Pensieve header: A graded-CU variant.

In[∘]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
$k = 1;
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
<< Engine.m
<< Objects.m
```

Out[∘]= {2021, 5, 27, 4, 27, 7.9530517}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

“Define” Code

In[∘]:=
```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ; op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

H

Symmetric Algebra Objects

In[∘]:=
```
H_i_ := Λ2𝔼_{{i}→{i}}[ℏ β_i b_i + ℏ τ_i t_i + ℏ α_i a_i + ℏ η_i y_i + ℏ ξ_i x_i];
```

In[∘]:=
```
sm_{i_,j_→k_} := Λ2𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) + y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_{i_→j_,k_} := Λ2𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) + η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_i_ := Λ2𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sη_i_ := Λ2𝔼_{{}→{i}}[0];
sϵ_i_ := Λ2𝔼_{{i}→{}}[0];
```

In[∘]:=

$$\mathtt{s}\sigma_{i\_\to j\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{j\}}\left[\beta_i\, b_j + \tau_i\, t_j + \alpha_i\, a_j + \eta_i\, y_j + \xi_i\, x_j\right];$$
$$\mathtt{s}\Upsilon_{i\_\to j\_,k\_,l\_,m\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{j,k,l,m\}}\left[\beta_i\, b_k + \tau_i\, t_k + \alpha_i\, a_l + \eta_i\, y_j + \xi_i\, x_m\right];$$

The CU Definitions

In[∘]:=

$$\mathtt{c}\Lambda = \left(\eta_i + \frac{e^{-\hbar\alpha_i - \hbar\epsilon\beta_i}\eta_j}{1+\hbar^2\epsilon\,\eta_j\,\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\mathrm{Log}\left[1+\hbar^2\epsilon\,\eta_j\,\xi_i\right]}{\hbar\,\epsilon}\right) b_k +$$
$$\left(\alpha_i + \alpha_j + \frac{\mathrm{Log}\left[1+\hbar^2\epsilon\,\eta_j\,\xi_i\right]}{\hbar}\right) a_k + \left(\frac{e^{-\hbar\alpha_j - \hbar\epsilon\beta_j}\xi_i}{1+\hbar^2\epsilon\,\eta_j\,\xi_i} + \xi_j\right) x_k;$$
$$\mathrm{Define}\left[\mathtt{cm}_{i,j\to k} = \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\left[\left(\eta_i + \frac{e^{-\hbar\alpha_i - \hbar\epsilon\beta_i}\eta_j}{1+\hbar^2\epsilon\,\eta_j\,\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\mathrm{Log}\left[1+\hbar^2\epsilon\,\eta_j\,\xi_i\right]}{\hbar\,\epsilon}\right) b_k +\right.\right.$$
$$\left.\left.\left(\alpha_i + \alpha_j + \frac{\mathrm{Log}\left[1+\hbar^2\epsilon\,\eta_j\,\xi_i\right]}{\hbar}\right) a_k + \left(\frac{e^{-\hbar\alpha_j - \hbar\epsilon\beta_j}\xi_i}{1+\hbar^2\epsilon\,\eta_j\,\xi_i} + \xi_j\right) x_k\right]\right]$$

