Pensieve header: The Objects. Continues pensieve://Projects/SL2Portfoilo2/Objects.nb.

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The Objects

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“Define” Code

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```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ; op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

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Symmetric Algebra Objects

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```
sm_{i_,j_→k_} := Λ2𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) + y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_{i_→j_,k_} := Λ2𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) + η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_{i_} := Λ2𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sη_{i_} := Λ2𝔼_{{}→{i}}[0];
sϵ_{i_} := Λ2𝔼_{{i}→{}}[0];
```

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```
sσ_{i_→j_} := Λ2𝔼_{{i}→{j}}[β_i b_j + τ_i t_j + α_i a_j + η_i y_j + ξ_i x_j];
sΥ_{i_→j_,k_,l_,m_} := Λ2𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l + η_i y_j + ξ_i x_m];
```

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The CU Definitions

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```
cΛ = (ηi + e^(-αi-ϵ βi) ηj/(1 + ϵ ηj ξi)) yk + (βi + βj + Log[1 + ϵ ηj ξi]/ϵ) bk +
   (αi + αj + Log[1 + ϵ ηj ξi]) ak + (e^(-αj-ϵ βj) ξi/(1 + ϵ ηj ξi) + ξj) xk;
Define[cm_{i,j→k} = Λ2E_{{i,j}→{k}}[(ηi + e^(-αi-ϵ βi) ηj/(1 + ϵ ηj ξi)) yk + (βi + βj + Log[1 + ϵ ηj ξi]/ϵ) bk +
   (αi + αj + Log[1 + ϵ ηj ξi]) ak + (e^(-αj-ϵ βj) ξi/(1 + ϵ ηj ξi) + ξj) xk]]
```

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```
Define[cσ_{i→j} = sσ_{i,j} /. τi → 0, cϵi = sϵi, cηi = sηi, cΔ_{i→j,k} = sΔ_{i→j,k},
  cSi = sSi // sϒ_{i→1,2,3,4} // cm_{4,3→i} // cm_{i,2→i} // cm_{i,1→i}];
```

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Booting Up QU

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```
Define[aσ_{i→j} = Λ2E_{{i}→{j}}[aj αi + xj ξi], bσ_{i→j} = Λ2E_{{i}→{j}}[bj βi + yj ηi]]
```

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```
Define[am_{i,j→k} = Λ2E_{{i,j}→{k}}[(αi + αj) ak + (𝒜j^-1 ξi + ξj) xk],
  bm_{i,j→k} = Λ2E_{{i,j}→{k}}[(βi + βj) bk + (ηi + e^(-ϵ βi) ηj) yk]]
```

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```
Define[R_{i,j} = Module[{k}, Λ2E_{{}→{i,j}}[ħ aj bi + Sum[(1 - e^(ϵ ħ))^k (ħ yi xj)^k/(k (1 - e^(k ϵ ħ))), {k, 1, $k+1}]]]]
```

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Three types of inverses appear below!

$\overline{R}$ is the inverse of R in the algebra $\mathbb{B}\otimes\mathbb{A}$.

P is the inverse of R as a quadratic form, like how an element of $V^*\otimes V^*$ can be the inverse of an element of $V\otimes V$.

$\overline{\text{aS}}$ is the inverse of aS as an operator form, like how an element of $V^*\otimes V$ can be the inverse of another element of $V^*\otimes V$.

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In[◦]:=
```
Define[R̄_{i,j} = If[$k == 0, 𝔼_{{}→{i,j}}[-ℏ a_j b_i - ℏ x_j y_i / B_i],
    Append[R̄_{{i,j},$k-1}, -Last[PadRight[R̄_{{i,j},0}, $k + 1] R_{1,2} PadRight[R̄_{{3,4},$k-1}, $k + 1] //
        (bm_{i,1→i} am_{j,2→j}) // (bm_{i,3→i} am_{j,4→j})]]]
]
```

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```
Define[P_{i,j} = If[$k == 0, 𝔼_{{i,j}→{}}[β_i α_j / ℏ + η_i ξ_j / ℏ], Append[P_{{i,j},$k-1},
    -Last[R_{1,2} // (PadRight[P_{{1,j},0}, $k + 1] * PadRight[P_{{i,2},$k-1}, $k + 1])]]]]
```

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```
Define[aS_i = (aσ_{i→2} R̄_{1,i}) // P_{1,2}]
```

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```
Define[aS̄_i = If[$k == 0, 𝔼_{{i}→{i}}[-a_i α_i - x_i 𝒜_i ξ_i],
    Append[aS̄_{{i},$k-1}, -Last[PadRight[aS̄_{{i},0}, $k + 1] // aS_i // PadRight[aS̄_{{i},$k-1}, $k + 1]]]
  ]]
```

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Booting Up QU

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```
Define[bS_i = bσ_{i→1} R_{i,2} // aS_2 // P_{1,2},
  bS̄_i = bσ_{i→1} R_{i,2} // aS̄_2 // P_{1,2},
  aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
  bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
```

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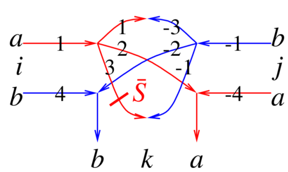
The Drinfel'd double: 

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```
Define[
  dm_{i,j→k} = ((sΥ_{i→4,4,1,1} // aΔ_{1→1,2} // aΔ_{2→2,3} // aS̄_3) (sΥ_{j→-1,-1,-4,-4} // bΔ_{-1→-1,-2} // bΔ_{-2→-2,-3})) //
    (P_{-1,3} P_{-3,1} am_{2,-4→k} bm_{4,-2→k})]
```

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```
Define[dσ_{i→j} = aσ_{i→j} bσ_{i→j},
  dϵ_i = sϵ_i, dη_i = sη_i,
  dS_i = sΥ_{i→1,1,2,2} // (bS̄_1 aS_2) // dm_{2,1→i},
  dS̄_i = sΥ_{i→1,1,2,2} // (bS_1 aS̄_2) // dm_{2,1→i},
  dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j})]
```

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```
Define[C_i = Λ2𝔼_{}→{i}[-ℏ/2 (b_i + ϵ a_i)],
  C̄_i = Λ2𝔼_{}→{i}[ℏ/2 (b_i + ϵ a_i)],
  Kink_i = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
  K̄ink_i = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
```

Not yet verified

Note. $t == \epsilon a - b$ and $b == -t + \epsilon a$.

```
Define[b2t_i = Λ2𝔼_{i}→{i}[α_i a_i + β_i (ϵ a_i - t_i) + ξ_i x_i + η_i y_i],
  t2b_i = Λ2𝔼_{i}→{i}[α_i a_i + τ_i (ϵ a_i - b_i) + ξ_i x_i + η_i y_i]]
```

The Knot Tensors

```
Define[kR_{i,j} = (R_{i,j} // (b2t_i b2t_j)) /. t_{i|j} → t,
  k̄R_{i,j} = (R̄_{i,j} // (b2t_i b2t_j)) /. {t_{i|j} → t, T_{i|j} → T},
  km_{i,j→k} = ((t2b_i t2b_j) // dm_{i,j→k} // b2t_k) /. {t_k → t, T_k → T, τ_{i|j} → 0},
  kC_i = (C_i // b2t_i) /. t_i → t,
  k̄C_i = (C̄_i // b2t_i) /. t_i → t,
  kKink_i = (Kink_i // b2t_i) /. {t_i → t, T_i → T},
  k̄Kink_i = (K̄ink_i // b2t_i) /. {t_i → t, T_i → T}]
```