

Pensieve header: Working in the LogKink basis.

Startup

```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"./Profile/Profile.m"];
BeginProfile[];
$k = 2;
<< Engine.m
<< Objects.m
<< KT.m
HL[_] := Style[_ , Background -> If[TrueQ@#, Green, Red]];
CF[sd_SeriesData] := MapAt[CF, sd, 3];
```

LK = Log[Kink_i] is imported from LogKink.nb.

In[]:= **Kink_i**

$$\text{Out[]} = \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[\frac{\hbar b_i}{2} + \hbar a_i b_i + \hbar x_i y_i, \frac{\hbar a_i}{2} - \frac{1}{4} \hbar^3 x_i^2 y_i^2, \frac{1}{9} \hbar^5 x_i^3 y_i^3 \right]$$

Timing[LK = Log_{dm}[Kink_i]]

$$\text{Out[]} = \left\{ 815.594, \mathbb{U}_{\{i\} \rightarrow \{i\}} \left[\frac{\hbar b_i}{2} + \hbar a_i b_i - \frac{\hbar^2 b_i x_i y_i}{-1 + B_i}, \right. \right. \\ \left. \frac{\hbar a_i}{2} - \frac{\hbar^2 a_i (-1 + B_i + \hbar b_i B_i) x_i y_i}{(-1 + B_i)^2} + \frac{\hbar^3 (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i^2 y_i^2}{2 (-1 + B_i)^3}, \right. \\ \left. - \frac{\hbar^3 a_i^2 B_i (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i y_i}{2 (-1 + B_i)^3} - \frac{\hbar^4 (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} + \right. \\ \left. \frac{\hbar^4 a_i (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} - \frac{\hbar^5 (-3 + \hbar b_i + 4 \hbar b_i B_i + 3 B_i^2 + \hbar b_i B_i^2) x_i^3 y_i^3}{3 (-1 + B_i)^5} \right\}$$

$$\begin{aligned}
 \text{In[*]}:= \text{LK} = \mathbb{U}_{\{i\} \rightarrow \{i\}} \left[\frac{\hbar b_i}{2} + \hbar a_i b_i - \frac{\hbar^2 b_i x_i y_i}{-1 + B_i}, \right. \\
 \frac{\hbar a_i}{2} - \frac{\hbar^2 a_i (-1 + B_i + \hbar b_i B_i) x_i y_i}{(-1 + B_i)^2} + \frac{\hbar^3 (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i^2 y_i^2}{2 (-1 + B_i)^3}, \\
 - \frac{\hbar^3 a_i^2 B_i (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i y_i}{2 (-1 + B_i)^3} - \frac{\hbar^4 (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} + \\
 \frac{\hbar^4 a_i (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} - \\
 \left. \frac{\hbar^5 (-3 + \hbar b_i + 4 \hbar b_i B_i + 3 B_i^2 + \hbar b_i B_i^2) x_i^3 y_i^3}{3 (-1 + B_i)^5} \right];
 \end{aligned}$$

$$\text{In[*]}:= \mathbf{lk} = \text{Sum}[\epsilon^k \text{LK}[\mathbf{k} + 1]], \{k, 0, \text{Length}[\text{LK}] - 1\} + \mathbf{0}[\epsilon]^{\text{Length}[\text{LK}]}$$

$$\begin{aligned}
 \text{Out[*]}:= \left(\frac{\hbar b_i}{2} + \hbar a_i b_i - \frac{\hbar^2 b_i x_i y_i}{-1 + B_i} \right) + \\
 \left(\frac{\hbar a_i}{2} - \frac{\hbar^2 a_i (-1 + B_i + \hbar b_i B_i) x_i y_i}{(-1 + B_i)^2} + \frac{\hbar^3 (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i^2 y_i^2}{2 (-1 + B_i)^3} \right) \epsilon + \\
 \left(- \frac{\hbar^3 a_i^2 B_i (-2 + \hbar b_i + 2 B_i + \hbar b_i B_i) x_i y_i}{2 (-1 + B_i)^3} - \frac{\hbar^4 (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} + \right. \\
 \frac{\hbar^4 a_i (-1 - 4 B_i + 4 \hbar b_i B_i + 5 B_i^2 + 2 \hbar b_i B_i^2) x_i^2 y_i^2}{2 (-1 + B_i)^4} - \\
 \left. \frac{\hbar^5 (-3 + \hbar b_i + 4 \hbar b_i B_i + 3 B_i^2 + \hbar b_i B_i^2) x_i^3 y_i^3}{3 (-1 + B_i)^5} \right) \epsilon^2 + \mathbf{0}[\epsilon]^3
 \end{aligned}$$

