t = ContractDomino[s, k1, k2]), s = t; Goto[Start]],
  {k2, 2, Length[s]}, {k1, k2 - 1}];
s
]
{s = Red[rels], Length /@ s, Length[s]}
Doing bigon: k=5; s[[k]] = EQ[m[z32, 0], m[z62, 0]];
s = {EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]], EQ[
m[z11, 2], m[z12, 1], m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z73, 2], m[z74, 1]], EQ[m[z32, 2],
m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z62, 2], m[z63, 1], m[z64, 2]], EQ[m[z23, 2],
m[z24, 1], m[z41, 2], m[z42, 1], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2]], EQ[m[z32, 0], m[
z62, 0]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], EQ[m[z51, 0], m[
z52, 0], m[z53, 0], m[z54, 0]], EQ[m[z31, 2], m[z32, 1], m[z43, 2], m[z44, 1], m[z61, 1], m[z62,
2], m[z63, 1], m[z64, 2]], EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z52, 1], m[z53, 2],
m[z61, 2], m[z64, 1]], EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z51, 2], m[z52, 1], m[
z63, 2], m[z64, 1]], EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]], EQ[m[z41, 0], m[z42, 0], m[
z43, 0], m[z44, 0]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71,
2], m[z72, 1]], EQ[m[z44, 0], m[z64, 0]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z14, 0],
m[z34, 0]], EQ[m[z42, 1], m[z43, 2], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2], m[z71, 2], m[z74,
1]], EQ[m[z21, 0], m[z73, 0]], EQ[m[z42, 0], m[z52, 0]], EQ[m[z13, 0], m[z31, 0], m[z41, 0],
m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34,
2], m[z62, 1], m[z63, 2]], EQ[m[z61, 0], m[z62, 0], m[z63, 0], m[z64, 0]], EQ[m[z21, 1], m[
z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1], m[z73, 2]], EQ[m[z12, 0], m[z22,
0], m[z72, 0]], EQ[m[z12, 1], m[z13, 2], m[z21, 2], m[z21, 2], m[z22, 1], m[z24, 1], m[z53, 2],
m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2], m[z73, 0], m[z73, 1],
m[z73, 1], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z21, 0], m[z22, 0], m[z23, 0], m[z24, 0]],
EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z32, 2], m[z33, 1], m[z34, 2], m[z61, 2], m[z62, 1]], EQ[m[
z31, 0], m[z32, 0], m[z33, 0], m[z34, 0]]}

```

Doing bigon: k=4; s[[k]]=EQ[m[z42, 0], m[z52, 0]];

s={EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z63, 2]], EQ[m[z11, 2], m[z12, 1], m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z73, 2], m[z74, 1]], EQ[m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1], m[z73, 2]], EQ[m[z42, 0], m[z52, 0]], EQ[m[z21, 0], m[z22, 0], m[z23, 0], m[z24, 0]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], EQ[m[z31, 2], m[z43, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z14, 0], m[z34, 0]], EQ[m[z44, 0], m[z64, 0]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]], EQ[m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z31, 0], m[z33, 0], m[z34, 0], m[z61, 0], m[z63, 0], m[z64, 0]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z42, 1], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2]], EQ[m[z41, 0], m[z42, 0], m[z43, 0], m[z44, 0]], EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]], EQ[m[z21, 0], m[z73, 0]], EQ[m[z12, 0], m[z22, 0], m[z72, 0]], EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z52, 1], m[z53, 2], m[z61, 2], m[z64, 1]], EQ[m[z41, 1], m[z42, 2], m[z43, 1], m[z44, 2], m[z51, 2], m[z52, 1], m[z63, 2], m[z64, 1]], EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z61, 2]], EQ[m[z42, 1], m[z43, 2], m[z51, 1], m[z52, 2], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]], EQ[m[z12, 1], m[z13, 2], m[z21, 2], m[z22, 1], m[z23, 1], m[z24, 1], m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z73, 0], m[z73, 1], m[z73, 1], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z51, 0], m[z52, 0], m[z53, 0], m[z54, 0]]}

Doing bigon: k=3; s[[k]]=EQ[m[z21, 0], m[z73, 0]];

s={EQ[m[z41, 0], m[z43, 0], m[z44, 0], m[z51, 0], m[z53, 0], m[z54, 0]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z21, 0], m[z73, 0]], EQ[m[z12, 0], m[z22, 0], m[z72, 0]], EQ[m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z63, 2]], EQ[m[z31, 0], m[z33, 0], m[z34, 0], m[z61, 0], m[z63, 0], m[z64, 0]], EQ[m[z43, 2], m[z51, 1], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]], EQ[m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1], m[z73, 2]], EQ[m[z12, 1], m[z13, 2], m[z21, 2], m[z22, 1], m[z24, 1], m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2], m[z73, 0], m[z73, 1], m[z73, 1], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z41, 1], m[z43, 1], m[z44, 2], m[z51, 2], m[z63, 2], m[z64, 1]], EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z61, 2]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]], EQ[m[z21, 0], m[z22, 0], m[z23, 0], m[z24, 0]], EQ[m[z11, 2], m[z12, 1], m[z21, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z73, 2], m[z74, 1]], EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]], EQ[m[z41, 1], m[z43, 1], m[z44, 2], m[z53, 2], m[z61, 2], m[z64, 1]], EQ[m[z14, 0], m[z34, 0]], EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z51, 1], m[z53, 1], m[z54, 2]], EQ[m[z44, 0], m[z64, 0]], EQ[m[z31, 2], m[z43, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]]}

Doing bigon: $k=3$; $s[[k]]=EQ[m[z44, 0], m[z64, 0]]$;