In[∘]:= $\mathtt{cm}_{1,2\to 1}$ // $\mathtt{cm}_{1,3\to 1}$

Out[∘]=

$$\mathbb{E}_{\{1,2,3\}\to\{1\}}\Big[a_1\,(\alpha_1+\alpha_2+\alpha_3) + b_1\,\beta_1 + b_1\,\beta_2 + b_1\,\beta_3 + y_1\,\eta_1 +$$
$$\frac{y_1\,\eta_2}{\mathcal{A}_1} + \frac{y_1\,\eta_3}{\mathcal{A}_1\,\mathcal{A}_2} + \frac{x_1\,\xi_1}{\mathcal{A}_2\,\mathcal{A}_3} + \hbar\, b_1\,\eta_2\,\xi_1 + \frac{\hbar\, b_1\,\eta_3\,\xi_1}{\mathcal{A}_2} + \frac{x_1\,\xi_2}{\mathcal{A}_3} + \hbar\, b_1\,\eta_3\,\xi_2 + x_1\,\xi_3,$$
$$-\frac{\hbar\, y_1\,\beta_1\,\eta_2}{\mathcal{A}_1} - \frac{\hbar\, y_1\,\beta_1\,\eta_3}{\mathcal{A}_1\,\mathcal{A}_2} - \frac{\hbar\, y_1\,\beta_2\,\eta_3}{\mathcal{A}_1\,\mathcal{A}_2} - \frac{\hbar\, x_1\,\beta_2\,\xi_1}{\mathcal{A}_2\,\mathcal{A}_3} - \frac{\hbar\, x_1\,\beta_3\,\xi_1}{\mathcal{A}_2\,\mathcal{A}_3} + \hbar\, a_1\,\eta_2\,\xi_1 - \frac{\hbar^2\, y_1\,\eta_2^2\,\xi_1}{\mathcal{A}_1} +$$
$$\frac{\hbar\, a_1\,\eta_3\,\xi_1}{\mathcal{A}_2} - \frac{\hbar^2\, b_1\,\beta_2\,\eta_3\,\xi_1}{\mathcal{A}_2} - \frac{2\,\hbar^2\, y_1\,\eta_2\,\eta_3\,\xi_1}{\mathcal{A}_1\,\mathcal{A}_2} - \frac{\hbar^2\, y_1\,\eta_3^2\,\xi_1}{\mathcal{A}_1\,\mathcal{A}_2^2} - \frac{\hbar^2\, x_1\,\eta_2\,\xi_1^2}{\mathcal{A}_2\,\mathcal{A}_3} -$$
$$\frac{1}{2}\,\hbar^3\, b_1\,\eta_2^2\,\xi_1^2 - \frac{\hbar^2\, x_1\,\eta_3\,\xi_1^2}{\mathcal{A}_2^2\,\mathcal{A}_3} - \frac{\hbar^3\, b_1\,\eta_2\,\eta_3\,\xi_1^2}{\mathcal{A}_2} - \frac{\hbar^3\, b_1\,\eta_3^2\,\xi_1^2}{2\,\mathcal{A}_2^2} - \frac{\hbar\, x_1\,\beta_3\,\xi_2}{\mathcal{A}_3} + \hbar\, a_1\,\eta_3\,\xi_2 -$$
$$\frac{\hbar^2\, y_1\,\eta_3^2\,\xi_2}{\mathcal{A}_1\,\mathcal{A}_2} - \frac{2\,\hbar^2\, x_1\,\eta_3\,\xi_1\,\xi_2}{\mathcal{A}_2\,\mathcal{A}_3} - \frac{\hbar^3\, b_1\,\eta_3^2\,\xi_1\,\xi_2}{\mathcal{A}_2} - \frac{\hbar^2\, x_1\,\eta_3\,\xi_2^2}{\mathcal{A}_3} - \frac{1}{2}\,\hbar^3\, b_1\,\eta_3^2\,\xi_2^2\Big]$$

In[∘]:= HL[($\mathtt{cm}_{1,2\to 1}$ // $\mathtt{cm}_{1,3\to 1}$) ≡ ($\mathtt{cm}_{2,3\to 2}$ // $\mathtt{cm}_{1,2\to 1}$)]

Out[∘]= True

In[∘]:=

$$\mathrm{Define}\left[\mathtt{c}\sigma_{i\to j} = \mathtt{s}\sigma_{i,j}\ /.\ \tau_i \to 0,\ \mathtt{c}\epsilon_i = \mathtt{s}\epsilon_i,\ \mathtt{c}\eta_i = \mathtt{s}\eta_i,\ \mathtt{c}\Delta_{i\to j,k} = \mathtt{s}\Delta_{i\to j,k},\right.$$
$$\left.\mathtt{cS}_i = \mathtt{sS}_i\ //\ \mathtt{s}\Upsilon_{i\to 1,2,3,4}\ //\ \mathtt{cm}_{4,3\to i}\ //\ \mathtt{cm}_{i,2\to i}\ //\ \mathtt{cm}_{i,1\to i}\right];$$

In[∘]:=
```
HL@Simplify[(cm_{1,2→1} // H_1 // U2l) ≡ ((H_1 H_2) // cm_{1,2→1} // U2l)]
```

