

Pensieve header: Testing suite for the FullDoPeGDO project.

```
In[*]:= Date []  
SetDirectory ["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];  
Once [<< KnotTheory`];  
Once [Get@"../Profile/Profile.m"];  
$k = 1;  
<< Engine.m  
<< Objects.m
```

```
Out[*]:= {2021, 1, 1, 9, 44, 49.1717724}
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

```
In[*]:= HL[ $\mathcal{E}$ ] := Style[ $\mathcal{E}$ , Background → If[TrueQ@ $\mathcal{E}$ , Green, Red]];
```

In[*]:= **\$k = 2; cm_{1,2→1} // cm_{1,3→1}**

$$\text{Out[*]} = \mathbb{E}_{\{1,2,3\} \rightarrow \{1\}} \left[\mathbf{a}_1 (\alpha_1 + \alpha_2 + \alpha_3) + \mathbf{b}_1 \beta_1 + \mathbf{b}_1 \beta_2 + \mathbf{b}_1 \beta_3 + \right. \\
 \mathbf{y}_1 \eta_1 + \frac{\mathbf{y}_1 \eta_2}{\mathcal{A}_1} + \frac{\mathbf{y}_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{x}_1 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \mathbf{b}_1 \eta_2 \xi_1 + \frac{\mathbf{b}_1 \eta_3 \xi_1}{\mathcal{A}_2} + \frac{\mathbf{x}_1 \xi_2}{\mathcal{A}_3} + \mathbf{b}_1 \eta_3 \xi_2 + \mathbf{x}_1 \xi_3, \\
 - \frac{\mathbf{y}_1 \beta_1 \eta_2}{\mathcal{A}_1} - \frac{\mathbf{y}_1 \beta_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{y}_1 \beta_2 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{x}_1 \beta_2 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} - \frac{\mathbf{x}_1 \beta_3 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \mathbf{a}_1 \eta_2 \xi_1 - \frac{\mathbf{y}_1 \eta_2^2 \xi_1}{\mathcal{A}_1} + \frac{\mathbf{a}_1 \eta_3 \xi_1}{\mathcal{A}_2} - \\
 \frac{\mathbf{b}_1 \beta_2 \eta_3 \xi_1}{\mathcal{A}_2} - \frac{2 \mathbf{y}_1 \eta_2 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{y}_1 \eta_3^2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2} - \frac{\mathbf{x}_1 \eta_2 \xi_1^2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{1}{2} \mathbf{b}_1 \eta_2^2 \xi_1^2 - \frac{\mathbf{x}_1 \eta_3 \xi_1^2}{\mathcal{A}_2^2 \mathcal{A}_3} - \frac{\mathbf{b}_1 \eta_2 \eta_3 \xi_1^2}{\mathcal{A}_2} - \\
 \frac{\mathbf{b}_1 \eta_3^2 \xi_1^2}{2 \mathcal{A}_2^2} - \frac{\mathbf{x}_1 \beta_3 \xi_2}{\mathcal{A}_3} + \mathbf{a}_1 \eta_3 \xi_2 - \frac{\mathbf{y}_1 \eta_3^2 \xi_2}{\mathcal{A}_1 \mathcal{A}_2} - \frac{2 \mathbf{x}_1 \eta_3 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{\mathbf{b}_1 \eta_3^2 \xi_1 \xi_2}{\mathcal{A}_2} - \frac{\mathbf{x}_1 \eta_3 \xi_2^2}{\mathcal{A}_3} - \frac{1}{2} \mathbf{b}_1 \eta_3^2 \xi_2^2, \\
 \frac{\mathbf{y}_1 \beta_1^2 \eta_2}{2 \mathcal{A}_1} + \frac{\mathbf{y}_1 \beta_1^2 \eta_3}{2 \mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{y}_1 \beta_1 \beta_2 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{y}_1 \beta_2^2 \eta_3}{2 \mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{x}_1 \beta_2^2 \xi_1}{2 \mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \beta_2 \beta_3 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \beta_3^2 \xi_1}{2 \mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{y}_1 \beta_1 \eta_2^2 \xi_1}{\mathcal{A}_1} - \\
 \frac{\mathbf{a}_1 \beta_2 \eta_3 \xi_1}{\mathcal{A}_2} + \frac{\mathbf{b}_1 \beta_2^2 \eta_3 \xi_1}{2 \mathcal{A}_2} + \frac{2 \mathbf{y}_1 \beta_1 \eta_2 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2} + \frac{2 \mathbf{y}_1 \beta_2 \eta_2 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{y}_1 \beta_1 \eta_3^2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2} + \frac{2 \mathbf{y}_1 \beta_2 \eta_3^2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2} + \\