$s=\{EQ[m[z41, 0], m[z43, 0], m[z44, 0], m[z51, 0], m[z53, 0], m[z54, 0]], EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z63, 2]], EQ[m[z44, 0], m[z64, 0]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]], EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z61, 2]], EQ[m[z41, 1], m[z43, 1], m[z44, 2], m[z51, 2], m[z63, 2], m[z64, 1]], EQ[m[z12, 0], m[z22, 0], m[z72, 0]], EQ[m[z12, 1], m[z13, 2], m[z22, 0], m[z22, 1]], m[z23, 0], m[z24, 0], m[z24, 1], m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z31, 2], m[z43, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z33, 2], m[z41, 2], m[z44, 1], m[z61, 1], m[z63, 1], m[z64, 2]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]], EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]], EQ[m[z41, 1], m[z43, 1], m[z44, 2], m[z53, 2], m[z61, 2], m[z64, 1]], EQ[m[z43, 2], m[z51, 1], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]], EQ[m[z11, 2], m[z12, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z74, 1]], EQ[m[z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z31, 0], m[z33, 0], m[z34, 0], m[z61, 0], m[z63, 0], m[z64, 0]], EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z51, 1], m[z53, 1], m[z54, 2]], EQ[m[z14, 0], m[z34, 0]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]]$

Doing bigon: $k=7$; $s[[k]]=EQ[m[z14, 0], m[z34, 0]]$;

$s=\{EQ[m[z12, 0], m[z22, 0], m[z72, 0]], EQ[m[z31, 0], m[z33, 0], m[z34, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z54, 0], m[z61, 0], m[z63, 0]], EQ[m[z43, 2], m[z51, 1], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]], EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z14, 0]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], EQ[m[z11, 2], m[z12, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z74, 1]], EQ[m[z14, 0], m[z34, 0]], EQ[m[z11, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z63, 2]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z22, 1], m[z23, 2], m[z31, 2], m[z34, 1]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z14, 2], m[z33, 2], m[z34, 1], m[z71, 2], m[z72, 1]], EQ[m[z33, 2], m[z41, 2], m[z61, 1], m[z63, 1]], EQ[m[z41, 1], m[z43, 1], m[z53, 2], m[z61, 2]], EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z51, 1], m[z53, 1], m[z54, 2]], EQ[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]], EQ[m[z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1]], EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z31, 2], m[z43, 2], m[z61, 1], m[z63, 1]], EQ[m[z12, 1], m[z13, 2], m[z22, 0], m[z22, 1], m[z23, 0], m[z24, 0], m[z24, 1], m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z13, 2], m[z14, 1], m[z31, 1], m[z33, 1], m[z34, 2], m[z61, 2]]$

Doing triangle: $k=7$; $s[[k]]=EQ[m[z24, 0], m[z54, 0], m[z74, 0]]$;

$s=\{EQ[m[z11, 0], m[z12, 0], m[z13, 0], m[z31, 0], m[z33, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z54, 0], m[z61, 0], m[z63, 0]], EQ[m[z33, 2], m[z41, 2], m[z61, 1], m[z63, 1]], EQ[m[z41, 1], m[z43, 1], m[z53, 2], m[z61, 2]], EQ[m[z11, 2], m[z31, 1], m[z33, 1], m[z63, 2]], EQ[m[z31, 2], m[z43, 2], m[z61, 1], m[z63, 1]], EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], EQ[m[z24, 0], m[z54, 0], m[z74, 0]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z33, 2], m[z71, 2], m[z72, 1]], EQ[m[z22, 2], m[z23, 1], m[z24, 2], m[z51, 2], m[z54, 1], m[z72, 1]], EQ[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]], EQ[m[z12, 0], m[z22, 0], m[z72, 0]], EQ[m[z13, 2], m[z31, 1], m[z33, 1], m[z61, 2]], EQ[m[z11, 1], m[z12, 2], m[z13, 1], m[z22, 1], m[z23, 2], m[z31, 2]], EQ[m[z11, 2], m[z12, 1], m[z22, 2], m[z23, 1], m[z24, 2], m[z74, 1]], EQ[m[z23, 2], m[z24, 1], m[z41, 2], m[z51, 1], m[z53, 1], m[z54, 2]], EQ[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], EQ[m[z12, 1], m[z13, 2], m[z22, 0], m[z22, 1], m[z23, 0], m[z24, 0], m[z24, 1], m[z53, 2], m[z54, 1], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2], m[z74, 0], m[z74, 2], m[z74, 2]], EQ[m[z43, 2], m[z51, 1], m[z53, 1], m[z54, 2], m[z71, 2], m[z74, 1]]$

Doing triangle: $k=6$; $s[[k]] = \text{EQ}[m[z12, 0], m[z22, 0], m[z72, 0]]$;

$s = \{ \text{EQ}[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]], \text{EQ}[m[z11, 1], m[z12, 2], m[z13, 1], m[z22, 1], m[z23, 2], m[z31, 2]], \text{EQ}[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[z31, 2], m[z43, 2], m[z61, 1], m[z63, 1]], \text{EQ}[m[z12, 0], m[z22, 0], m[z72, 0]], \text{EQ}[m[z13, 2], m[z31, 1], m[z33, 1], m[z61, 2]], \text{EQ}[m[w\$718, 1], m[z43, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[z11, 1], m[z12, 2], m[z13, 1], m[z33, 2], m[z71, 2], m[z72, 1]], \text{EQ}[-1, m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[z12, 1], m[z13, 2], m[z22, 0], m[z22, 1], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1], m[z72, 0], m[z72, 2], m[z72, 2]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[z11, 0], m[z12, 0], m[z13, 0], m[z31, 0], m[z33, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z61, 0], m[z63, 0]], \text{EQ}[m[z11, 2], m[z31, 1], m[z33, 1], m[z63, 2]], \text{EQ}[m[w\$718, 0], m[z22, 2], m[z23, 1], m[z51, 2], m[z72, 1]], \text{EQ}[m[w\$719, 1], m[z11, 2], m[z12, 1], m[z22, 2], m[z23, 1]], \text{EQ}[m[z41, 1], m[z43, 1], m[z53, 2], m[z61, 2]], \text{EQ}[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], \text{EQ}[m[z33, 2], m[z41, 2], m[z61, 1], m[z63, 1]] \}$