Out[∘]=
$$\hbar\left(\left(-e^{-\hbar\alpha_1}+e^{-\hbar^2\alpha_1}\right)y_1\eta_2+\left(\left(-e^{-\hbar\alpha_2}+e^{-\hbar^2\alpha_2}\right)x_1+(-1+\hbar)\hbar b_1\eta_2\right)\xi_1\right)=0\ \&\&$$
$$e^{-\hbar(1+\hbar)(\alpha_1+\alpha_2)}\hbar\left(2e^{\hbar(1+\hbar)\alpha_2}y_1\eta_2\left(\left(e^{\hbar^2\alpha_1}-e^{\hbar\alpha_1}\hbar\right)\beta_1+\hbar\left(e^{\hbar^2\alpha_1}-e^{\hbar\alpha_1}\hbar^2\right)\eta_2\xi_1\right)+e^{\hbar(1+\hbar)\alpha_1}\xi_1\left(2x_1\left(\left(e^{\hbar^2\alpha_2}-e^{\hbar\alpha_2}\hbar\right)\beta_2+\hbar\left(e^{\hbar^2\alpha_2}-e^{\hbar\alpha_2}\hbar^2\right)\eta_2\xi_1\right)-e^{\hbar(1+\hbar)\alpha_2}(-1+\hbar)\eta_2\left(-2a_1+\hbar^2\left(1+\hbar+\hbar^2\right)b_1\eta_2\xi_1\right)\right)\right)=0$$

Booting Up QU

Upper to lower and lower to Upper:

In[∘]:=
```
U2l[ℰ_] :=
  ℰ /. {B_{i_}^{p_.} :→ e^{-p ħ b_i}, B^{p_.} :→ e^{-p ħ b}, T_{i_}^{p_.} :→ e^{p ħ t_i}, T^{p_.} :→ e^{p ħ t}, 𝒜_{i_}^{p_.} :→ e^{p ħ α_i}, 𝒜^{p_.} :→ e^{p ħ α}};
l2U[ℰ_] := ℰ //. {e^{c_. b_{i_}+d_.} :→ B_i^{-c/ħ} e^d, e^{c_. b+d_.} :→ B^{-c/ħ} e^d,
    e^{c_. t_{i_}+d_.} :→ T_i^{c/ħ} e^d, e^{c_. t+d_.} :→ T^{c/ħ} e^d,
    e^{c_. α_{i_}+d_.} :→ 𝒜_i^{c/ħ} e^d, e^{c_. α+d_.} :→ 𝒜^{c/ħ} e^d, e^{𝒳_} :→ e^{Expand@𝒳}};
l2U[r_Rule] := Module[{U = r⟦1⟧ /. {b → B, t → T, α → 𝒜}}, U → l2U[U2l[U] /. r]];
AlsoUpper[rs_List] := rs ⋃ (l2U /@ rs);
```

Derivatives in the presence of exponentiated variables:

In[∘]:=
```
D_b[f_] := ∂_b f - ħ B ∂_B f; D_{b_i_}[f_] := ∂_{b_i} f - ħ B_i ∂_{B_i} f;
D_t[f_] := ∂_t f + ħ T ∂_T f; D_{t_i_}[f_] := ∂_{t_i} f + ħ T_i ∂_{T_i} f;
D_α[f_] := ∂_α f + ħ 𝒜 ∂_𝒜 f; D_{α_i_}[f_] := ∂_{α_i} f + ħ 𝒜_i ∂_{𝒜_i} f;
D_{v_}[f_] := ∂_v f;
```

In[∘]:=
```
Define[aσ_{i→j} = Λ2𝔼_{{i}→{j}}[a_j α_i + x_j ξ_i], bσ_{i→j} = Λ2𝔼_{{i}→{j}}[b_j β_i + y_j η_i]]
```

In[∘]:=
```
Define[am_{i,j→k} = Λ2𝔼_{{i,j}→{k}}[(α_i + α_j) a_k + (𝒜_j^{-1} ξ_i + ξ_j) x_k],
  bm_{i,j→k} = Λ2𝔼_{{i,j}→{k}}[(β_i + β_j) b_k + (η_i + e^{-ħ ϵ β_i} η_j) y_k]]
```

In[∘]:=
```
𝒜_j^{-1} // U2l
```

Out[∘]= $e^{-\hbar\alpha_j}$

In[∘]:=
```
{am_{1,2→3}, bm_{1,2→3}}
```

Out[∘]=
$$\left\{\mathbb{E}_{\{1,2\}\to\{3\}}\left[a_3(\alpha_1+\alpha_2)+x_3\left(\frac{\xi_1}{\mathcal{A}_2}+\xi_2\right),0\right],\ \mathbb{E}_{\{1,2\}\to\{3\}}\left[b_3(\beta_1+\beta_2)+y_3(\eta_1+\eta_2),-\hbar y_3\beta_1\eta_2\right]\right\}$$