 \frac{\mathbf{x}_1 \beta_2 \eta_2 \xi_1^2}{\mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \beta_3 \eta_2 \xi_1^2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{1}{2} \mathbf{a}_1 \eta_2^2 \xi_1^2 + \frac{\mathbf{y}_1 \eta_2^3 \xi_1^2}{\mathcal{A}_1} + \frac{2 \mathbf{x}_1 \beta_2 \eta_3 \xi_1^2}{\mathcal{A}_2^2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \beta_3 \eta_3 \xi_1^2}{\mathcal{A}_2^2 \mathcal{A}_3} - \frac{\mathbf{a}_1 \eta_2 \eta_3 \xi_1^2}{\mathcal{A}_2} + \\
 \frac{\mathbf{b}_1 \beta_2 \eta_2 \eta_3 \xi_1^2}{\mathcal{A}_2} + \frac{3 \mathbf{y}_1 \eta_2^2 \eta_3 \xi_1^2}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{a}_1 \eta_3^2 \xi_1^2}{2 \mathcal{A}_2^2} + \frac{\mathbf{b}_1 \beta_2 \eta_3^2 \xi_1^2}{\mathcal{A}_2^2} + \frac{3 \mathbf{y}_1 \eta_2 \eta_3^2 \xi_1^2}{\mathcal{A}_1 \mathcal{A}_2^2} + \frac{\mathbf{y}_1 \eta_3^3 \xi_1^2}{\mathcal{A}_1 \mathcal{A}_2^2} + \frac{\mathbf{x}_1 \eta_2^2 \xi_1^3}{\mathcal{A}_2 \mathcal{A}_3} + \\
 \frac{1}{3} \mathbf{b}_1 \eta_2^3 \xi_1^3 + \frac{2 \mathbf{x}_1 \eta_2 \eta_3 \xi_1^3}{\mathcal{A}_2^2 \mathcal{A}_3} + \frac{\mathbf{b}_1 \eta_2^2 \eta_3 \xi_1^3}{\mathcal{A}_2} + \frac{\mathbf{x}_1 \eta_3^2 \xi_1^3}{\mathcal{A}_2^2 \mathcal{A}_3} + \frac{\mathbf{b}_1 \eta_2 \eta_3^2 \xi_1^3}{\mathcal{A}_2^2} + \frac{\mathbf{b}_1 \eta_3^3 \xi_1^3}{3 \mathcal{A}_2^3} + \frac{\mathbf{x}_1 \beta_3^2 \xi_2}{2 \mathcal{A}_3} + \\
 \frac{\mathbf{y}_1 \beta_1 \eta_3^2 \xi_2}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{y}_1 \beta_2 \eta_3^2 \xi_2}{\mathcal{A}_1 \mathcal{A}_2} + \frac{2 \mathbf{x}_1 \beta_2 \eta_3 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3} + \frac{2 \mathbf{x}_1 \beta_3 \eta_3 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{\mathbf{a}_1 \eta_3^2 \xi_1 \xi_2}{\mathcal{A}_2} + \frac{\mathbf{b}_1 \beta_2 \eta_3^2 \xi_1 \xi_2}{\mathcal{A}_2} + \\
 \frac{2 \mathbf{y}_1 \eta_2 \eta_3^2 \xi_1 \xi_2}{\mathcal{A}_1 \mathcal{A}_2} + \frac{2 \mathbf{y}_1 \eta_3^3 \xi_1 \xi_2}{\mathcal{A}_1 \mathcal{A}_2^2} + \frac{2 \mathbf{x}_1 \eta_2 \eta_3 \xi_1^2 \xi_2}{\mathcal{A}_2 \mathcal{A}_3} + \frac{3 \mathbf{x}_1 \eta_3^2 \xi_1^2 \xi_2}{\mathcal{A}_2^2 \mathcal{A}_3} + \frac{\mathbf{b}_1 \eta_2 \eta_3^2 \xi_1^2 \xi_2}{\mathcal{A}_2} + \frac{\mathbf{b}_1 \eta_3^3 \xi_1^2 \xi_2}{\mathcal{A}_2^2} + \\
 \left. \frac{\mathbf{x}_1 \beta_3 \eta_3 \xi_2^2}{\mathcal{A}_3} - \frac{1}{2} \mathbf{a}_1 \eta_3^2 \xi_2^2 + \frac{\mathbf{y}_1 \eta_3^3 \xi_2^2}{\mathcal{A}_1 \mathcal{A}_2} + \frac{3 \mathbf{x}_1 \eta_3^2 \xi_1 \xi_2^2}{\mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{b}_1 \eta_3^3 \xi_1 \xi_2^2}{\mathcal{A}_2} + \frac{\mathbf{x}_1 \eta_3^2 \xi_2^3}{\mathcal{A}_3} + \frac{1}{3} \mathbf{b}_1 \eta_3^3 \xi_2^3 \right]$$

In[*]:= **Timing@Block[{\$k = 2}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]**

Out[*]:= {0.609375, True}

In[*]:= **Timing@Block[{\$k = 3}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]**

Out[*]:= {1.375, True}

In[*]:= **Timing@Block[{\$k = 4}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]**

Out[*]:= {2.5625, True}

In[*]:= **Timing@Block[{\$k = 5}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]**

Out[*]:= {6.21875, True}

In[]:= **Timing@Block**[{**\$k = 6**}, **HL**[(**cm_{1,2→1}** // **cm_{1,3→1}**) ≡ (**cm_{2,3→2}** // **cm_{1,2→1}**)]]

Out[]:= {11.8125, **True**}

In[]:= **Timing@Block**[{**\$k = 7**}, **HL**[(**cm_{1,2→1}** // **cm_{1,3→1}**) ≡ (**cm_{2,3→2}** // **cm_{1,2→1}**)]]

Out[]:= {23.5625, **True**}

In[]:= **Timing@Block**[{**\$k = 8**}, **HL**[(**cm_{1,2→1}** // **cm_{1,3→1}**) ≡ (**cm_{2,3→2}** // **cm_{1,2→1}**)]]

Out[]:= {40., **True**}

In[]:= **ao_{1→2}**

Out[]:= $E_{\{1\} \rightarrow \{2\}} [a_2 \alpha_1 + x_2 \xi_1, \theta, \theta]$

In[]:= **am_{1,2→3}**

Out[]:= $E_{\{1,2\} \rightarrow \{3\}} \left[a_3 (\alpha_1 + \alpha_2) + x_3 \left(\frac{\xi_1}{\mathcal{A}_2} + \xi_2 \right), \theta, \theta \right]$

In[]:= **bm_{1,2→3}**