Contracting dominos; $s =$

$\{ \text{EQ}[m[z31, 2], m[z43, 2], m[z61, 1], m[z63, 1]], \text{EQ}[m[z13, 2], m[z31, 1], m[z33, 1], m[z61, 2]], \text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[z13, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[z11, 0], m[z13, 0], m[z31, 0], m[z33, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z61, 0], m[z63, 0]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[w\$718, 1], m[z43, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[z13, 0], m[z31, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0]], \text{EQ}[m[z33, 2], m[z41, 2], m[z61, 1], m[z63, 1]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[z23, 1], m[z51, 2]], \text{EQ}[m[w\$720, 0], m[z11, 1], m[z13, 1], m[z23, 2], m[z31, 2]], \text{EQ}[m[z11, 2], m[z31, 1], m[z33, 1], m[z63, 2]], \text{EQ}[m[z41, 1], m[z43, 1], m[z53, 2], m[z61, 2]], \text{EQ}[m[w\$721, 1], m[z11, 1], m[z13, 1], m[z33, 2], m[z71, 2]], \text{EQ}[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z61, 0]], \text{EQ}[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]] \}$

$\{k1, k2\} = \{1, 2\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[z31, 2], m[z43, 2], m[z61, 1], m[z63, 1]], \text{EQ}[m[z13, 2], m[z31, 1], m[z33, 1], m[z61, 2]] \}$

Found domino at $k1=1, k2=2$

Doing triangle: $k=2$; $s[[k]] = \text{EQ}[m[z13, 2], m[z33, 1], m[z\$723, 0]]$;

$s = \{ \text{EQ}[m[z43, 2], m[z63, 1], m[z\$724, 0]], \text{EQ}[m[z13, 2], m[z33, 1], m[z\$723, 0]], \text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[z13, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[z11, 0], m[z13, 0], m[z33, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z63, 0], m[z\$722, 0]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[w\$718, 1], m[z43, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[z13, 0], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0], m[z\$723, 2], m[z\$724, 1]], \text{EQ}[m[z33, 2], m[z41, 2], m[z63, 1], m[z\$722, 1], m[z\$723, 2]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[z23, 1], m[z51, 2]], \text{EQ}[m[w\$720, 0], m[z11, 1], m[z13, 1], m[z23, 2], m[z\$722, 2], m[z\$723, 1]], \text{EQ}[m[z11, 2], m[z33, 1], m[z63, 2], m[z\$722, 1], m[z\$724, 2]], \text{EQ}[m[z41, 1], m[z43, 1], m[z53, 2], m[z\$722, 2], m[z\$724, 1]], \text{EQ}[m[w\$721, 1], m[z11, 1], m[z13, 1], m[z33, 2], m[z71, 2]], \text{EQ}[m[z11, 0], m[z23, 0], m[z33, 0], m[z43, 0], m[z51, 0], m[z\$723, 1], m[z\$724, 2]], \text{EQ}[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]], \text{EQ}[m[z\$722, 0], m[z\$723, 0], m[z\$724, 0]] \}$

Doing triangle: $k=1$; $s[[k]] = \text{EQ}[m[z43, 2], m[z63, 1], m[z\$724, 0]]$;

$s = \{ \text{EQ}[m[z43, 2], m[z63, 1], m[z\$724, 0]], \text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[w\$726, 0], m[z11, 0], m[z41, 0], m[z43, 0], m[z51, 0], m[z53, 0], m[z63, 0], m[z\$722, 0]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[w\$718, 1], m[z43, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[w\$725, 2], m[z41, 0], m[z53, 0], m[z63, 0], m[z71, 0], m[z\$724, 1]], \text{EQ}[m[w\$726, 2], m[z41, 2], m[z63, 1], m[z\$722, 1]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[z23, 1], m[z51, 2]], \text{EQ}[m[w\$720, 0], m[w\$726, 1], m[z11, 1], m[z23, 2], m[z\$722, 2]], \text{EQ}[m[w\$725, 2], m[w\$726, 1], m[z11, 2], m[z63, 2], m[z\$722, 1], m[z\$724, 2]], \text{EQ}[m[z41, 1], m[z43, 1], m[z53, 2], m[z\$722, 2], m[z\$724, 1]], \text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]], \text{EQ}[m[w\$725, 1], m[z11, 0], m[z23, 0], m[z43, 0], m[z51, 0], m[z\$724, 2]], \text{EQ}[m[z41, 1], m[z43, 1], m[z51, 2], m[z63, 2]], \text{EQ}[m[w\$725, 0], m[w\$726, 0], m[z\$722, 0], m[z\$724, 0]] \}$

$t = \{ \text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[w\$726, 0], m[w\$728, 0], m[z11, 0], m[z41, 0], m[z51, 0], m[z53, 0], m[z\$722, 0]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[w\$718, 1], m[w\$727, 1], m[w\$728, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[w\$725, 2], m[w\$727, 1], m[z41, 0], m[z53, 0], m[z71, 0]], \text{EQ}[m[w\$726, 2], m[w\$727, 2], m[w\$728, 1], m[z41, 2], m[z\$722, 1]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[z23, 1], m[z51, 2]], \text{EQ}[m[w\$720, 0], m[w\$726, 1], m[z11, 1], m[z23, 2], m[z\$722, 2]], \text{EQ}[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[z11, 2], m[z\$722, 1]], \text{EQ}[m[w\$728, 1], m[z41, 1], m[z53, 2], m[z\$722, 2]], \text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]], \text{EQ}[m[w\$725, 1], m[w\$727, 2], m[z11, 0], m[z23, 0], m[z51, 0]], \text{EQ}[m[w\$727, 0], m[z41, 1], m[z51, 2]], \text{EQ}[m[w\$725, 0], m[w\$726, 0], m[w\$727, 0], m[w\$728, 0], m[z\$722, 0]] \}$

Doing triangle: $k=8$; $s[[k]] = \text{EQ}[m[w\$727, 0], m[z41, 1], m[z51, 2]]$;