In[∘]:=
```
HL /@ {(am_{1,2→1} // am_{1,3→1}) ≡ (am_{2,3→2} // am_{1,2→1}), (bm_{1,2→1} // bm_{1,3→1}) ≡ (bm_{2,3→2} // bm_{1,2→1})}
```

Out[∘]= {True, True}

In[∘]:= $bm_{1,2\to1}$ // H_1 // U2l
$(H_1 H_2)$ // $bm_{1,2\to1}$ // U2l

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{1\}}\left[\hbar\, b_1\,\beta_1+\hbar\, b_1\,\beta_2+\hbar\, y_1\,\eta_1+\hbar\, y_1\,\eta_2,\ -\hbar^2\, y_1\,\beta_1\,\eta_2\right]$

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{1\}}\left[\hbar\, b_1\,\beta_1+\hbar\, b_1\,\beta_2+\hbar\, y_1\,\eta_1+\hbar\, y_1\,\eta_2,\ -\hbar^3\, y_1\,\beta_1\,\eta_2\right]$

In[∘]:= HL@Simplify[($bm_{1,2\to1}$ // H_1 // U2l) ≡ (($H_1 H_2$) // $bm_{1,2\to1}$ // U2l)]

Out[∘]= $(-1+\hbar)\,\hbar\, y_1\,\beta_1\,\eta_2 == 0$

In[∘]:= HL@Simplify[($am_{1,2\to1}$ // H_1 // U2l) ≡ (($H_1 H_2$) // $am_{1,2\to1}$ // U2l)]

Out[∘]= $\left(e^{-\hbar\,\alpha_2}-e^{-\hbar^2\,\alpha_2}\right)\hbar\, x_1\,\xi_1 == 0$

In[∘]:= Define$\left[R_{i,j} = \text{Module}\left[\{k\},\ \Lambda 2\mathbb{E}_{\{\}\to\{i,j\}}\left[\hbar\, a_j\, b_i + \sum_{k=1}^{\$k+1}\frac{\left(1-e^{\epsilon\,\hbar^3}\right)^k\left(\hbar\, y_i\, x_j\right)^k}{k\left(1-e^{k\,\epsilon\,\hbar^3}\right)}\right]\right]\right]$

In[∘]:= $R_{i,j}$

Out[∘]= $\mathbb{E}_{\{\}\to\{i,j\}}\left[\hbar\, a_j\, b_i+\hbar\, x_j\, y_i,\ -\frac{1}{4}\hbar^5\, x_j^2\, y_i^2\right]$

In[∘]:= Define$\Big[\overline{R}_{i,j}$ = If$\big[\$k == 0,\ \mathbb{E}_{\{\}\to\{i,j\}}\left[-\hbar\, a_j\, b_i - \hbar\, x_j\, y_i / B_i\right],$
Append$\big[\overline{R}_{\{i,j\},\$k-1}$, −Last$\big[$PadRight$\big[\overline{R}_{\{i,j\},0}, \$k+1\big]\ R_{1,2}$ PadRight$\big[\overline{R}_{\{3,4\},\$k-1}, \$k+1\big]$ //
$(bm_{i,1\to i}\ am_{j,2\to j})$ // $(bm_{i,3\to i}\ am_{j,4\to j})\big]\big]\big]$
$\Big]$

In[∘]:= $\overline{R}_{i,j}$

Out[∘]= $\mathbb{E}_{\{\}\to\{i,j\}}\left[-\hbar\, a_j\, b_i-\frac{\hbar\, x_j\, y_i}{B_i},\ -\hbar^3\, a_j\, B_i^{-\hbar}\, x_j\, y_i-\frac{1}{4}\hbar^4\, B_i^{-2-2\hbar}\left(-\hbar\, B_i^2-4\, B_i^{2\hbar}+4\,\hbar\, B_i^{2\hbar}+4\, B_i^{1+\hbar}\right)x_j^2\, y_i^2\right]$