Out[]:= $E_{\{1,2\} \rightarrow \{3\}} \left[b_3 (\beta_1 + \beta_2) + y_3 (\eta_1 + \eta_2), -y_3 \beta_1 \eta_2, \frac{1}{2} y_3 \beta_1^2 \eta_2 \right]$

In[]:= **Block**[{**\$k = 8**}, **R_{1,2}**]

Out[]:= $E_{\{\} \rightarrow \{1,2\}} \left[\hbar a_2 b_1 + \hbar x_2 y_1, -\frac{1}{4} \hbar^3 x_2^2 y_1^2, \frac{1}{9} \hbar^5 x_2^3 y_1^3, \frac{1}{48} (\hbar^5 x_2^2 y_1^2 - 3 \hbar^7 x_2^4 y_1^4), \right.$
 $-\frac{1}{36} \hbar^7 x_2^3 y_1^3 + \frac{1}{25} \hbar^9 x_2^5 y_1^5, -\frac{1}{480} \hbar^7 x_2^2 y_1^2 + \frac{1}{32} \hbar^9 x_2^4 y_1^4 - \frac{1}{36} \hbar^{11} x_2^6 y_1^6,$
 $\frac{7 \hbar^9 x_2^3 y_1^3}{1080} - \frac{1}{30} \hbar^{11} x_2^5 y_1^5 + \frac{1}{49} \hbar^{13} x_2^7 y_1^7, \frac{17 \hbar^9 x_2^2 y_1^2}{80640} - \frac{17 \hbar^{11} x_2^4 y_1^4}{1280} + \frac{5}{144} \hbar^{13} x_2^6 y_1^6 - \frac{1}{64} \hbar^{15} x_2^8 y_1^8,$
 $\left. -\frac{809 \hbar^{11} x_2^3 y_1^3}{544320} + \frac{9}{400} \hbar^{13} x_2^5 y_1^5 - \frac{1}{28} \hbar^{15} x_2^7 y_1^7 + \frac{1}{81} \hbar^{17} x_2^9 y_1^9 \right]$

In[]:= **Timing@Block**[{**\$k = 8**},

HL /@ {(**am_{1,2→1}** // **am_{1,3→1}**) ≡ **Echo**@(**am_{2,3→2}** // **am_{1,2→1}**),
 (**bm_{1,2→1}** // **bm_{1,3→1}**) ≡ **Echo**@(**bm_{2,3→2}** // **bm_{1,2→1}**) }

]

$$\gg \mathbb{E}_{\{1,2,3\} \rightarrow \{1\}} \left[\mathbf{a}_1 (\alpha_1 + \alpha_2 + \alpha_3) + \frac{\mathbf{x}_1 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \frac{\mathbf{x}_1 \xi_2}{\mathcal{A}_3} + \mathbf{x}_1 \xi_3, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0} \right]$$

$$\gg \mathbb{E}_{\{1,2,3\} \rightarrow \{1\}} \left[\mathbf{b}_1 \beta_1 + \mathbf{b}_1 \beta_2 + \mathbf{b}_1 \beta_3 + \mathbf{y}_1 \eta_1 + \mathbf{y}_1 \eta_2 + \mathbf{y}_1 \eta_3, \right. \\ -\mathbf{y}_1 \beta_1 \eta_2 - \mathbf{y}_1 \beta_1 \eta_3 - \mathbf{y}_1 \beta_2 \eta_3, \frac{1}{2} \mathbf{y}_1 \beta_1^2 \eta_2 + \frac{1}{2} \mathbf{y}_1 \beta_1^2 \eta_3 + \mathbf{y}_1 \beta_1 \beta_2 \eta_3 + \frac{1}{2} \mathbf{y}_1 \beta_2^2 \eta_3, \\ -\frac{1}{6} \mathbf{y}_1 \beta_1^3 \eta_2 - \frac{1}{6} \mathbf{y}_1 \beta_1^3 \eta_3 - \frac{1}{2} \mathbf{y}_1 \beta_1^2 \beta_2 \eta_3 - \frac{1}{2} \mathbf{y}_1 \beta_1 \beta_2^2 \eta_3 - \frac{1}{6} \mathbf{y}_1 \beta_2^3 \eta_3, \\ \frac{1}{24} \mathbf{y}_1 \beta_1^4 \eta_2 + \frac{1}{24} \mathbf{y}_1 \beta_1^4 \eta_3 + \frac{1}{6} \mathbf{y}_1 \beta_1^3 \beta_2 \eta_3 + \frac{1}{4} \mathbf{y}_1 \beta_1^2 \beta_2^2 \eta_3 + \frac{1}{6} \mathbf{y}_1 \beta_1 \beta_2^3 \eta_3 + \frac{1}{24} \mathbf{y}_1 \beta_2^4 \eta_3, \\ -\frac{1}{120} \mathbf{y}_1 \beta_1^5 \eta_2 - \frac{1}{120} \mathbf{y}_1 \beta_1^5 \eta_3 - \frac{1}{24} \mathbf{y}_1 \beta_1^4 \beta_2 \eta_3 - \frac{1}{12} \mathbf{y}_1 \beta_1^3 \beta_2^2 \eta_3 - \frac{1}{12} \mathbf{y}_1 \beta_1^2 \beta_2^3 \eta_3 - \frac{1}{24} \mathbf{y}_1 \beta_1 \beta_2^4 \eta_3 - \frac{1}{120} \mathbf{y}_1 \beta_2^5 \eta_3, \\ \frac{1}{720} \mathbf{y}_1 \beta_1^6 \eta_2 + \frac{1}{720} \mathbf{y}_1 \beta_1^6 \eta_3 + \frac{1}{120} \mathbf{y}_1 \beta_1^5 \beta_2 \eta_3 + \frac{1}{48} \mathbf{y}_1 \beta_1^4 \beta_2^2 \eta_3 + \frac{1}{36} \mathbf{y}_1 \beta_1^3 \beta_2^3 \eta_3 + \frac{1}{48} \mathbf{y}_1 \beta_1^2 \beta_2^4 \eta_3 + \\ \frac{1}{120} \mathbf{y}_1 \beta_1 \beta_2^5 \eta_3 + \frac{1}{720} \mathbf{y}_1 \beta_2^6 \eta_3, -\frac{\mathbf{y}_1 \beta_1^7 \eta_2}{5040} - \frac{\mathbf{y}_1 \beta_1^7 \eta_3}{5040} - \frac{1}{720} \mathbf{y}_1 \beta_1^6 \beta_2 \eta_3 - \frac{1}{240} \mathbf{y}_1 \beta_1^5 \beta_2^2 \eta_3 - \frac{1}{144} \mathbf{y}_1 \beta_1^4 \beta_2^3 \eta_3 - \\ \frac{1}{144} \mathbf{y}_1 \beta_1^3 \beta_2^4 \eta_3 - \frac{1}{240} \mathbf{y}_1 \beta_1^2 \beta_2^5 \eta_3 - \frac{1}{720} \mathbf{y}_1 \beta_1 \beta_2^6 \eta_3 - \frac{\mathbf{y}_1 \beta_2^7 \eta_3}{5040}, \frac{\mathbf{y}_1 \beta_1^8 \eta_2}{40320} + \frac{\mathbf{y}_1 \beta_1^8 \eta_3}{40320} + \frac{\mathbf{y}_1 \beta_1^7 \beta_2 \eta_3}{5040} + \\ \left. \frac{\mathbf{y}_1 \beta_1^6 \beta_2^2 \eta_3}{1440} + \frac{1}{720} \mathbf{y}_1 \beta_1^5 \beta_2^3 \eta_3 + \frac{1}{576} \mathbf{y}_1 \beta_1^4 \beta_2^4 \eta_3 + \frac{1}{720} \mathbf{y}_1 \beta_1^3 \beta_2^5 \eta_3 + \frac{\mathbf{y}_1 \beta_1^2 \beta_2^6 \eta_3}{1440} + \frac{\mathbf{y}_1 \beta_1 \beta_2^7 \eta_3}{5040} + \frac{\mathbf{y}_1 \beta_2^8 \eta_3}{40320} \right]$$

