Pensieve header: Profile with NO encapsulation of Zip3-Inner. Time to K817@$k=2: 3905.91.

Startup

In[•]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
$k = 1;
<< Objects.m
<< KT.m
```

Out[•]= {2021, 1, 3, 6, 28, 9.5249954}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

Engine

Canonical Forms:

In[•]:=
```
CCF[ℰ_] := PPCCF@ExpandDenominator@ExpandNumerator@Together[ℰ]; (*Coefficient Canonical Form *)
CF[ℰ_] := PPCF@Module[
    {vs = Cases[ℰ, (y | a | x | η | β | τ | ξ)_, ∞] ∪ {y, a, x, η, β, τ, ξ}},
    Total[(CCF[#[[2]]] × (Times @@ vs^#[[1]])) & /@ CoefficientRules[ℰ, vs]]
   ];
CF[ℰ_List] := CF /@ ℰ;
CF[𝔼sp___[𝒮___]] := CF /@ 𝔼sp[𝒮];
```

Variables and their duals:

In[•]:=
```
{t*, b*, y*, a*, x*, z*, τ*, β*, η*, α*, ξ*, ξ*} = {τ, β, η, α, ξ, ξ, t, b, y, a, x, z};
(vs_List)* := (v ↦ v*) /@ vs;
(u_i_)* := (u*)i;
```

Weights:

In[•]:=
```
Clear[Wt];
Evaluate[Wt /@ {y, b, t, a, x, η, β, τ, α, ξ}] = {1, 0, 0, 2, 1, 1, 2, 2, 0, 1};
Wt[u_i_] := Wt[u];
```

The maximal weight $n, i.e. the *n* of gl(*n*). Initially and for a long while this will not be tested beyond $n == 2.

In[•]:=
```
$n = 2;
```

Upper to lower and lower to Upper:

```
U2l[ℰ_] := ℰ /. {B_i^p_. :> e^(-p ħ b_i), B^p_. :> e^(-p ħ b), T_i^p_. :> e^(p ħ t_i), T^p_. :> e^(p ħ t), 𝒜_i^p_. :> e^(p α_i), 𝒜^p_. :> e^(p α)};
l2U[ℰ_] := ℰ //. {e^(c_. b_i_ + d_.) :> B_i^(-c/ħ) e^d, e^(c_. b + d_.) :> B^(-c/ħ) e^d, e^(c_. t_i_ + d_.) :> T_i^(c/ħ) e^d, e^(c_. t + d_.) :> T^(c/ħ) e^d,
    e^(c_. α_i_ + d_.) :> 𝒜_i^c e^d, e^(c_. α + d_.) :> 𝒜^c e^d, e^𝒳_ :> e^Expand@𝒳};
l2U[r_Rule] := Module[{U = r[[1]] /. {b → B, t → T, α → 𝒜}}, U → l2U[U2l[U] /. r]];
AlsoUpper[rs_List] := rs ⋃ (l2U /@ rs);
```

Derivatives in the presence of exponentiated variables:

```
D_b[f_] := ∂_b f - ħ B ∂_B f; D_b_i_[f_] := ∂_b_i f - ħ B_i ∂_B_i f;
D_t[f_] := ∂_t f + ħ T ∂_T f; D_t_i_[f_] := ∂_t_i f + ħ T_i ∂_T_i f;
D_α[f_] := ∂_α f + 𝒜 ∂_𝒜 f; D_α_i_[f_] := ∂_α_i f + 𝒜_i ∂_𝒜_i f;
D_v_[f_] := ∂_v f;
```

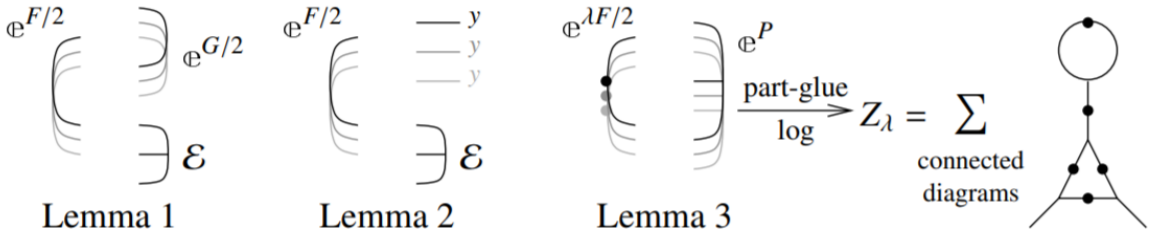
𝔼 operations:

```
ℰ_𝔼[$] := Length[ℰ] - 1; 𝔼_[ℰs___][$] := 𝔼[ℰs][$];
ℰ_𝔼[k_Integer] := ℰ[[k + 1]]; 𝔼_[ℰs___][k_Integer] := {ℰs}[[k + 1]];
𝔼 /: ℰ1_𝔼 ≡ ℰ2_𝔼 := Inner[CF@#1 == CF@#2 &, ℰ1, ℰ2, And];
𝔼_(d1_→r1_)[ℰ1s___] ≡ 𝔼_(d2_→r2_)[ℰ2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[ℰ1s] ≡ 𝔼[ℰ2s]);
𝔼 /: ℰ1_𝔼 * ℰ2_𝔼 := 𝔼 @@ Table[CF[ℰ1[kk] + ℰ2[kk]], {kk, 0, Min[ℰ1[$], ℰ2[$]]}];
𝔼_(d1_→r1_)[ℰ1s___] 𝔼_(d2_→r2_)[ℰ2s___] ^:= 𝔼_((d1⋃d2)→(r1⋃r2)) @@ (𝔼[ℰ1s] × 𝔼[ℰ2s]);
```

```
𝔼_(d1_→r1_)[ℰ1s___] // 𝔼_(d2_→r2_)[ℰ2s___] := Module[{is = r1 ⋂ d2, lvs},
  lvs = Flatten@Table[{y_$@i, b_$@i, t_$@i, a_$@i, x_$@i}, {i, is}];
  𝔼_(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is]) @@ (Zip_lvs⋃lvs*[{lvs*.lvs, Times[
       𝔼[ℰ1s] /. Table[(v : b | B | t | T | a | x | y)_i → v_$@i, {i, is}],
       𝔼[ℰ2s] /. Table[(v : β | τ | α | 𝒜 | ξ | η)_i → v_$@i, {i, is}]
      ]}])
 ]
```

```
Λ2𝔼_(d_→r_)[Λ_] := Module[{k}, 𝔼_(d→r) @@ l2U@Table[SeriesCoefficient[Λ, {ϵ, 0, k}], {k, 0, $k}]];
```

Zipping! Lemmas 2 and 3 are combined, yet they must be applied first to the middle weight variables and then to the heavy and light variables.




```
Zip_vs_[{ℱ_, ℰ_}] := {ℱ, ℰ} // Zip1_vs // Zip2_Select[vs, (0<Wt[#]<$n)&] // Zip3_Select[vs, (0<Wt[#]<$n)&] //
    Zip2_Select[vs, (Wt[#]==0∨Wt[#]==$n)&] // Zip3_Select[vs, (Wt[#]==0∨Wt[#]==$n)&] // Last;
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E}\, e^{\frac{1}{2}\sum_{i,j\in B} G_{ij} z_i z_j} \right\rangle_B = \det(1-GF)^{-1/2} \left\langle F(1-GF)^{-1} : \mathcal{E} \right\rangle_B$$