$s = \{ \text{EQ}[m[w\$728, 1], m[z41, 1], m[z53, 2], m[z\$722, 2]], \text{EQ}[m[w\$725, 2], m[w\$727, 1], m[z41, 0], m[z53, 0], m[z71, 0]], \text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]], \text{EQ}[m[w\$726, 2], m[w\$727, 2], m[w\$728, 1], m[z41, 2], m[z\$722, 1]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[z23, 1], m[z51, 2]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]], \text{EQ}[m[w\$718, 1], m[w\$727, 1], m[w\$728, 2], m[z51, 1], m[z53, 1], m[z71, 2]], \text{EQ}[m[w\$727, 0], m[z41, 1], m[z51, 2]], \text{EQ}[m[w\$719, 0], m[z23, 2], m[z41, 2], m[z51, 1], m[z53, 1]], \text{EQ}[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[z11, 2], m[z\$722, 1]], \text{EQ}[m[w\$725, 1], m[w\$727, 2], m[z11, 0], m[z23, 0], m[z51, 0]], \text{EQ}[m[w\$720, 0], m[w\$726, 1], m[z11, 1], m[z23, 2], m[z\$722, 2]], \text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[w\$726, 0], m[w\$728, 0], m[z11, 0], m[z41, 0], m[z51, 0], m[z53, 0], m[z\$722, 0]], \text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]], \text{EQ}[m[w\$725, 0], m[w\$726, 0], m[w\$727, 0], m[w\$728, 0], m[z\$722, 0]] \}$

Contracting dominos; $s = \{ \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$730, 0], m[z23, 1]],$

$\text{EQ}[m[w\$719, 0], m[w\$719, 2], m[w\$719, 2], m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[z23, 0], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1]],$
 $\text{EQ}[m[w\$725, 0], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$730, 2], m[z\$722, 0]],$
 $\text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]],$
 $\text{EQ}[m[w\$719, 0], m[w\$730, 2], m[z23, 2], m[z53, 1]],$
 $\text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]],$
 $\text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$730, 1], m[z53, 2], m[z\$722, 2]],$
 $\text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]],$
 $\text{EQ}[m[w\$720, 0], m[w\$726, 1], m[z11, 1], m[z23, 2], m[z\$722, 2]],$
 $\text{EQ}[m[w\$725, 2], m[w\$730, 0], m[z53, 0], m[z71, 0]],$
 $\text{EQ}[m[w\$718, 2], m[w\$719, 1], m[w\$720, 1], m[w\$721, 2], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[z11, 0], m[z53, 0], m[z\$722, 0]], \text{EQ}[m[w\$725, 1], m[w\$730, 1], m[z11, 0], m[z23, 0]],$
 $\text{EQ}[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[z11, 2], m[z\$722, 1]],$
 $\text{EQ}[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[z53, 1], m[z71, 2]] \}$

$\{k1, k2\} = \{4, 5\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], \text{EQ}[m[w\$719, 0], m[w\$730, 2], m[z23, 2], m[z53, 1]] \}$

$\{k1, k2\} = \{4, 6\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], \text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]] \}$

$\{k1, k2\} = \{5, 6\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$719, 0], m[w\$730, 2], m[z23, 2], m[z53, 1]], \text{EQ}[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]] \}$

$\{k1, k2\} = \{4, 8\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]] \}$

$\{k1, k2\} = \{5, 8\}$;

$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$719, 0], m[w\$730, 2], m[z23, 2], m[z53, 1]], \text{EQ}[m[w\$719, 1], m[w\$721, 0], m[z11, 2], m[z23, 1]] \}$

Found domino at $k1=5, k2=8$

Doing triangle: k=8; s[[k]]=EQ[m[w\$721, 0], m[z11, 2], m[z\$732, 0]];

s={EQ[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$730, 0], m[z\$731, 1], m[z\$733, 2]], EQ[m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1], m[z\$731, 0], m[z\$731, 1], m[z\$732, 1], m[z\$732, 2], m[z\$733, 2]], EQ[m[w\$725, 0], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$730, 2], m[z\$722, 0]], EQ[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], EQ[m[w\$730, 2], m[z53, 1], m[z\$733, 0]], EQ[m[w\$721, 1], m[w\$725, 0], m[z11, 1], m[z71, 2]], EQ[m[w\$728, 1], m[w\$729, 2], m[w\$730, 1], m[z53, 2], m[z\$722, 2]], EQ[m[w\$721, 0], m[z11, 2], m[z\$732, 0]], EQ[m[w\$720, 0], m[w\$726, 1], m[z11, 1], m[z\$722, 2], m[z\$731, 2], m[z\$732, 1]], EQ[m[w\$725, 2], m[w\$730, 0], m[z53, 0], m[z71, 0]], EQ[m[w\$718, 2], m[w\$720, 1], m[w\$721, 2], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[z11, 0], m[z53, 0], m[z\$722, 0], m[z\$731, 2], m[z\$733, 1]], EQ[m[w\$725, 1], m[w\$730, 1], m[z11, 0], m[z\$732, 2], m[z\$733, 1]], EQ[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[z11, 2], m[z\$722, 1]], EQ[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[z53, 1], m[z71, 2]], EQ[m[z\$731, 0], m[z\$732, 0], m[z\$733, 0]]}

Doing triangle: k=5; s[[k]]=EQ[m[w\$730, 2], m[z53, 1], m[z\$733, 0]];

s={EQ[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$730, 0], m[z\$731, 1], m[z\$733, 2]], EQ[-1, m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[w\$734, 1], m[w\$734, 2], m[w\$735, 1], m[w\$735, 2], m[z53, 2], m[z71, 0], m[z71, 1], m[z71, 1], m[z\$731, 0], m[z\$731, 1], m[z\$733, 2]], EQ[m[w\$725, 0], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$730, 2], m[z\$722, 0]], EQ[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], EQ[m[w\$730, 2], m[z53, 1], m[z\$733, 0]], EQ[m[w\$725, 0], m[w\$735, 0], m[z71, 2]], EQ[m[w\$728, 1], m[w\$729, 2], m[w\$730, 1], m[z53, 2], m[z\$722, 2]], EQ[m[w\$720, 0], m[w\$726, 1], m[w\$734, 1], m[z\$722, 2], m[z\$731, 2]], EQ[m[w\$725, 2], m[w\$730, 0], m[z53, 0], m[z71, 0]], EQ[m[w\$718, 2], m[w\$720, 1], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$734, 0], m[z53, 0], m[z\$722, 0], m[z\$731, 2], m[z\$733, 1]], EQ[m[w\$725, 1], m[w\$730, 1], m[w\$735, 2], m[z\$733, 1]], EQ[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[w\$734, 2], m[w\$735, 1], m[z\$722, 1]], EQ[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[z53, 1], m[z71, 2]], EQ[m[w\$734, 0], m[w\$735, 0], m[z\$731, 0], m[z\$733, 0]]}