Out[*]= {0.453125, {True, True}}

In[*]= \$k = 1; R_{i,j}\$

$$\text{Out}[*]= \mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[\hbar \mathbf{a}_j \mathbf{b}_i + \hbar \mathbf{x}_j \mathbf{y}_i, -\frac{1}{4} \hbar^3 \mathbf{x}_j^2 \mathbf{y}_i^2 \right]$$

In[*]= \$k = 3; \bar{R}_{i,j}\$

$$\text{Out}[*]= \mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[-\hbar \mathbf{a}_j \mathbf{b}_i - \frac{\hbar \mathbf{x}_j \mathbf{y}_i}{\mathbf{B}_i}, -\frac{\hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{y}_i}{\mathbf{B}_i} - \frac{3 \hbar^3 \mathbf{x}_j^2 \mathbf{y}_i^2}{4 \mathbf{B}_i^2}, \right. \\ -\frac{\hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{y}_i}{2 \mathbf{B}_i} + \frac{\hbar^4 \mathbf{x}_j^2 \mathbf{y}_i^2}{2 \mathbf{B}_i^2} - \frac{3 \hbar^4 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{y}_i^2}{2 \mathbf{B}_i^2} - \frac{10 \hbar^5 \mathbf{x}_j^3 \mathbf{y}_i^3}{9 \mathbf{B}_i^3}, \\ \left. -\frac{\hbar^4 \mathbf{a}_j^3 \mathbf{x}_j \mathbf{y}_i}{6 \mathbf{B}_i} - \frac{3 \hbar^5 \mathbf{x}_j^2 \mathbf{y}_i^2}{16 \mathbf{B}_i^2} + \frac{\hbar^5 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{y}_i^2}{\mathbf{B}_i^2} - \frac{3 \hbar^5 \mathbf{a}_j^2 \mathbf{x}_j^2 \mathbf{y}_i^2}{2 \mathbf{B}_i^2} + \frac{2 \hbar^6 \mathbf{x}_j^3 \mathbf{y}_i^3}{\mathbf{B}_i^3} - \frac{10 \hbar^6 \mathbf{a}_j \mathbf{x}_j^3 \mathbf{y}_i^3}{3 \mathbf{B}_i^3} - \frac{35 \hbar^7 \mathbf{x}_j^4 \mathbf{y}_i^4}{16 \mathbf{B}_i^4} \right]$$

In[*]= \$k = 3; P_{i,j}\$

$$\text{Out}[*]= \mathbb{E}_{\{i,j\} \rightarrow \{\}} \left[\frac{\alpha_j \beta_i}{\hbar} + \frac{\eta_i \xi_j}{\hbar}, \frac{\eta_i^2 \xi_j^2}{4 \hbar}, \frac{1}{8} \eta_i^2 \xi_j^2 + \frac{5 \eta_i^3 \xi_j^3}{36 \hbar}, \frac{1}{24} \hbar \eta_i^2 \xi_j^2 + \frac{1}{6} \eta_i^3 \xi_j^3 + \frac{5 \eta_i^4 \xi_j^4}{48 \hbar} \right]$$

$ln[*]:= \$k = 3; aS_i$

$$Out[*]:= E_{\{i\} \rightarrow \{i\}} \left[-a_i \alpha_i - x_i \mathcal{A}_i \xi_i, -\hbar a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2} \hbar x_i^2 \mathcal{A}_i^2 \xi_i^2, \right. \\ \left. -\frac{1}{2} \hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + \frac{1}{4} \hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3, -\frac{1}{6} \hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i - \right. \\ \left. \frac{1}{12} \hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{2}{3} \hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2} \hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3} \hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4 \right]$$

$ln[*]:= \$k = 3; \overline{aS}_i$

$$Out[*]:= E_{\{i\} \rightarrow \{i\}} \left[-a_i \alpha_i - x_i \mathcal{A}_i \xi_i, \hbar x_i \mathcal{A}_i \xi_i - \hbar a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2} \hbar x_i^2 \mathcal{A}_i^2 \xi_i^2, \right. \\ \left. -\frac{1}{2} \hbar^2 x_i \mathcal{A}_i \xi_i + \hbar^2 a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2} \hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + \frac{5}{4} \hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3, \right. \\ \left. \frac{1}{6} \hbar^3 x_i \mathcal{A}_i \xi_i - \frac{1}{2} \hbar^3 a_i x_i \mathcal{A}_i \xi_i + \frac{1}{2} \hbar^3 a_i^2 x_i \mathcal{A}_i \xi_i - \frac{1}{6} \hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i - \frac{19}{12} \hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \right. \\ \left. \frac{5}{2} \hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{13}{6} \hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2} \hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3} \hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4 \right]$$