In[∘]:= $\left(\overline{R}_{1,2}\, R_{3,4}\right)$ // $(dm_{1,3\to1}\ dm_{2,4\to2})$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[-\frac{\hbar\left(-B_1+B_1^{\hbar}\right)x_2\, y_1}{B_1},\ -\left((-1+\hbar)\,\hbar^4\, B_1^{-2+2\hbar}\, x_2^2\, y_1^2\right)\right]$

Import from Testing.nb

In[∘]:= **$k = 3; $\overline{\mathrm{R}}_{i,j}$**

Out[∘]= $$\mathbb{E}_{\{\}\to\{i,j\}}\Big[-\hbar\, a_j\, b_i - \frac{\hbar\, x_j\, y_i}{B_i},\; -\frac{\hbar^2 a_j x_j y_i}{B_i} - \frac{3\hbar^3 x_j^2 y_i^2}{4 B_i^2},$$
$$-\frac{\hbar^3 a_j^2 x_j y_i}{2 B_i} + \frac{\hbar^4 x_j^2 y_i^2}{2 B_i^2} - \frac{3\hbar^4 a_j x_j^2 y_i^2}{2 B_i^2} - \frac{10\hbar^5 x_j^3 y_i^3}{9 B_i^3},$$
$$-\frac{\hbar^4 a_j^3 x_j y_i}{6 B_i} - \frac{3\hbar^5 x_j^2 y_i^2}{16 B_i^2} + \frac{\hbar^5 a_j x_j^2 y_i^2}{B_i^2} - \frac{3\hbar^5 a_j^2 x_j^2 y_i^2}{2 B_i^2} + \frac{2\hbar^6 x_j^3 y_i^3}{B_i^3} - \frac{10\hbar^6 a_j x_j^3 y_i^3}{3 B_i^3} - \frac{35\hbar^7 x_j^4 y_i^4}{16 B_i^4}\Big]$$

In[∘]:= **$k = 3; $\mathrm{P}_{i,j}$**

Out[∘]= $$\mathbb{E}_{\{i,j\}\to\{\}}\Big[\frac{\alpha_j \beta_i}{\hbar} + \frac{\eta_i \xi_j}{\hbar},\; \frac{\eta_i^2 \xi_j^2}{4\hbar},\; \frac{1}{8}\eta_i^2 \xi_j^2 + \frac{5\eta_i^3 \xi_j^3}{36\hbar},\; \frac{1}{24}\hbar\,\eta_i^2 \xi_j^2 + \frac{1}{6}\eta_i^3 \xi_j^3 + \frac{5\eta_i^4 \xi_j^4}{48\hbar}\Big]$$

In[∘]:= **$k = 3; aS_i**

Out[∘]= $$\mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i \alpha_i - x_i \mathcal{A}_i \xi_i,\; -\hbar\, a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2}\hbar\, x_i^2 \mathcal{A}_i^2 \xi_i^2,$$
$$-\frac{1}{2}\hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + \frac{1}{4}\hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2}\hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3,\; -\frac{1}{6}\hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i -$$
$$\frac{1}{12}\hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2}\hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{2}{3}\hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2}\hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3}\hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4\Big]$$

In[∘]:= **$k = 3; $\overline{\mathrm{aS}}_i$**

Out[∘]= $$\mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i \alpha_i - x_i \mathcal{A}_i \xi_i,\; \hbar\, x_i \mathcal{A}_i \xi_i - \hbar\, a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2}\hbar\, x_i^2 \mathcal{A}_i^2 \xi_i^2,$$
$$-\frac{1}{2}\hbar^2 x_i \mathcal{A}_i \xi_i + \hbar^2 a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2}\hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + \frac{5}{4}\hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2}\hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3,$$
$$\frac{1}{6}\hbar^3 x_i \mathcal{A}_i \xi_i - \frac{1}{2}\hbar^3 a_i x_i \mathcal{A}_i \xi_i + \frac{1}{2}\hbar^3 a_i^2 x_i \mathcal{A}_i \xi_i - \frac{1}{6}\hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i - \frac{19}{12}\hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 +$$
$$\frac{5}{2}\hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{13}{6}\hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2}\hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3}\hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4\Big]$$

In[∘]:= **$\left(\overline{\mathrm{aS}}_1 \mathbin{//} \mathrm{aS}_1\right)$**

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1 \alpha_1 + x_1 \xi_1, 0, 0, 0]$