t={EQ[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$736, 2], m[z\$731, 1]], EQ[-1, m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[w\$734, 1], m[w\$734, 2], m[w\$735, 1], m[w\$735, 2], m[w\$737, 2], m[z71, 0], m[z71, 1], m[z71, 1], m[z\$731, 0], m[z\$731, 1]], EQ[m[w\$725, 0], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$736, 1], m[w\$737, 2], m[z\$722, 0]], EQ[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], EQ[m[w\$725, 0], m[w\$735, 0], m[z71, 2]], EQ[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]], EQ[m[w\$720, 0], m[w\$726, 1], m[w\$734, 1], m[z\$722, 2], m[z\$731, 2]], EQ[m[w\$725, 2], m[w\$737, 0], m[z71, 0]], EQ[m[w\$718, 2], m[w\$720, 1], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$734, 0], m[w\$736, 1], m[z\$722, 0], m[z\$731, 2]], EQ[m[w\$725, 1], m[w\$735, 2], m[w\$737, 1]], EQ[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[w\$734, 2], m[w\$735, 1], m[z\$722, 1]], EQ[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[w\$736, 2], m[w\$737, 1], m[z71, 2]], EQ[m[w\$734, 0], m[w\$735, 0], m[w\$736, 0], m[w\$737, 0], m[z\$731, 0]]}

Doing triangle: $k=1$; $s[[k]] = \text{EQ}[m[w\$725, 2], m[w\$737, 0], m[z71, 0]]$;

$$s = \{ \text{EQ}[m[w\$725, 2], m[w\$737, 0], m[z71, 0]], \text{EQ}[m[w\$718, 2], m[w\$720, 1], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$734, 0], m[w\$736, 1], m[z\$722, 0], m[z\$731, 2]], \text{EQ}[m[w\$725, 2], m[w\$726, 1], m[w\$728, 2], m[w\$734, 2], m[w\$735, 1], m[z\$722, 1]], \text{EQ}[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[w\$736, 2], m[w\$737, 1], m[z71, 2]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$736, 2], m[z\$731, 1]], \text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]], \text{EQ}[-1, m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$725, 1], m[w\$726, 2], m[w\$734, 1], m[w\$734, 2], m[w\$735, 1], m[w\$735, 2], m[w\$737, 2], m[z71, 0], m[z71, 1], m[z71, 1], m[z\$731, 0], m[z\$731, 1]], \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], \text{EQ}[m[w\$734, 0], m[w\$735, 0], m[w\$736, 0], m[w\$737, 0], m[z\$731, 0]], \text{EQ}[m[w\$725, 1], m[w\$735, 2], m[w\$737, 1]], \text{EQ}[m[w\$720, 0], m[w\$726, 1], m[w\$734, 1], m[z\$722, 2], m[z\$731, 2]], \text{EQ}[m[w\$725, 0], m[w\$735, 0], m[z71, 2]], \text{EQ}[m[w\$725, 0], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$736, 1], m[w\$737, 2], m[z\$722, 0]] \}$$

Doing bigon: $k=5$; $s[[k]] = \text{EQ}[m[w\$735, 2], m[w\$738, 0]]$;

$$s = \{ \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]], \text{EQ}[m[w\$726, 1], m[w\$728, 2], m[w\$734, 2], m[w\$735, 1], m[w\$738, 1], m[w\$739, 2], m[z\$722, 1]], \text{EQ}[m[w\$734, 0], m[w\$735, 0], m[w\$736, 0], m[w\$738, 2], m[w\$739, 1], m[z\$731, 0]], \text{EQ}[-1, m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$726, 2], m[w\$734, 1], m[w\$734, 2], m[w\$735, 1], m[w\$735, 2], m[w\$738, 0], m[w\$738, 1], m[w\$739, 0], m[w\$739, 1], m[z\$731, 0], m[z\$731, 1]], \text{EQ}[m[w\$735, 2], m[w\$738, 0]], \text{EQ}[m[w\$718, 2], m[w\$720, 1], m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$734, 0], m[w\$736, 1], m[z\$722, 0], m[z\$731, 2]], \text{EQ}[m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$736, 1], m[w\$739, 0], m[z\$722, 0]], \text{EQ}[m[w\$735, 0], m[w\$738, 2]], \text{EQ}[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$736, 2], m[z\$731, 1]], \text{EQ}[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[w\$736, 2], m[w\$739, 2]], \text{EQ}[m[w\$720, 0], m[w\$726, 1], m[w\$734, 1], m[z\$722, 2], m[z\$731, 2]], \text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]] \}$$

Contracting dominos; $s = \{ \text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]],$

$$\text{EQ}[m[w\$726, 1], m[w\$728, 2], m[w\$734, 2], m[w\$739, 2], m[z\$722, 1]],$$

$$\text{EQ}[m[w\$718, 0], m[w\$720, 1], m[w\$729, 0], m[w\$736, 2], m[z\$731, 1]],$$

$$\text{EQ}[m[w\$734, 0], m[w\$736, 0], m[w\$739, 1], m[z\$731, 0]], \text{EQ}[m[w\$718, 2], m[w\$720, 1],$$

$$m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$734, 0], m[w\$736, 1], m[z\$722, 0], m[z\$731, 2]],$$

$$\text{EQ}[m[w\$726, 0], m[w\$728, 0], m[w\$729, 1], m[w\$736, 1], m[w\$739, 0], m[z\$722, 0]],$$

$$\text{EQ}[m[w\$720, 0], m[w\$726, 1], m[w\$734, 1], m[z\$722, 2], m[z\$731, 2]],$$

$$\text{EQ}[m[w\$718, 1], m[w\$728, 2], m[w\$729, 2], m[w\$736, 2], m[w\$739, 2]],$$

$$\text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]],$$

$$\text{EQ}[-1, m[w\$720, 0], m[w\$720, 2], m[w\$720, 2], m[w\$726, 2], m[w\$734, 1],$$

$$m[w\$734, 2], m[w\$739, 0], m[w\$739, 1], m[z\$731, 0], m[z\$731, 1]] \}$$

$\{k1, k2\} = \{1, 4\}$;

$$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]], \text{EQ}[m[w\$734, 0], m[w\$736, 0], m[w\$739, 1], m[z\$731, 0]] \}$$

$\{k1, k2\} = \{1, 9\}$;

$$s[[\{k1, k2\}]] = \{ \text{EQ}[m[w\$728, 1], m[w\$729, 2], m[w\$736, 0], m[z\$722, 2]], \text{EQ}[m[w\$726, 2], m[w\$728, 1], m[w\$729, 0], m[z\$722, 1]] \}$$

Found domino at $k1=1, k2=9$