$$\text{In}[*]:= \text{CF}\left[\text{lk} + \epsilon \text{c} \frac{\hbar^2 \mathbf{B}_i \mathbf{a}_i \mathbf{x}_i \mathbf{y}_i \text{lk}}{(\mathbf{B}_i - 1)^2}\right]$$

$$\begin{aligned} \text{Out}[*]= & \left(\frac{\hbar \mathbf{b}_i}{2} + \hbar \mathbf{a}_i \mathbf{b}_i - \frac{\hbar^2 \mathbf{b}_i \mathbf{x}_i \mathbf{y}_i}{-1 + \mathbf{B}_i} \right) + \\ & \left(\frac{\hbar \mathbf{a}_i}{2} + \frac{\text{c} \hbar^3 \mathbf{a}_i^2 \mathbf{b}_i \mathbf{B}_i \mathbf{x}_i \mathbf{y}_i}{(-1 + \mathbf{B}_i)^2} + \frac{\hbar^2 \mathbf{a}_i (2 - 2 \mathbf{B}_i - 2 \hbar \mathbf{b}_i \mathbf{B}_i + \text{c} \hbar \mathbf{b}_i \mathbf{B}_i) \mathbf{x}_i \mathbf{y}_i}{2 (-1 + \mathbf{B}_i)^2} - \frac{\text{c} \hbar^4 \mathbf{a}_i \mathbf{b}_i \mathbf{B}_i \mathbf{x}_i^2 \mathbf{y}_i^2}{(-1 + \mathbf{B}_i)^3} + \right. \\ & \left. \frac{\hbar^3 (-2 + \hbar \mathbf{b}_i + 2 \mathbf{B}_i + \hbar \mathbf{b}_i \mathbf{B}_i) \mathbf{x}_i^2 \mathbf{y}_i^2}{2 (-1 + \mathbf{B}_i)^3} \right) \epsilon + \\ & \left(- \frac{\hbar^3 \mathbf{a}_i^2 \mathbf{B}_i (-2 + \text{c} + \hbar \mathbf{b}_i + 2 \mathbf{B}_i - \text{c} \mathbf{B}_i + \hbar \mathbf{b}_i \mathbf{B}_i) \mathbf{x}_i \mathbf{y}_i}{2 (-1 + \mathbf{B}_i)^3} - \frac{\text{c} \hbar^4 \mathbf{a}_i^2 \mathbf{B}_i (-1 + \mathbf{B}_i + \hbar \mathbf{b}_i \mathbf{B}_i) \mathbf{x}_i^2 \mathbf{y}_i^2}{(-1 + \mathbf{B}_i)^4} - \right. \\ & \frac{\hbar^4 (-1 - 4 \mathbf{B}_i + 4 \hbar \mathbf{b}_i \mathbf{B}_i + 5 \mathbf{B}_i^2 + 2 \hbar \mathbf{b}_i \mathbf{B}_i^2) \mathbf{x}_i^2 \mathbf{y}_i^2}{2 (-1 + \mathbf{B}_i)^4} + \\ & \frac{\hbar^4 \mathbf{a}_i (-1 - 4 \mathbf{B}_i + 4 \hbar \mathbf{b}_i \mathbf{B}_i + 5 \mathbf{B}_i^2 + 2 \hbar \mathbf{b}_i \mathbf{B}_i^2) \mathbf{x}_i^2 \mathbf{y}_i^2}{2 (-1 + \mathbf{B}_i)^4} + \frac{\text{c} \hbar^5 \mathbf{a}_i \mathbf{B}_i (-2 + \hbar \mathbf{b}_i + 2 \mathbf{B}_i + \hbar \mathbf{b}_i \mathbf{B}_i) \mathbf{x}_i^3 \mathbf{y}_i^3}{2 (-1 + \mathbf{B}_i)^5} - \\ & \left. \frac{\hbar^5 (-3 + \hbar \mathbf{b}_i + 4 \hbar \mathbf{b}_i \mathbf{B}_i + 3 \mathbf{B}_i^2 + \hbar \mathbf{b}_i \mathbf{B}_i^2) \mathbf{x}_i^3 \mathbf{y}_i^3}{3 (-1 + \mathbf{B}_i)^5} \right) \epsilon^2 + \mathbf{0}[\epsilon]^3 \end{aligned}$$