In[•]:=
```
Zip1{} = Identity;
Zip1vs_@{ℱ_, 𝔼[Q_, P___]} := PPZip1@Module[{ℐ, F, G, u, v},
   ℐ = IdentityMatrix@Length@vs;
   F = Table[If[Wt[u] + Wt[v] == $n, ∂u*,v* ℱ, 0], {u, vs}, {v, vs}];
   G = Table[If[Wt[u] + Wt[v] == $n, ∂u,v Q, 0], {u, vs}, {v, vs}];
   {CF[vs*.(F.Inverse[ℐ - G.F]).vs* / 2], 𝔼[CF[Q - Log[Det[ℐ - G.F]] / 2 - vs.G.vs / 2], P]}
  ]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F:\, \mathcal{E}\, \mathbb{P}^{\sum_{i\in B} y_i z_i}\right\rangle_B = \mathbb{P}^{\frac{1}{2}\sum_{i,j\in B} F_{ij} y_i y_j} \left\langle F:\ \mathcal{E}|_{z_B \to z_B + F y_B}\right\rangle_B$.

In[•]:=
```
Zip2{} = Identity;
Zip2vs_@{ℱ_, 𝔼[Q_, P___]} := PPZip2@Module[{F, Y, u, v},
   F = Table[If[Wt[u] + Wt[v] == $n, ∂u*,v* ℱ, 0], {u, vs}, {v, vs}];
   Y = Table[∂v Q, {v, vs}] /. AlsoUpper@Table[v → 0, {v, vs}];
   CF /@ ({ℱ, 𝔼[Q - Y.vs + Y.F.Y / 2, P]} /. AlsoUpper@Thread[vs → vs + F.Y])
  ]
```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ, $Z_\lambda := \log[\lambda F:\, \mathbb{e}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2}\sum_{i,j\in B} F_{ij}\left(\partial_{z_i}\partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)\right).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m]\,\lambda^m$.

```
In[•]:= Zip3vs_@{ℱ_, ℰ_𝔼} := PPZip3@Module[{F, u, v, Z, $k, kk, jj, $m = 0, m, n},
  $k = Length[ℰ] - 1;
  Do[Z[0, kk] = ℰ[[kk + 1]], {kk, 0, $k}];
  F[u_, v_] := F[u, v] = CF@If[Wt[u] + Wt[v] == $n, ∂u*,v* ℱ, 0];
  Z[m_, kk_, u_] := Z[m, kk, u] = Du[Z[m, kk]];
  Z[m_, kk_, u_, v_] := Z[m, kk, u, v] = Dv[Z[m, kk, u]];
  For[m = 0, m ≤ 2 $m, ++m, For[kk = 0, kk ≤ $k, ++kk,
    Z[m + 1, kk] = CF@Sum[
      If[F[u, v] === 0, 0, F[u, v]/(2 (m + 1))
        (Z[m, kk, u, v] + Sum[Z[n, jj, u] * Z[m - n, kk - jj, v], {n, 0, m}, {jj, 0, kk}])],
      {u, vs}, {v, vs}];
    If[Z[m + 1, kk] =!= 0, $m = m + 1]
   ]];
  CF /@ ({
     ℱ - Sum[F[u, v] u* v*/2, {u, vs}, {v, vs}],
     𝔼 @@ Table[Sum[Z[m, kk], {m, 0, $m}], {kk, 0, $k}]
    } /. AlsoUpper@Table[v → 0, {v, vs}])
 ]
```

Encapsulation.

```
In[•]:= EZip3vs_@{ℱ_, ℰ_𝔼} := PPEZip3@Module[
  {nℰ, nℱ, j = 0, ps, rr = {(*release rules*)}},
  nℰ = Total[
     CoefficientRules[#, vs] /. (ps_ → c_) :> (AppendTo[rr, cℰ[++j] → c]; cℰ[j] × (Times @@ vs^ps))
    ] & /@ ℰ;
  nℱ = Total[CoefficientRules[ℱ, vs*] /.
    (ps_ → c_) :> (AppendTo[rr, cℱ[++j] → c]; cℱ[j] × (Times @@ (vs*)^ps))];
  CF[Expand[{nℱ, nℰ} // Zip3vs] /. rr]
 ]
```

Profiling