```
Doing triangle: k=9; s[[k]] = EQ[m[w$726, 2], m[w$728, 1], m[z$741, 0]];
s = {EQ[m[w$728, 1], m[w$736, 0], m[z$742, 0]], EQ[m[w$726, 1], m[w$728, 2], m[w$734, 2], m[w$739,
2], m[z$740, 1], m[z$742, 2]], EQ[m[w$718, 0], m[w$720, 1], m[w$736, 2], m[z$731, 1], m[z$740,
2], m[z$742, 1]], EQ[m[w$734, 0], m[w$736, 0], m[w$739, 1], m[z$731, 0]], EQ[m[w$718, 2], m[
w$720, 1], m[w$726, 0], m[w$728, 0], m[w$734, 0], m[w$736, 1], m[z$731, 2], m[z$740, 0]], EQ[m[
w$726, 0], m[w$728, 0], m[w$736, 1], m[w$739, 0], m[z$740, 0]], EQ[m[w$720, 0], m[w$726, 1], m[
w$734, 1], m[z$731, 2], m[z$740, 2], m[z$741, 1]], EQ[m[w$718, 1], m[w$728, 2], m[w$736, 2], m[
w$739, 2], m[z$740, 1], m[z$741, 2]], EQ[m[w$726, 2], m[w$728, 1], m[z$741, 0]], EQ[-1, m[w$720
, 0], m[w$720, 2], m[w$720, 2], m[w$726, 2], m[w$734, 1], m[w$734, 2], m[w$739, 0], m[w$739, 1],
m[z$731, 0], m[z$731, 1]], EQ[m[z$740, 0], m[z$741, 0], m[z$742, 0]]}
```