In[∘]:= **$\left(\overline{\mathrm{aS}}_1 \mathbin{//} \mathrm{aS}_1\right)$**

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1 \alpha_1 + x_1 \xi_1, 0, 0, 0]$

In[∘]:= **$\left(\overline{\mathrm{bS}}_1 \mathbin{//} \mathrm{bS}_1\right)$**

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[b_1 \beta_1 + y_1 \eta_1, 0, 0, 0]$

In[∘]:= **$k = 1**

Out[∘]= 1

In[∘]:= $\mathbf{dS_1}$

Out[∘]=
$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1-b_1\,\beta_1-\frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1}-x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$
$$\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$
$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2}-$$
$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

In[∘]:=
$$\mathbb{F}=\mathbb{E};\ \mathbf{dS_1}\equiv\mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1-b_1\,\beta_1-\frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1}-x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$
$$\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$
$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2}-$$
$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]= True

In[∘]:=
$$\overline{\mathbf{dS}}_1\equiv\mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1-b_1\,\beta_1-\frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1}-x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$
$$-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}+\hbar\,x_1\,\mathcal{A}_1\,\xi_1-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$
$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2}-$$
$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]= True

In[∘]:= $\mathbf{dS_1}\ //\ \overline{\mathbf{dS}}_1$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1\,\alpha_1+b_1\,\beta_1+y_1\,\eta_1+x_1\,\xi_1,\ 0]$

In[∘]:=
$$\mathbb{E}_{\{1\}\to\{1\}}\Bigg[-a_1\,\alpha_1-b_1\,\beta_1-\frac{{\color{blue}\blacksquare}\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-{\color{green}\blacksquare}\,x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$
$$\Bigg(\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$
$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\eta_1^2\,\xi_1}{2\,B_1^2}-$$
$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Bigg)\Bigg]\;//$$
$$\mathbb{E}_{\{1\}\to\{1\}}\Bigg[-a_1\,\alpha_1-b_1\,\beta_1-\frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1}-x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$
$$-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}+\hbar\,x_1\,\mathcal{A}_1\,\xi_1-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{{\color{red}\blacksquare}\,a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$
$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{{\color{purple}\blacksquare}\,(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\eta_1^2\,\xi_1}{2\,B_1^2}-$$
$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Bigg]$$

Out[∘]=
$$\mathbb{E}_{\{1\}\to\{1\}}\Bigg[a_1\,\alpha_1+b_1\,\beta_1+{\color{blue}\blacksquare}\,y_1\,\eta_1+{\color{green}\blacksquare}\,x_1\,\xi_1+\frac{(-\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1+B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar},$$
$$(-\hbar+\hbar\,{\color{blue}\blacksquare})\,y_1\,\eta_1+(1-{\color{blue}\blacksquare})\,y_1\,\beta_1\,\eta_1+\frac{1}{2}\left(-\hbar+\hbar\,{\color{blue}\blacksquare}^2\right)y_1^2\,\eta_1^2+(\hbar-\hbar\,{\color{green}\blacksquare})\,x_1\,\xi_1+(-\hbar+\hbar\,{\color{green}\blacksquare})\,a_1\,x_1\,\xi_1+$$
$$(1-{\color{green}\blacksquare})\,x_1\,\beta_1\,\xi_1+(-\hbar+\hbar\,{\color{blue}\blacksquare})\,x_1\,y_1\,\eta_1\,\xi_1+a_1\,(-{\color{blue}\blacksquare}\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{red}\blacksquare}\,\mathcal{A}_1-B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1+$$
$$\Big(({\color{blue}\blacksquare}\,\mathcal{A}_1-{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1+$$
$$(-\mathcal{A}_1+{\color{blue}\blacksquare}\,\mathcal{A}_1+{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1+B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1\Big)\Big/\hbar+$$
$$\frac{1}{2}\,y_1\,\Big(\mathcal{A}_1-2\,{\color{blue}\blacksquare}\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}^2\,\mathcal{A}_1-2\,{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1-3\,B_1\,\mathcal{A}_1+$$
$$6\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1-2\,{\color{blue}\blacksquare}^2\,B_1\,\mathcal{A}_1+2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-2\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1\Big)\,\eta_1^2\,\xi_1+$$
$$\frac{1}{2}\left(-\hbar+2\,\hbar\,{\color{green}\blacksquare}-\hbar\,{\color{green}\blacksquare}^2\right)x_1^2\,\xi_1^2+\frac{1}{2}\,x_1\,\Big(\mathcal{A}_1-2\,{\color{blue}\blacksquare}\,\mathcal{A}_1-2\,{\color{green}\blacksquare}\,\mathcal{A}_1+4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}^2\,\mathcal{A}_1-$$
$$3\,B_1\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1+6\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1\Big)\,\eta_1\,\xi_1^2+$$
$$\frac{1}{4\,\hbar}\,\Big(-\mathcal{A}_1^2+2\,{\color{blue}\blacksquare}\,\mathcal{A}_1^2-2\,{\color{blue}\blacksquare}^2\,\mathcal{A}_1^2+2\,{\color{green}\blacksquare}\,\mathcal{A}_1^2-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,\mathcal{A}_1^2-2\,{\color{green}\blacksquare}^2\,\mathcal{A}_1^2-3\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1^2+$$
$$4\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,B_1\,\mathcal{A}_1^2-8\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2+12\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2+4\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1^2+$$
$$4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2-3\,B_1^2\,\mathcal{A}_1^2+6\,{\color{blue}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-2\,{\color{blue}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2+6\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-$$
$$8\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-2\,{\color{green}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2\Big)\,\eta_1^2\,\xi_1^2\Bigg]$$