```
In[•]:= BeginProfile[];
```

```
In[•]:= Timing@Block[{$k = 1}, Z[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[•]= {16.1563, 𝔼{}→{0}[1/2 × (-4 t ℏ - Log[(1/T^3 - 2/T^2 + 2/T)^2] - Log[(1 + T/(1 - 2 T + 2 T^2) - T^2/(1 - 2 T + 2 T^2))^2]),
  a (-2 ℏ + 2 T^2 ℏ)/(1 - T + T^2) + (-2 ℏ + 3 T ℏ - 2 T^2 ℏ + T^3 ℏ)/(1 - 2 T + 3 T^2 - 2 T^3 + T^4) + x y (-2 ℏ^2 - 2 T ℏ^2)/(1 - T + T^2)]}
```

In[•]:=
```
PrintProfile[]
```

Out[•]=
```
ProfileRoot is root. Profiled time: 16.141
  (    1)  0.063/ 16.140 above Z
  (    1)     0/    0 above RVK
CCF: called 8772 times, time in 6.295/6.295
  ( 8772)  6.295/ 6.295 under CF
CF: called 12930 times, time in 5.342/11.637
  (   84)  0.187/ 0.359 under Z
  (   76)  0.078/ 0.109 under Boot
  (   90)  0.109/ 0.267 under Zip1
  (  270)  1.623/ 5.968 under Zip2
  ( 12410)  3.345/ 4.934 under Zip3
  ( 8772)  6.295/ 6.295 above CCF
Zip3: called 90 times, time in 2.488/7.422
  (   44)  1.252/ 4.296 under Z
  (   46)  1.236/ 3.126 under Boot
  ( 12410)  3.345/ 4.934 above CF
Zip1: called 45 times, time in 1.021/1.288
  (   22)  0.345/ 0.471 under Z
  (   23)  0.676/ 0.817 under Boot
  (   90)  0.109/ 0.267 above CF
Zip2: called 90 times, time in 0.761/6.729
  (   44)  0.327/ 5.889 under Z
  (   46)  0.434/ 0.840 under Boot
  (  270)  1.623/ 5.968 above CF
Boot: called 23 times, time in 0.171/14.11
  (    5)  0.046/ 5.063 under Z
  (   18)  0.125/ 9.047 under Boot
  (   18)  0.125/ 9.047 above Boot
  (   76)  0.078/ 0.109 above CF
  (   23)  0.676/ 0.817 above Zip1
  (   46)  0.434/ 0.840 above Zip2
  (   46)  1.236/ 3.126 above Zip3
Z: called 1 times, time in 0.063/16.141
  (    1)  0.063/ 16.140 under ProfileRoot
  (    5)  0.046/ 5.063 above Boot
  (   84)  0.187/ 0.359 above CF
  (   22)  0.345/ 0.471 above Zip1
  (   44)  0.327/ 5.889 above Zip2
  (   44)  1.252/ 4.296 above Zip3
RVK: called 1 times, time in 0./0.
  (    1)     0/    0 under ProfileRoot
```

In[•]:=
```
Timing@Block[{$k = 1}, Z[Knot[8, 17]]]
```

Out[•]=

$$\left\{61.6875,\ \mathbb{E}_{\{\}\to\{0\}}\left[\frac{1}{2}\times\left(-2\,t\,\hbar-\text{Log}\left[\left(-1-\frac{1}{T^4}+\frac{4}{T^3}-\frac{6}{T^2}+\frac{5}{T}\right)^2\right]-\right.\right.\right.$$

$$\text{Log}\left[\left(1+\frac{T}{1-4\,T+6\,T^2-5\,T^3+T^4}-\frac{2\,T^2}{1-4\,T+6\,T^2-5\,T^3+T^4}+\frac{T^3}{1-4\,T+6\,T^2-5\,T^3+T^4}\right)^2\right]-$$

$$\text{Log}\left[\left(1-\frac{T}{1-3\,T+4\,T^2-4\,T^3+T^4}+\frac{4\,T^2}{1-3\,T+4\,T^2-4\,T^3+T^4}-\frac{7\,T^3}{1-3\,T+4\,T^2-4\,T^3+T^4}+\right.\right.$$