$ln[*]:= (\overline{aS}_1 // aS_1)$

$$Out[*]:= E_{\{1\} \rightarrow \{1\}} [a_1 \alpha_1 + x_1 \xi_1, \theta, \theta, \theta]$$

$ln[*]:= (\overline{aS}_1 // aS_1)$

$$Out[*]:= E_{\{1\} \rightarrow \{1\}} [a_1 \alpha_1 + x_1 \xi_1, \theta, \theta, \theta]$$

$ln[*]:= (\overline{bS}_1 // bS_1)$

$$Out[*]:= E_{\{1\} \rightarrow \{1\}} [b_1 \beta_1 + y_1 \eta_1, \theta, \theta, \theta]$$

$ln[*]:= \$k = 1$

$$Out[*]:= 1$$

$ln[*]:= dS_1$

$$Out[*]:= E_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \right. \\ \left. \frac{\hbar y_1 \mathcal{A}_1 \eta_1}{B_1} - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{B_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 B_1^2} - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{B_1} - \right. \\ \left. \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1) \eta_1 \xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} + \frac{y_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 B_1^2} - \right. \\ \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 B_1} + \frac{(-3 \mathcal{A}_1^2 + 4 B_1 \mathcal{A}_1^2 - B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar B_1^2} \right]$$

$$\begin{aligned} \text{In[*]} := \mathbb{F} = \mathbb{E}; \quad \mathbf{dS}_1 \equiv \mathbb{F}_{\{1\} \rightarrow \{1\}} \left[-\mathbf{a}_1 \alpha_1 - \mathbf{b}_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{\mathbf{B}_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - \mathbf{B}_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar \mathbf{B}_1}, \right. \\ \frac{\hbar y_1 \mathcal{A}_1 \eta_1}{\mathbf{B}_1} - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{\mathbf{B}_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 \mathbf{B}_1^2} - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{\mathbf{B}_1} - \\ \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{\mathbf{B}_1} + \frac{(-\mathcal{A}_1 + \mathbf{B}_1 \mathcal{A}_1) \eta_1 \xi_1}{\mathbf{B}_1} + \frac{(\mathcal{A}_1 - \mathbf{B}_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar \mathbf{B}_1} + \frac{y_1 (3 \mathcal{A}_1^2 - \mathbf{B}_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 \mathbf{B}_1^2} - \\ \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - \mathbf{B}_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 \mathbf{B}_1} + \frac{(-3 \mathcal{A}_1^2 + 4 \mathbf{B}_1 \mathcal{A}_1^2 - \mathbf{B}_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar \mathbf{B}_1^2} \right] \end{aligned}$$

Out[*] = True

$$\begin{aligned} \text{In[*]} := \overline{\mathbf{dS}_1} \equiv \mathbb{F}_{\{1\} \rightarrow \{1\}} \left[-\mathbf{a}_1 \alpha_1 - \mathbf{b}_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{\mathbf{B}_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - \mathbf{B}_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar \mathbf{B}_1}, \right. \\ - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{\mathbf{B}_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 \mathbf{B}_1^2} + \hbar x_1 \mathcal{A}_1 \xi_1 - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{\mathbf{B}_1} - \\ \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{\mathbf{B}_1} + \frac{(-\mathcal{A}_1 + \mathbf{B}_1 \mathcal{A}_1) \eta_1 \xi_1}{\mathbf{B}_1} + \frac{(\mathcal{A}_1 - \mathbf{B}_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar \mathbf{B}_1} + \frac{y_1 (3 \mathcal{A}_1^2 - \mathbf{B}_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 \mathbf{B}_1^2} - \\ \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - \mathbf{B}_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 \mathbf{B}_1} + \frac{(-3 \mathcal{A}_1^2 + 4 \mathbf{B}_1 \mathcal{A}_1^2 - \mathbf{B}_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar \mathbf{B}_1^2} \right] \end{aligned}$$