```
t=Failed
```

```
{k1, k2} = {4, 9};
```

```
s[{{k1, k2}}] = {EQ[m[w$734, 0], m[w$736, 0], m[w$739, 1], m[z$731, 0]], EQ[m[w$726, 2], m[w$728, 1]
, m[w$729, 0], m[z$722, 1]]}
```

```
{EQ[m[w$728, 1], m[w$729, 2], m[w$736, 0], m[z$722, 2]],
EQ[m[w$726, 1], m[w$728, 2], m[w$734, 2], m[w$739, 2], m[z$722, 1]],
EQ[m[w$718, 0], m[w$720, 1], m[w$729, 0], m[w$736, 2], m[z$731, 1]],
EQ[m[w$734, 0], m[w$736, 0], m[w$739, 1], m[z$731, 0]],
EQ[m[w$718, 2], m[w$720, 1], m[w$726, 0], m[w$728, 0],
m[w$729, 1], m[w$734, 0], m[w$736, 1], m[z$722, 0], m[z$731, 2]],
EQ[m[w$726, 0], m[w$728, 0], m[w$729, 1], m[w$736, 1], m[w$739, 0], m[z$722, 0]],
EQ[m[w$720, 0], m[w$726, 1], m[w$734, 1], m[z$722, 2], m[z$731, 2]],
EQ[m[w$718, 1], m[w$728, 2], m[w$729, 2], m[w$736, 2], m[w$739, 2]],
EQ[m[w$726, 2], m[w$728, 1], m[w$729, 0], m[z$722, 1]],
EQ[-1, m[w$720, 0], m[w$720, 2], m[w$720, 2], m[w$726, 2], m[w$734, 1], m[w$734, 2],
m[w$739, 0], m[w$739, 1], m[z$731, 0], m[z$731, 1]]}, {4, 5, 5, 4, 9, 6, 5, 5, 4, 11}, 10}
```

```
MatchQ[
```

```
{EQ[m[z11, 0], m[z23, 0], m[z33, 0], m[z41, 0]],
EQ[m[z13, 2], m[z31, 1], m[z33, 1], m[z41, 2]],
{EQ[m[Z1_, p1_], m[Z2_, p2_], m[W1_, pa_], m[W2_, pb_]],
EQ[m[W1_, pc_], m[W2_, pd_], m[Z3_, p3_], m[Z4_, p4_]]}
```

```
]
```

```
True
```

```
Length /@ s
```

```
{4, 5, 10, 12, 4, 4, 5, 5, 4, 4, 4, 5}
```

```
RandomPermutation[
```

```
Flatten@Table[T[k, i, j], {k, Length[s]}, {i, 2, Length[s[[k]]}], {j, i}]
{T[2, 4, 2], T[2, 4, 4], T[1, 6, 3], T[1, 6, 5], T[1, 5, 1], T[1, 6, 1],
T[1, 5, 2], T[2, 4, 3], T[1, 4, 4], T[2, 4, 1], T[1, 5, 5], T[2, 6, 6],
T[1, 2, 1], T[1, 4, 2], T[1, 5, 4], T[1, 6, 4], T[1, 4, 1], T[1, 2, 2], T[1, 3, 2],
T[1, 6, 2], T[1, 3, 3], T[2, 6, 5], T[2, 3, 3], T[2, 6, 4], T[1, 3, 1], T[2, 3, 1],
T[2, 3, 2], T[2, 5, 3], T[2, 6, 3], T[2, 2, 2], T[1, 4, 3], T[2, 6, 1], T[2, 5, 5],
T[1, 6, 6], T[2, 5, 2], T[2, 6, 2], T[2, 2, 1], T[2, 5, 1], T[1, 5, 3], T[2, 5, 4]}
```

```
eqns = (Times @@ # == 1) & /@ Rest[Reverse[s]] /.
  {m[w_, 0] => w, m[w_, 1] => 1 - 1/w, m[w_, 2] => 1/(1 - w)}
  {
    
$$\left\{ \frac{\left(1 - \frac{1}{w\$20}\right) \left(1 - \frac{1}{w\$22}\right)^2 w\$22}{(1 - w\$20)^2} == 1 \right\}$$

  }
```

```
vars = Complement[Union[Cases[eqns, _Symbol, Infinity]], {z1}]
{w$20, w$22}
```

```
{
  sol = FindRoot @@ Join[{eqns /. z1 -> 1},
    ({#, Exp[Random[] Pi I/2]} & /@ vars), {MaxIterations -> 1000}],
  Max[Abs[({(First /@ (eqns /. z1 -> 1)) /. sol) - 1}],
  Plus @@ (Vol /@ (vars /. sol))
}
```

FindRoot::frnum: Function {-0.950343 + 0.0208871 i} is not a length 2 list of numbers at {w\$20, w\$22} = {0.709676 + 0.704528 i, 0.979475 + 0.201566 i}.

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FindRoot::frnum: Function {-0.950343 + 0.0208871 i} is not a length 2 list of numbers at {w\$20, w\$22} = {0.709676 + 0.704528 i, 0.979475 + 0.201566 i}.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps: {FindRoot[$\left\{ \frac{\left(1 - \frac{1}{w\$20}\right) \left(1 - \frac{1}{w\$22}\right)^2 w\$22}{(1 - w\$20)^2} == 1 \right\}$, {w\$20, 0.709676 + 0.704528 i}, {<<1>>}, MaxIterations->1000]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps: {FindRoot[$\left\{ \frac{\left(1 - \frac{1}{w\$20}\right) \left(1 - \frac{1}{w\$22}\right)^2 w\$22}{(1 - w\$20)^2} == 1 \right\}$, {w\$20, 0.709676 + 0.704528 i}, {<<1>>}, MaxIterations->1000]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

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General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

NIntegrate::nlim: u = {Arg[w\$20], Arg[w\$22]} is not a valid limit of integration.

NIntegrate::nlim: u = {Arg[1 - 1/w\$20], Arg[1 - 1/w\$22]} is not a valid limit of integration.

NIntegrate::nlim: u = {Arg[1/(1 - 1/w\$20)], Arg[1/(1 - 1/w\$22)]} is not a valid limit of integration.

General::stop: Further output of NIntegrate::nlim will be suppressed during this calculation.