$$\left.\left.\left.\left.\frac{7\,T^4}{1-3\,T+4\,T^2-4\,T^3+T^4}-\frac{4\,T^5}{1-3\,T+4\,T^2-4\,T^3+T^4}+\frac{T^6}{1-3\,T+4\,T^2-4\,T^3+T^4}\right)^2\right]\right),$$

$$\frac{-3\,\hbar+8\,T\,\hbar-8\,T^2\,\hbar+8\,T^4\,\hbar-8\,T^5\,\hbar+3\,T^6\,\hbar}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}+\frac{a\left(-6\,\hbar+16\,T\,\hbar-16\,T^2\,\hbar+16\,T^4\,\hbar-16\,T^5\,\hbar+6\,T^6\,\hbar\right)}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}+$$

$$\left.\left.\frac{x\,y\left(-6\,\hbar^2+10\,T\,\hbar^2-6\,T^2\,\hbar^2-6\,T^3\,\hbar^2+10\,T^4\,\hbar^2-6\,T^5\,\hbar^2\right)}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}\right]\right\}$$

In[•]:=
```
PrintProfile[]
```

Out[•]=
```
ProfileRoot is root. Profiled time: 77.812
  (    2)   0.188/ 77.812 above Z
  (    2)      0/     0 above RVK
CCF: called 25936 times, time in 36.892/36.892
  ( 25936)  36.892/ 36.892 under CF
CF: called 26978 times, time in 29.935/66.827
  (  298)   1.034/  2.286 under Z
  (   88)   0.108/  0.170 under Boot
  (  212)   0.344/  0.721 under Zip1
  (  636)  15.436/ 40.996 under Zip2
  ( 25744)  13.013/ 22.654 under Zip3
  ( 25936)  36.892/ 36.892 above CCF
Zip3: called 212 times, time in 6.586/29.24
  (  158)   5.086/ 25.458 under Z
  (   54)   1.500/  3.782 under Boot
  ( 25744)  13.013/ 22.654 above CF
Zip1: called 106 times, time in 2.092/2.813
  (   79)   1.339/  1.887 under Z
  (   27)   0.753/  0.926 under Boot
  (  212)   0.344/  0.721 above CF
Zip2: called 212 times, time in 1.932/42.928
  (  158)   1.388/ 41.868 under Z
  (   54)   0.544/  1.060 under Boot
  (  636)  15.436/ 40.996 above CF
Z: called 2 times, time in 0.188/77.812
  (    2)   0.188/ 77.812 under ProfileRoot
  (    7)   0.046/  6.125 above Boot
  (  298)   1.034/  2.286 above CF
  (   79)   1.339/  1.887 above Zip1
  (  158)   1.388/ 41.868 above Zip2
  (  158)   5.086/ 25.458 above Zip3
Boot: called 27 times, time in 0.187/15.735
  (    7)   0.046/  6.125 under Z
  (   20)   0.141/  9.610 under Boot
  (   20)   0.141/  9.610 above Boot
  (   88)   0.108/  0.170 above CF
  (   27)   0.753/  0.926 above Zip1
  (   54)   0.544/  1.060 above Zip2
  (   54)   1.500/  3.782 above Zip3
RVK: called 2 times, time in 0./0.
  (    2)      0/     0 under ProfileRoot
```

In[•]:=
```
Timing@Block[{$k = 2}, Z[Knot[3, 1]]]
```

Out[•]=