Out[*] = True

$$\text{In[*]} := \mathbf{dS}_1 // \overline{\mathbf{dS}_1}$$

$$\text{Out[*]} := \mathbb{E}_{\{1\} \rightarrow \{1\}} [\mathbf{a}_1 \alpha_1 + \mathbf{b}_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1, \theta]$$

$$\begin{aligned} \text{In[*]} = & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \right. \\ & \left(\frac{\hbar y_1 \mathcal{A}_1 \eta_1}{B_1} - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{B_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 B_1^2} - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{B_1} - \right. \\ & \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1) \eta_1 \xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} + \frac{y_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 B_1^2} - \\ & \left. \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 B_1} + \frac{(-3 \mathcal{A}_1^2 + 4 B_1 \mathcal{A}_1^2 - B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar B_1^2} \right) \right] // \end{aligned}$$

$$\begin{aligned} & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \right. \\ & - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{B_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 B_1^2} + \hbar x_1 \mathcal{A}_1 \xi_1 - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{B_1} - \\ & \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1) \eta_1 \xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} + \frac{y_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 B_1^2} - \\ & \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 B_1} + \frac{(-3 \mathcal{A}_1^2 + 4 B_1 \mathcal{A}_1^2 - B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar B_1^2} \right] \end{aligned}$$

$$\begin{aligned} \text{Out[*]} = & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[a_1 \alpha_1 + b_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1 + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar}, \right. \\ & (-\hbar + \hbar) y_1 \eta_1 + (1 -) y_1 \beta_1 \eta_1 + \frac{1}{2} (-\hbar + \hbar) y_1^2 \eta_1^2 + (\hbar - \hbar) x_1 \xi_1 + (-\hbar + \hbar) a_1 x_1 \xi_1 + \\ & (1 -) x_1 \beta_1 \xi_1 + (-\hbar + \hbar) x_1 y_1 \eta_1 \xi_1 + a_1 (-\mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1 + \\ & (B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1 + \\ & \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1 + B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar} + \\ & \frac{1}{2} y_1 (\mathcal{A}_1 - 2 B_1 \mathcal{A}_1 + 2 B_1^2 \mathcal{A}_1 - 2 B_1 \mathcal{A}_1 - B_1^2 \mathcal{A}_1 + 2 B_1^2 B_1 \mathcal{A}_1 - 3 B_1 \mathcal{A}_1 + \\ & 6 B_1 B_1 \mathcal{A}_1 - 2 B_1^2 B_1 \mathcal{A}_1 + 2 B_1 B_1 \mathcal{A}_1 - B_1^2 B_1 \mathcal{A}_1 - 2 B_1^2 B_1 \mathcal{A}_1) \eta_1^2 \xi_1 + \\ & \frac{1}{2} (-\hbar + 2 \hbar - \hbar) x_1^2 \xi_1^2 + \frac{1}{2} x_1 (\mathcal{A}_1 - 2 B_1 \mathcal{A}_1 - 2 B_1 \mathcal{A}_1 + 4 B_1 B_1 \mathcal{A}_1 - B_1^2 \mathcal{A}_1 - \\ & 3 B_1 \mathcal{A}_1 + 2 B_1 B_1 \mathcal{A}_1 + 6 B_1 B_1 \mathcal{A}_1 - 4 B_1 B_1 \mathcal{A}_1 - B_1^2 B_1 \mathcal{A}_1) \eta_1 \xi_1^2 + \\ & \frac{1}{4 \hbar} (-\mathcal{A}_1^2 + 2 B_1 \mathcal{A}_1^2 - 2 B_1^2 \mathcal{A}_1^2 + 2 B_1 \mathcal{A}_1^2 - 4 B_1 B_1 \mathcal{A}_1^2 + 4 B_1^2 B_1 \mathcal{A}_1^2 - 2 B_1^2 B_1^2 \mathcal{A}_1^2 - 3 B_1^2 B_1^2 \mathcal{A}_1^2 + 4 B_1^2 B_1^2 B_1 \mathcal{A}_1^2 + \\ & 4 B_1 \mathcal{A}_1^2 - 