In[*]:= **tab5 = Table[Echo[K → Z[K]], {K, AllKnots[{3, 5]}]}**

KnotTheory: Loading precomputed data in PD4Knots`.

» 4

» Knot[3, 1] →

$$\begin{aligned} \mathbb{E}_{\{1\} \rightarrow \{1\}} & \left[-2 \mathbf{t} \hbar + \text{Log}\left[\frac{\mathbf{T}^3}{1 - \mathbf{T} + \mathbf{T}^2}\right], \frac{2 \mathbf{a} (-1 + \mathbf{T}) \times (1 + \mathbf{T}) \hbar}{1 - \mathbf{T} + \mathbf{T}^2} + \frac{(-1 + \mathbf{T}) \times (2 - \mathbf{T} + \mathbf{T}^2) \hbar}{(1 - \mathbf{T} + \mathbf{T}^2)^2} - \frac{2 \times (1 + \mathbf{T}) \mathbf{x} \mathbf{y} \hbar^2}{1 - \mathbf{T} + \mathbf{T}^2}, \right. \\ & \frac{2 \mathbf{a}^2 \mathbf{T} (1 - 4 \mathbf{T} + \mathbf{T}^2) \hbar^2}{(1 - \mathbf{T} + \mathbf{T}^2)^2} + \frac{2 \mathbf{a} \mathbf{T} (1 - 7 \mathbf{T} + 6 \mathbf{T}^2 - 3 \mathbf{T}^3 + \mathbf{T}^4) \hbar^2}{(1 - \mathbf{T} + \mathbf{T}^2)^3} + \frac{(-1 + \mathbf{T}) \mathbf{T} (-1 + 10 \mathbf{T} - 6 \mathbf{T}^2 + 6 \mathbf{T}^3 - 2 \mathbf{T}^4 + \mathbf{T}^5) \hbar^2}{2 (1 - \mathbf{T} + \mathbf{T}^2)^4} - \\ & \left. \frac{4 \mathbf{a} \mathbf{T} (-2 + 2 \mathbf{T} + \mathbf{T}^2) \mathbf{x} \mathbf{y} \hbar^3}{(1 - \mathbf{T} + \mathbf{T}^2)^2} + \frac{2 \times (1 + \mathbf{T}) \times (-1 + \mathbf{T} - 2 \mathbf{T}^2 - \mathbf{T}^3 + \mathbf{T}^4) \mathbf{x} \mathbf{y} \hbar^3}{(1 - \mathbf{T} + \mathbf{T}^2)^3} + \frac{(1 + 5 \mathbf{T} + \mathbf{T}^2) \mathbf{x}^2 \mathbf{y}^2 \hbar^4}{(1 - \mathbf{T} + \mathbf{T}^2)^2} \right] \end{aligned}$$