In[∘]:=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$

$$\Big(\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1} - \frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1} - \frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2} - \hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1 - x_1\,\mathcal{A}_1\,\beta_1\,\xi_1 + \frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1} -$$

$$\frac{{\color{blue}\blacksquare}\,\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1\,\mathcal{A}_1)\,{\color{red}\blacksquare}\,\eta_1\,\xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1} + \frac{y_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2} -$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2 + \frac{x_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1} + \frac{\left(-3\,\mathcal{A}_1^2 + 4\,B_1\,\mathcal{A}_1^2 - B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big)\Big]\ //$$

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$

$$-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1} - \frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2} + \hbar\,x_1\,\mathcal{A}_1\,\xi_1 - \hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1 - x_1\,\mathcal{A}_1\,\beta_1\,\xi_1 + \frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1} -$$

$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1} + \frac{{\color{green}\blacksquare}\,(-\mathcal{A}_1 + B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1} + \frac{{\color{purple}\blacksquare}\,(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1} + \frac{y_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2} -$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2 + \frac{x_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1} + \frac{\left(-3\,\mathcal{A}_1^2 + 4\,B_1\,\mathcal{A}_1^2 - B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\,\alpha_1 + b_1\,\beta_1 + y_1\,\eta_1 + x_1\,\xi_1,$$

$$(\hbar - \hbar\,{\color{blue}\blacksquare})\,x_1\,y_1\,\eta_1\,\xi_1 + (-{\color{blue}\blacksquare}\,\mathcal{A}_1 - {\color{green}\blacksquare}\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{red}\blacksquare}\,\mathcal{A}_1 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1 + {\color{green}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{red}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1 +$$

$$\frac{(\mathcal{A}_1 - {\color{purple}\blacksquare}\,\mathcal{A}_1 - B_1\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar} +$$

$$y_1\,(-{\color{blue}\blacksquare}\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1^2\,\xi_1 + x_1\,(\mathcal{A}_1 - {\color{blue}\blacksquare}\,\mathcal{A}_1 - B_1\,\mathcal{A}_1 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1^2 +$$

$$\frac{\left(-{\color{blue}\blacksquare}\,\mathcal{A}_1^2 + {\color{purple}\blacksquare}\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2 + 2\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1^2 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2 + B_1^2\,\mathcal{A}_1^2 - {\color{blue}\blacksquare}\,B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{\hbar}\Big]$$

In[∘]:=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\,\alpha_1 + b_1\,\beta_1 + y_1\,\eta_1 + x_1\,\xi_1,$$

$$(\hbar - \hbar\,{\color{blue}\blacksquare})\,x_1\,y_1\,\eta_1\,\xi_1 + \left(-{\color{green}\blacksquare}\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{red}\blacksquare}\,\mathcal{A}_1 + {\color{green}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{red}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{blue}\blacksquare}\,\mathcal{A}_1^2 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1 +$$

$$\frac{(\mathcal{A}_1 - {\color{purple}\blacksquare}\,\mathcal{A}_1 - B_1\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar} +$$