8 B_1 B_1 \mathcal{A}_1^2 + 4 B_1^2 B_1 \mathcal{A}_1^2 - 8 B_1 B_1 \mathcal{A}_1^2 + 12 B_1 B_1 \mathcal{A}_1^2 - 8 B_1^2 B_1 \mathcal{A}_1^2 + 4 B_1^2 B_1 \mathcal{A}_1^2 + \\ & 4 B_1^2 B_1 \mathcal{A}_1^2 + 4 B_1 B_1 \mathcal{A}_1^2 - 8 B_1^2 B_1 \mathcal{A}_1^2 - 3 B_1^2 B_1^2 \mathcal{A}_1^2 + 6 B_1^2 B_1^2 \mathcal{A}_1^2 - 2 B_1^2 B_1^2 \mathcal{A}_1^2 + 6 B_1^2 B_1^2 \mathcal{A}_1^2 - \\ & 8 B_1^2 B_1^2 \mathcal{A}_1^2 + 4 B_1^2 B_1^2 \mathcal{A}_1^2 - 2 B_1^2 B_1^2 \mathcal{A}_1^2 - B_1^2 B_1^2 \mathcal{A}_1^2 - 4 B_1^2 B_1^2 \mathcal{A}_1^2 + 4 B_1^2 B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2 \left. \right] \end{aligned}$$

$$\begin{aligned} In[*]:= & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \right. \\ & \left(\frac{\hbar y_1 \mathcal{A}_1 \eta_1}{B_1} - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{B_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 B_1^2} - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{B_1} - \right. \\ & \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1) \eta_1 \xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} + \frac{y_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 B_1^2} - \\ & \left. \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 B_1} + \frac{(-3 \mathcal{A}_1^2 + 4 B_1 \mathcal{A}_1^2 - B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar B_1^2} \right) \right] // \end{aligned}$$

$$\begin{aligned} \mathbb{E}_{\{1\} \rightarrow \{1\}} & \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \right. \\ & - \frac{y_1 \mathcal{A}_1 \beta_1 \eta_1}{B_1} - \frac{\hbar y_1^2 \mathcal{A}_1^2 \eta_1^2}{2 B_1^2} + \hbar x_1 \mathcal{A}_1 \xi_1 - \hbar a_1 x_1 \mathcal{A}_1 \xi_1 - x_1 \mathcal{A}_1 \beta_1 \xi_1 + \frac{a_1 \mathcal{A}_1 \eta_1 \xi_1}{B_1} - \\ & \frac{\hbar x_1 y_1 \mathcal{A}_1^2 \eta_1 \xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1 \mathcal{A}_1) \eta_1 \xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} + \frac{y_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1}{2 B_1^2} - \\ & \left. \frac{1}{2} \hbar x_1^2 \mathcal{A}_1^2 \xi_1^2 + \frac{x_1 (3 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2) \eta_1 \xi_1^2}{2 B_1} + \frac{(-3 \mathcal{A}_1^2 + 4 B_1 \mathcal{A}_1^2 - B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{4 \hbar B_1^2} \right] \end{aligned}$$

$$\begin{aligned} Out[*]:= & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[a_1 \alpha_1 + b_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1, \right. \\ & (\hbar - \hbar \text{Blue}) x_1 y_1 \eta_1 \xi_1 + (-\text{Blue} \mathcal{A}_1 - \text{Green} \mathcal{A}_1 + \text{Purple} \mathcal{A}_1 + \text{Red} \mathcal{A}_1 + \text{Blue} B_1 \mathcal{A}_1 + \text{Green} B_1 \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1 - \text{Red} B_1 \mathcal{A}_1) \eta_1 \xi_1 + \\ & \frac{(\mathcal{A}_1 - \text{Purple} \mathcal{A}_1 - B_1 \mathcal{A}_1 + \text{Purple} B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar} + \\ & y_1 (-\text{Blue} \mathcal{A}_1 + \text{Purple} \mathcal{A}_1 + \text{Blue} B_1 \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1) \eta_1^2 \xi_1 + x_1 (\mathcal{A}_1 - \text{Blue} \mathcal{A}_1 - B_1 \mathcal{A}_1 + \text{Blue} B_1 \mathcal{A}_1) \eta_1 \xi_1^2 + \\ & \left. \frac{(-\text{Blue} \mathcal{A}_1^2 + \text{Purple} \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2 + 2 \text{Blue} B_1 \mathcal{A}_1^2 - \text{Purple} B_1 \mathcal{A}_1^2 + B_1^2 \mathcal{A}_1^2 - \text{Blue} B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{\hbar} \right] \end{aligned}$$