$$\left\{320.688,\ \mathbb{E}_{\{\}\to\{0\}}\left[\frac{1}{2}\times\left(-4\,t\,\hbar-\text{Log}\left[\left(\frac{1}{T^3}-\frac{2}{T^2}+\frac{2}{T}\right)^2\right]-\text{Log}\left[\left(1+\frac{T}{1-2T+2T^2}-\frac{T^2}{1-2T+2T^2}\right)^2\right]\right),\right.\right.$$

$$\frac{a\left(-2\hbar+2T^2\hbar\right)}{1-T+T^2}+\frac{-2\hbar+3T\hbar-2T^2\hbar+T^3\hbar}{1-2T+3T^2-2T^3+T^4}+\frac{xy\left(-2\hbar^2-2T\hbar^2\right)}{1-T+T^2},\ \frac{a^2\left(2T\hbar^2-8T^2\hbar^2+2T^3\hbar^2\right)}{1-2T+3T^2-2T^3+T^4}+$$

$$\frac{a\left(2T\hbar^2-14T^2\hbar^2+12T^3\hbar^2-6T^4\hbar^2+2T^5\hbar^2\right)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6}+\frac{T\hbar^2-11T^2\hbar^2+16T^3\hbar^2-12T^4\hbar^2+8T^5\hbar^2-3T^6\hbar^2+T^7\hbar^2}{2-8T+20T^2-32T^3+38T^4-32T^5+20T^6-8T^7+2T^8}+$$

$$\left.\left.\frac{axy\left(8T\hbar^3-8T^2\hbar^3-4T^3\hbar^3\right)}{1-2T+3T^2-2T^3+T^4}+\frac{xy\left(-2\hbar^3-2T^2\hbar^3-6T^3\hbar^3+2T^5\hbar^3\right)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6}+\frac{x^2y^2\left(\hbar^4+5T\hbar^4+T^2\hbar^4\right)}{1-2T+3T^2-2T^3+T^4}\right]\right\}$$

In[•]:=
```
PrintProfile[]
```

Out[•]=
```
ProfileRoot is root. Profiled time: 398.499
  (    3)    0.329/ 398.499 above Z
  (    3)       0/      0 above RVK
CCF: called 51756 times, time in 209.633/209.633
  ( 51756)  209.633/ 209.633 under CF
CF: called 40772 times, time in 165.855/375.488
  (  424)    1.544/   3.361 under Z
  (  202)    0.265/   0.453 under Boot
  (  302)    0.456/   1.022 under Zip1
  (  996)   34.351/  99.917 under Zip2
  ( 38848)  129.239/ 270.735 under Zip3
  ( 51756)  209.633/ 209.633 above CCF
Zip3: called 302 times, time in 16.504/287.239
  (  202)   13.428/ 279.492 under Z
  (  100)    3.076/   7.747 under Boot
  ( 38848)  129.239/ 270.735 above CF
Zip1: called 151 times, time in 3.043/4.065
  (  101)    1.697/   2.388 under Z
  (   50)    1.346/   1.677 under Boot
  (  302)    0.456/   1.022 above CF
Zip2: called 302 times, time in 2.793/102.71
  (  202)    1.889/ 100.398 under Z
  (  100)    0.904/   2.312 under Boot
  (  996)   34.351/  99.917 above CF
Boot: called 47 times, time in 0.342/32.388
  (   12)    0.062/  12.531 under Z
  (   35)    0.280/  19.857 under Boot
  (   35)    0.280/  19.857 above Boot
  (  202)    0.265/   0.453 above CF
  (   50)    1.346/   1.677 above Zip1
  (  100)    0.904/   2.312 above Zip2
  (  100)    3.076/   7.747 above Zip3
Z: called 3 times, time in 0.329/398.499
  (    3)    0.329/ 398.499 under ProfileRoot
  (   12)    0.062/  12.531 above Boot
  (  424)    1.544/   3.361 above CF
  (  101)    1.697/   2.388 above Zip1