$$y_1\,(-{\color{blue}\blacksquare}\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1^2\,\xi_1 + x_1\,(\mathcal{A}_1 - {\color{blue}\blacksquare}\,\mathcal{A}_1 - B_1\,\mathcal{A}_1 + {\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1^2 +$$

$$\frac{\left(-{\color{blue}\blacksquare}\,\mathcal{A}_1^2 + {\color{purple}\blacksquare}\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2 + 2\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1^2 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2 + B_1^2\,\mathcal{A}_1^2 - {\color{blue}\blacksquare}\,B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{\hbar}\Big]\ /.\ \{\text{Red} \mid \text{Green} \mid \text{Blue} \to 1\}$$

Out[∘]=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\,\alpha_1 + b_1\,\beta_1 + y_1\,\eta_1 + x_1\,\xi_1,$$

$$\left({\color{purple}\blacksquare}\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1 - \mathcal{A}_1^2 + B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1 + \frac{(\mathcal{A}_1 - {\color{purple}\blacksquare}\,\mathcal{A}_1 - B_1\,\mathcal{A}_1 + {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar} +$$

$$y_1\,(-\mathcal{A}_1 + {\color{purple}\blacksquare}\,\mathcal{A}_1 + B_1\,\mathcal{A}_1 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1^2\,\xi_1 + \frac{\left(-\mathcal{A}_1^2 + {\color{purple}\blacksquare}\,\mathcal{A}_1^2 + B_1\,\mathcal{A}_1^2 - {\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{\hbar}\Big]$$

In[∘]:= $$\mathbb{E}_{\{1\}\to\{1\}}\left[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{{\color{lime}\blacksquare}\, y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\, B_1},\ 0\right] \ //$$
$$\mathbb{E}_{\{1\}\to\{1\}}\left[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\, B_1},\ \frac{{\color{purple}\blacksquare}\,(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\, B_1}\right]$$

Out[∘]= $$\mathbb{E}_{\{1\}\to\{1\}}\left[a_1\,\alpha_1 + b_1\,\beta_1 + {\color{lime}\blacksquare}\, y_1\,\eta_1 + x_1\,\xi_1 + \frac{(-\mathcal{A}_1 + {\color{lime}\blacksquare}\,\mathcal{A}_1 + B_1\,\mathcal{A}_1 - {\color{lime}\blacksquare}\, B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar},\right.$$
$$({\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1 - {\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\, B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1 + \frac{(-{\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\, B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar} + y_1\left({\color{lime}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1 - {\color{lime}\blacksquare}^2\,{\color{purple}\blacksquare}\, B_1\,\mathcal{A}_1\right)\eta_1^2\,\xi_1 +$$
$$\left.\frac{\left({\color{lime}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1^2 + {\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\, B_1\,\mathcal{A}_1^2 - 2\,{\color{lime}\blacksquare}^2\,{\color{purple}\blacksquare}\, B_1\,\mathcal{A}_1^2 - {\color{lime}\blacksquare}\,{\color{purple}\blacksquare}\, B_1^2\,\mathcal{A}_1^2 + {\color{lime}\blacksquare}^2\,{\color{purple}\blacksquare}\, B_1^2\,\mathcal{A}_1^2\right)\eta_1^2\,\xi_1^2}{\hbar}\right]$$

In[∘]:= $\left(kR_{1,4}\,\overline{kR}_{5,2}\,\overline{kC}_3\right)$ // $km_{2,4\to 2}$ // $km_{1,3\to 1}$ // $km_{1,5\to 1}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ \hbar\, a_1,\ 0]$

In[∘]:= $\overline{kC}_1\, d\eta_2$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[-\frac{\hbar\, t_1}{2},\ \hbar\, a_1,\ 0\right]$

In[∘]:= $(\mathbb{E}_{\{\}\to\{1,2\}}[0,\ a_2\, x_1]$ // $am_{1,2\to 1})$

Out[∘]= $\mathbb{E}_{\{\}\to\{1\}}[0,\ -x_1 + a_1\, x_1]$

In[∘]:= $\$k = 2;\ \mathbb{E}2\Lambda[\mathbb{E}_{\{\}\to\{1,2\}}[0,\ y_2\, b_1]$ // $bm_{1,2\to 1}]$

Out[∘]= $b_1\, y_1$

In[∘]:= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ y_2\, b_1]$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ b_1\, y_2]$