$$\begin{aligned} In[*]:= & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[a_1 \alpha_1 + b_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1, \right. \\ & (\hbar - \hbar \text{Blue}) x_1 y_1 \eta_1 \xi_1 + (-\text{Green} \mathcal{A}_1 + \text{Purple} \mathcal{A}_1 + \text{Red} \mathcal{A}_1 + \text{Green} B_1 \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1 - \text{Red} B_1 \mathcal{A}_1 - \text{Blue} \mathcal{A}_1^2 + \text{Blue} B_1 \mathcal{A}_1^2) \eta_1 \xi_1 + \\ & \frac{(\mathcal{A}_1 - \text{Purple} \mathcal{A}_1 - B_1 \mathcal{A}_1 + \text{Purple} B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar} + \\ & y_1 (-\text{Blue} \mathcal{A}_1 + \text{Purple} \mathcal{A}_1 + \text{Blue} B_1 \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1) \eta_1^2 \xi_1 + x_1 (\mathcal{A}_1 - \text{Blue} \mathcal{A}_1 - B_1 \mathcal{A}_1 + \text{Blue} B_1 \mathcal{A}_1) \eta_1 \xi_1^2 + \\ & \left. \frac{(-\text{Blue} \mathcal{A}_1^2 + \text{Purple} \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2 + 2 \text{Blue} B_1 \mathcal{A}_1^2 - \text{Purple} B_1 \mathcal{A}_1^2 + B_1^2 \mathcal{A}_1^2 - \text{Blue} B_1^2 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{\hbar} \right] /. \{ \text{Red} | \text{Green} | \text{Blue} \rightarrow 1 \} \end{aligned}$$

$$\begin{aligned} Out[*]:= & \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[a_1 \alpha_1 + b_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1, \right. \\ & (\text{Purple} \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1 - \mathcal{A}_1^2 + B_1 \mathcal{A}_1^2) \eta_1 \xi_1 + \frac{(\mathcal{A}_1 - \text{Purple} \mathcal{A}_1 - B_1 \mathcal{A}_1 + \text{Purple} B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar} + \\ & y_1 (-\mathcal{A}_1 + \text{Purple} \mathcal{A}_1 + B_1 \mathcal{A}_1 - \text{Purple} B_1 \mathcal{A}_1) \eta_1^2 \xi_1 + \frac{(-\mathcal{A}_1^2 + \text{Purple} \mathcal{A}_1^2 + B_1 \mathcal{A}_1^2 - \text{Purple} B_1 \mathcal{A}_1^2) \eta_1^2 \xi_1^2}{\hbar} \left. \right] \end{aligned}$$

$$\begin{aligned}
 In[*] &:= \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \theta \right] // \\
 &\mathbb{E}_{\{1\} \rightarrow \{1\}} \left[-a_1 \alpha_1 - b_1 \beta_1 - \frac{y_1 \mathcal{A}_1 \eta_1}{B_1} - x_1 \mathcal{A}_1 \xi_1 + \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar B_1}, \frac{(\mathcal{A}_1 - B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar B_1} \right] \\
 Out[*] &:= \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[a_1 \alpha_1 + b_1 \beta_1 + y_1 \eta_1 + x_1 \xi_1 + \frac{(-\mathcal{A}_1 + \mathcal{A}_1 + B_1 \mathcal{A}_1 - B_1 \mathcal{A}_1) \eta_1 \xi_1}{\hbar}, \right. \\
 &\left. \left(\mathcal{A}_1 - B_1 \mathcal{A}_1 \right) \eta_1 \xi_1 + \frac{(-\mathcal{A}_1 + \mathcal{A}_1 + B_1 \mathcal{A}_1) \beta_1 \eta_1 \xi_1}{\hbar} + y_1 \left(\mathcal{A}_1^2 - B_1 \mathcal{A}_1^2 \right) \eta_1^2 \xi_1 + \right. \\
 &\left. \frac{\left(\mathcal{A}_1^2 + B_1 \mathcal{A}_1^2 - 2 \mathcal{A}_1^2 - B_1 \mathcal{A}_1^2 + B_1^2 \mathcal{A}_1^2 + B_1^2 \mathcal{A}_1^2 \right) \eta_1^2 \xi_1^2}{\hbar} \right]
 \end{aligned}$$

$$In[*] := (\overline{kR}_{1,4} \overline{kR}_{5,2} \overline{kC}_3) // km_{2,4 \rightarrow 2} // km_{1,3 \rightarrow 1} // km_{1,5 \rightarrow 1}$$

$$Out[*] := \mathbb{E}_{\{\} \rightarrow \{1,2\}} [\theta, \hbar a_1, \theta]$$

$$In[*] := \overline{kC}_1 d\eta_2$$

$$Out[*] := \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[-\frac{\hbar t_1}{2}, \hbar a_1, \theta \right]$$

$$In[*] := (\mathbb{E}_{\{\} \rightarrow \{1,2\}} [\theta, a_2 x_1] // am_{1,2 \rightarrow 1})$$

$$Out[*] := \mathbb{E}_{\{\} \rightarrow \{1\}} [\theta, -x_1 + a_1 x_1]$$

$$In[*] := \$k = 2; \mathbb{E}2\Lambda[\mathbb{E}_{\{\} \rightarrow \{1,2\}} [\theta, y_2 b_1] // bm_{1,2 \rightarrow 1}]$$

$$Out[*] := b_1 y_1$$

$$In[*] := \mathbb{E}_{\{\} \rightarrow \{1,2\}} [\theta, y_2 b_1]$$

$$Out[*] := \mathbb{E}_{\{\} \rightarrow \{1,2\}} [\theta, b_1 y_2]$$