» 4

» Knot[4, 1] →

$$\begin{aligned} \mathbb{E}_{\{1\} \rightarrow \{1\}} & \left[-\mathbf{t} \hbar + \text{Log}\left[-\frac{\sqrt{\frac{\mathbf{T}^4}{(-1+2\mathbf{T})^2}} (-1 + 2 \mathbf{T})}{1 - 3 \mathbf{T} + \mathbf{T}^2}\right], \frac{(-1 + \mathbf{T}) \times (1 + \mathbf{T}) \hbar}{1 - 3 \mathbf{T} + \mathbf{T}^2} + \frac{2 \mathbf{a} (-1 + \mathbf{T}) \times (1 + \mathbf{T}) \hbar}{1 - 3 \mathbf{T} + \mathbf{T}^2} - \frac{2 \times (1 + \mathbf{T}) \mathbf{x} \mathbf{y} \hbar^2}{1 - 3 \mathbf{T} + \mathbf{T}^2}, \right. \\ & \frac{(-1 + \mathbf{T})^2 \mathbf{T} (3 - 5 \mathbf{T} + 3 \mathbf{T}^2) \hbar^2}{2 (1 - 3 \mathbf{T} + \mathbf{T}^2)^3} + \frac{2 \mathbf{a} \mathbf{T} (3 - 4 \mathbf{T} + 3 \mathbf{T}^2) \hbar^2}{(1 - 3 \mathbf{T} + \mathbf{T}^2)^2} + \frac{2 \mathbf{a}^2 \mathbf{T} (3 - 4 \mathbf{T} + 3 \mathbf{T}^2) \hbar^2}{(1 - 3 \mathbf{T} + \mathbf{T}^2)^2} + \\ & \left. \frac{2 \times (-1 + \mathbf{T}) (1 + \mathbf{T})^2 \mathbf{x} \mathbf{y} \hbar^3}{(1 - 3 \mathbf{T} + \mathbf{T}^2)^2} - \frac{4 \mathbf{a} \mathbf{T} (-4 + 2 \mathbf{T} + \mathbf{T}^2) \mathbf{x} \mathbf{y} \hbar^3}{(1 - 3 \mathbf{T} + \mathbf{T}^2)^2} + \frac{(1 + 7 \mathbf{T} + \mathbf{T}^2) \mathbf{x}^2 \mathbf{y}^2 \hbar^4}{(1 - 3 \mathbf{T} + \mathbf{T}^2)^2} \right] \end{aligned}$$

» 4

$$\begin{aligned} & \text{Knot}[5, 1] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-3 t \hbar + \text{Log} \left[\frac{T^5}{1 - T + T^2 - T^3 + T^4} \right], \right. \\ & \frac{2 a (-1 + T) \times (1 + T) \times (2 - T + 2 T^2) \hbar}{1 - T + T^2 - T^3 + T^4} + \frac{(-1 + T) \times (4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6) \hbar}{(1 - T + T^2 - T^3 + T^4)^2} - \\ & \frac{2 \times (1 + T) \times (2 - T + 2 T^2) x y \hbar^2}{1 - T + T^2 - T^3 + T^4}, \frac{2 a^2 T (1 - 3 T + T^2) \times (1 - T + 6 T^2 - T^3 + T^4) \hbar^2}{(1 - T + T^2 - T^3 + T^4)^2} + \\ & \frac{2 a T (1 - 7 T + 21 T^2 - 51 T^3 + 57 T^4 - 48 T^5 + 33 T^6 - 19 T^7 + 9 T^8 - 3 T^9 + T^{10}) \hbar^2}{(1 - T + T^2 - T^3 + T^4)^3} + \frac{1}{2 (1 - T + T^2 - T^3 + T^4)^4} \\ & (-1 + T) T (-1 + 10 T - 31 T^2 + 85 T^3 - 100 T^4 + 114 T^5 - 86 T^6 + 70 T^7 - 42 T^8 + 28 T^9 - 13 T^{10} + 7 T^{11} - 2 T^{12} + T^{13}) \\ & \hbar^2 - \frac{4 a T (-3 + 2 T - 10 T^2 + 10 T^3 + 2 T^5 + 2 T^6) x y \hbar^3}{(1 - T + T^2 - T^3 + T^4)^2} + \\ & \left. \frac{2 \times (1 + T) \times (-4 + 8 T - 15 T^2 + 19 T^3 - 26 T^4 + 18 T^5 - 10 T^6 - 7 T^7 + 11 T^8 - 8 T^9 + 4 T^{10}) x y \hbar^3}{(1 - T + T^2 - T^3 + T^4)^3} + \right. \\ & \left. \frac{(2 + 7 T + 5 T^2 + 25 T^3 + 5 T^4 + 7 T^5 + 2 T^6) x^2 y^2 \hbar^4}{(1 - T + T^2 - T^3 + T^4)^2} \right] \end{aligned}$$