```

{FindRoot[{{(1 - 1/w$20) (1 - 1/w$22)^2 w$22
(1 - w$20)^2 == 1},
{w$20, 0.709676 + 0.704528 i}, {w$22, 0.979475 + 0.201566 i}, MaxIterations -> 1000},
Abs[-1 + ( (1 - 1/w$20) (1 - 1/w$22)^2 w$22 / (1 - w$20)^2 ) /. FindRoot[{{(1 - 1/w$20) (1 - 1/w$22)^2 w$22
(1 - w$20)^2 == 1},
{w$20, 0.709676 + 0.704528 i}, {w$22, 0.979475 + 0.201566 i}, MaxIterations -> 1000}]]],
-NIntegrate[Log[Abs[2 Sin[u]]], {u, 0,
Arg[1 - 1/FindRoot[{{(1 - 1/w$20) (1 - 1/w$22)^2 w$22
(1 - w$20)^2 == 1}, {w$20, 0.709676 + 0.704528 i},
{w$22, 0.979475 + 0.201566 i}, MaxIterations -> 1000}]]] - NIntegrate[
Log[Abs[2 Sin[u]]], {u, 0, Arg[1 / (1 - FindRoot[{{(1 - 1/w$20) (1 - 1/w$22)^2 w$22
(1 - w$20)^2 == 1}, {w$20,
0.709676 + 0.704528 i}, {w$22, 0.979475 + 0.201566 i}, MaxIterations -> 1000}]]]}] -
NIntegrate[Log[Abs[2 Sin[u]]], {u, 0, Arg[FindRoot[{{(1 - 1/w$20) (1 - 1/w$22)^2 w$22
(1 - w$20)^2 == 1},
{w$20, 0.709676 + 0.704528 i}, {w$22, 0.979475 + 0.201566 i}, MaxIterations -> 1000}]]]}] -
NIntegrate[Log[Abs[2 Sin[u]]], {u, 0, {Arg[1 - 1/w$20], Arg[1 - 1/w$22]}]}] -
NIntegrate[Log[Abs[2 Sin[u]]],
{u, 0, {Arg[1/(1 - w$20)], Arg[1/(1 - w$22)]}}]}] -
NIntegrate[Log[Abs[2 Sin[u]]], {u, 0, {Arg[w$20], Arg[w$22]}]}]

```

sols = vars /. Solve[eqns /. z1 -> 1, vars]

General::spell1: Possible spelling error: new symbol name "sols" is similar to existing symbol "sol".

Solve::svars: Equations may not give solutions for all "solve" variables.

$$\left\{ \left\{ \frac{\sqrt{w\$22} - \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{2 \sqrt{w\$22}}, w\$22 \right\}, \left\{ \frac{\sqrt{w\$22} + \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{2 \sqrt{w\$22}}, w\$22 \right\} \right\}$$

N[sols]

$$\left\{ \left\{ \frac{0.5 \left(\sqrt{w\$22} - 1. \sqrt{4. - 7. w\$22 + 4. w\$22^2} \right)}{\sqrt{w\$22}}, w\$22 \right\}, \right. \\ \left. \left\{ \frac{0.5 \left(\sqrt{w\$22} + \sqrt{4. - 7. w\$22 + 4. w\$22^2} \right)}{\sqrt{w\$22}}, w\$22 \right\} \right\}$$

(Plus @@ Vol /@ #) & /@ sols

NIntegrate::nlim: u = Arg $\left[\frac{\sqrt{w\$22} - 1. \sqrt{4. - 7. w\$22 + 4. w\$22^2}}{\sqrt{w\$22}}\right]$ is not a valid limit of integration.

NIntegrate::nlim: u = Arg $\left[1. - \frac{2. \sqrt{w\$22}}{\sqrt{w\$22} - 1. \sqrt{4. - 7. w\$22 + 4. w\$22^2}}\right]$ is not a valid limit of integration.

NIntegrate::nlim: u = Arg $\left[\frac{1}{1. - \frac{0.5 \left(\sqrt{w\$22} - 1. \sqrt{4. - 7. w\$22 + 4. w\$22^2} \right)}{\sqrt{w\$22}}}\right]$ is not a valid limit of integration.

General::stop: Further output of NIntegrate::nlim will be suppressed during this calculation.

$$\left\{ -\text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[1 - \frac{1}{w\$22}\right]\right\}\right] - \right. \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{1}{1 - w\$22}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[w\$22\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{\sqrt{w\$22} - \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{\sqrt{w\$22}}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[1 - \frac{2 \sqrt{w\$22}}{\sqrt{w\$22} - \sqrt{4 - 7 w\$22 + 4 w\$22^2}}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{1}{1 - \frac{\sqrt{w\$22} - \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{2 \sqrt{w\$22}}}\right]\right\}\right], \\ -\text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[1 - \frac{1}{w\$22}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{1}{1 - w\$22}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[w\$22\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{\sqrt{w\$22} + \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{\sqrt{w\$22}}\right]\right\}\right] - \\ \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[1 - \frac{2 \sqrt{w\$22}}{\sqrt{w\$22} + \sqrt{4 - 7 w\$22 + 4 w\$22^2}}\right]\right\}\right] - \\ \left. \text{NIntegrate}\left[\text{Log}\left[\text{Abs}\left[2 \text{Sin}\left[u\right]\right]\right], \left\{u, 0, \text{Arg}\left[\frac{1}{1 - \frac{\sqrt{w\$22} + \sqrt{4 - 7 w\$22 + 4 w\$22^2}}{2 \sqrt{w\$22}}}\right]\right\}\right] \right\}$$

```
Mod[Range[10, 20][[{2, 3}]], 7]
```

```
{4, 5}
```