» 4

$$\begin{aligned} & \text{Knot}[5, 2] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-3 t \hbar + \text{Log} \left[\frac{T^2 \sqrt{\frac{T^4}{(-1+2T)^2}} (-1+2T)}{2-3T+2T^2} \right], \right. \\ & \frac{4 a (-1 + T) \times (1 + T) \hbar}{2 - 3 T + 2 T^2} - \frac{(-1 + T) \times (-9 + 11 T - 7 T^2 + T^3) \hbar}{(2 - 3 T + 2 T^2)^2} - \frac{4 \times (1 + T) x y \hbar^2}{2 - 3 T + 2 T^2}, \\ & \frac{4 a^2 T (3 - 8 T + 3 T^2) \hbar^2}{(2 - 3 T + 2 T^2)^2} + \frac{4 a T (7 - 30 T + 36 T^2 - 20 T^3 + 5 T^4) \hbar^2}{(2 - 3 T + 2 T^2)^3} + \\ & \frac{(-1 + T) \times (-1 - 29 T + 148 T^2 - 194 T^3 + 146 T^4 - 60 T^5 + 13 T^6 + T^7) \hbar^2}{2 (2 - 3 T + 2 T^2)^4} - \frac{8 a T (-5 + 4 T + 2 T^2) x y \hbar^3}{(2 - 3 T + 2 T^2)^2} + \\ & \left. \frac{4 \times (1 + T) \times (-4 + 7 T - 5 T^2 - 5 T^3 + 4 T^4) x y \hbar^3}{(2 - 3 T + 2 T^2)^3} + \frac{2 \times (2 + 11 T + 2 T^2) x^2 y^2 \hbar^4}{(2 - 3 T + 2 T^2)^2} \right] \end{aligned}$$

$$\text{Out[*]} = \left\{ \text{Knot}[3, 1] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-2 t \hbar + \text{Log} \left[\frac{T^3}{1 - T + T^2} \right], \right.$$

$$\begin{aligned} & \frac{2 a (-1 + T) \times (1 + T) \hbar}{1 - T + T^2} + \frac{(-1 + T) \times (2 - T + T^2) \hbar}{(1 - T + T^2)^2} - \frac{2 \times (1 + T) x y \hbar^2}{1 - T + T^2}, \frac{2 a^2 T (1 - 4 T + T^2) \hbar^2}{(1 - T + T^2)^2} + \\ & \frac{2 a T (1 - 7 T + 6 T^2 - 3 T^3 + T^4) \hbar^2}{(1 - T + T^2)^3} + \frac{(-1 + T) T (-1 + 10 T - 6 T^2 + 6 T^3 - 2 T^4 + T^5) \hbar^2}{2 (1 - T + T^2)^4} - \end{aligned}$$

$$\left. \begin{aligned} & \frac{4 a T (-2 + 2 T + T^2) x y \hbar^3}{(1 - T + T^2)^2} + \frac{2 \times (1 + T) \times (-1 + T - 2 T^2 - T^3 + T^4) x y \hbar^3}{(1 - T + T^2)^3} + \frac{(1 + 5 T + T^2) x^2 y^2 \hbar^4}{(1 - T + T^2)^2} \right], \\ \text{Knot}[4, 1] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-t \hbar + \text{Log} \left[-\frac{\sqrt{\frac{T^4}{(-1+2T)^2}} (-1 + 2 T)}{1 - 3 T + T^2} \right], \right. \\ & \frac{(-1 + T) \times (1 + T) \hbar}{1 - 3 T + T^2} + \frac{2 a (-1 + T) \times (1 + T) \hbar}{1 - 3 T + T^2} - \frac{2 \times (1 + T) x y \hbar^2}{1 - 3 T + T^2}, \\ & \frac{(-1 + T)^2 T (3 - 5 T + 3 T^2) \hbar^2}{2 (1 - 3 T + T^2)^3} + \frac{2 a T (3 - 4 T + 3 T^2) \hbar^2}{(1 - 3 T + T^2)^2} + \frac{2 a^2 T (3 - 4 T + 3 T^2) \hbar^2}{(1 - 3 T + T^2)^2} + \\ & \left. \frac{2 \times (-1 + T) (1 + T)^2 x y \hbar^3}{(1 - 3 T + T^2)^2} - \frac{4 a T (-4 + 2 T + T^2) x y \hbar^3}{(1 - 3 T + T^2)^2} + \frac{(1 + 7 T + T^2) x^2 y^2 \hbar^4}{(1 - 3 T + T^2)^2} \right], \\ \text{Knot}[5, 1] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-3 t \hbar + \text{Log} \left[\frac{T^5}{1 - T + T^2 - T^3 + T^4} \right], \right. \\ & \frac{2 a (-1 + T) \times (1 + T) \times (2 - T + 2 T^2) \hbar}{1 - T + T^2 - T^3 + T^4} + \frac{(-1 + T) \times (4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6) \hbar}{(1 - T + T^2 - T^3 + T^4)^2} - \\ & \frac{2 \times (1 + T) \times (2 - T + 2 T^2) x y \hbar^2}{1 - T + T^2 - T^3 + T^4}, \frac{2 a^2 T (1 - 3 T + T^2) \times (1 - T + 6 T^2 - T^3 + T^4) \hbar^2}{(1 - T + T^2 - T^3 + T^4)^2} + \\ & \frac{2 a T (1 - 7 T + 21 T^2 - 51 T^3 + 57 T^4 - 48 T^5 + 33 T^6 - 19 T^7 + 9 T^8 - 3 T^9 + T^{10}) \hbar^2}{(1 - T + T^2 - T^3 + T^4)^3} + \\ & \frac{1}{2 (1 - T + T^2 - T^3 + T^4)^4} (-1 + T) T (-1 + 10 T - 31 T^2 + 85 T^3 - 100 T^4 + 114 T^5 - 86 T^6 + 70 T^7 - 42 T^8 + \\ & 28 T^9 - 13 T^{10} + 7 T^{11} - 2 T^{12} + T^{13}) \hbar^2 - \frac{4 a T (-3 + 2 T - 10 T^2 + 10 T^3 + 2 T^5 + 2 T^6) x y \hbar^3}{(1 - T + T^2 - T^3 + T^4)^2} + \\ & \frac{2 \times (1 + T) \times (-4 + 8 T - 15 T^2 + 19 T^3 - 26 T^4 + 18 T^5 - 10 T^6 - 7 T^7 + 11 T^8 - 8 T^9 + 4 T^{10}) x y \hbar^3}{(1 - T + T^2 - T^3 + T^4)^3} + \\ & \left. \frac{(2 + 7 T + 5 T^2 + 25 T^3 + 5 T^4 + 7 T^5 + 2 T^6) x^2 y^2 \hbar^4}{(1 - T + T^2 - T^3 + T^4)^2} \right], \\ \text{Knot}[5, 2] \rightarrow \mathbb{E}_{\{\} \rightarrow \{1\}} \left[-3 t \hbar + \text{Log} \left[\frac{T^2 \sqrt{\frac{T^4}{(-1+2T)^2}} (-1 + 2 T)}{2 - 3 T + 2 T^2} \right], \right. \\ & \frac{4 a (-1 + T) \times (1 + T) \hbar}{2 - 3 T + 2 T^2} - \frac{(-1 + T) \times (-9 + 11 T - 7 T^2 + T^3) \hbar}{(2 - 3 T + 2 T^2)^2} - \frac{4 \times (1 + T) x y \hbar^2}{2 - 3 T + 2 T^2}, \end{aligned}$$

$$\begin{aligned}
& \frac{4 a^2 T (3 - 8 T + 3 T^2) \hbar^2}{(2 - 3 T + 2 T^2)^2} + \frac{4 a T (7 - 30 T + 36 T^2 - 20 T^3 + 5 T^4) \hbar^2}{(2 - 3 T + 2 T^2)^3} + \\
& \frac{(-1 + T) \times (-1 - 29 T + 148 T^2 - 194 T^3 + 146 T^4 - 60 T^5 + 13 T^6 + T^7) \hbar^2}{2 (2 - 3 T + 2 T^2)^4} - \\
& \frac{8 a T (-5 + 4 T + 2 T^2) x y \hbar^3}{(2 - 3 T + 2 T^2)^2} + \frac{4 \times (1 + T) \times (-4 + 7 T - 5 T^2 - 5 T^3 + 4 T^4) x y \hbar^3}{(2 - 3 T + 2 T^2)^3} + \\
& \left. \frac{2 \times (2 + 11 T + 2 T^2) x^2 y^2 \hbar^4}{(2 - 3 T + 2 T^2)^2} \right\}
\end{aligned}$$