  (  202)    1.889/ 100.398 above Zip2
  (  202)   13.428/ 279.492 above Zip3
RVK: called 3 times, time in 0./0.
  (    3)       0/      0 under ProfileRoot
```

In[•]:=
```
Timing@Block[{$k = 2}, Z[Knot[8, 17]]]
```

Out[•]=

$$\left\{3507.41,\ \mathbb{E}_{\{\}\to\{0\}}\left[\frac{1}{2}\times\left(-2\,t\,\hbar-\text{Log}\left[\left(-1-\frac{1}{T^4}+\frac{4}{T^3}-\frac{6}{T^2}+\frac{5}{T}\right)^2\right]-\right.\right.\right.$$

$$\text{Log}\left[\left(1+\frac{T}{1-4\,T+6\,T^2-5\,T^3+T^4}-\frac{2\,T^2}{1-4\,T+6\,T^2-5\,T^3+T^4}+\frac{T^3}{1-4\,T+6\,T^2-5\,T^3+T^4}\right)^2\right]-$$

$$\text{Log}\left[\left(1-\frac{T}{1-3\,T+4\,T^2-4\,T^3+T^4}+\frac{4\,T^2}{1-3\,T+4\,T^2-4\,T^3+T^4}-\frac{7\,T^3}{1-3\,T+4\,T^2-4\,T^3+T^4}+\right.\right.$$

$$\left.\left.\left.\left.\frac{7\,T^4}{1-3\,T+4\,T^2-4\,T^3+T^4}-\frac{4\,T^5}{1-3\,T+4\,T^2-4\,T^3+T^4}+\frac{T^6}{1-3\,T+4\,T^2-4\,T^3+T^4}\right)^2\right]\right)\right],$$

$$\frac{-3\,\hbar+8\,T\,\hbar-8\,T^2\,\hbar+8\,T^4\,\hbar-8\,T^5\,\hbar+3\,T^6\,\hbar}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}+\frac{a\left(-6\,\hbar+16\,T\,\hbar-16\,T^2\,\hbar+16\,T^4\,\hbar-16\,T^5\,\hbar+6\,T^6\,\hbar\right)}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}+$$

$$\frac{x\,y\left(-6\,\hbar^2+10\,T\,\hbar^2-6\,T^2\,\hbar^2-6\,T^3\,\hbar^2+10\,T^4\,\hbar^2-6\,T^5\,\hbar^2\right)}{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6},$$

$$\left(a\left(8\,T\,\hbar^2-64\,T^2\,\hbar^2+262\,T^3\,\hbar^2-608\,T^4\,\hbar^2+952\,T^5\,\hbar^2-1096\,T^6\,\hbar^2+952\,T^7\,\hbar^2-608\,T^8\,\hbar^2+262\,T^9\,\hbar^2-64\,T^{10}\,\hbar^2+8\,T^{11}\,\hbar^2\right)\right)\Big/\left(1-8\,T+32\,T^2-86\,T^3+168\,T^4-248\,T^5+283\,T^6-248\,T^7+168\,T^8-86\,T^9+32\,T^{10}-8\,T^{11}+T^{12}\right)+$$

$$\left(a^2\left(8\,T\,\hbar^2-64\,T^2\,\hbar^2+262\,T^3\,\hbar^2-608\,T^4\,\hbar^2+952\,T^5\,\hbar^2-1096\,T^6\,\hbar^2+952\,T^7\,\hbar^2-608\,T^8\,\hbar^2+262\,T^9\,\hbar^2-64\,T^{10}\,\hbar^2+8\,T^{11}\,\hbar^2\right)\right)\Big/\left(1-8\,T+32\,T^2-86\,T^3+168\,T^4-248\,T^5+283\,T^6-248\,T^7+168\,T^8-86\,T^9+32\,T^{10}-8\,T^{11}+T^{12}\right)+$$

$$\left(4\,T\,\hbar^2-50\,T^2\,\hbar^2+307\,T^3\,\hbar^2-1160\,T^4\,\hbar^2+3062\,T^5\,\hbar^2-6127\,T^6\,\hbar^2+9760\,T^7\,\hbar^2-12\,754\,T^8\,\hbar^2+13\,916\,T^9\,\hbar^2-12\,754\,T^{10}\,\hbar^2+9760\,T^{11}\,\hbar^2-6127\,T^{12}\,\hbar^2+3062\,T^{13}\,\hbar^2-1160\,T^{14}\,\hbar^2+307\,T^{15}\,\hbar^2-50\,T^{16}\,\hbar^2+4\,T^{17}\,\hbar^2\right)\Big/$$

$$\left(2-24\,T+144\,T^2-578\,T^3+1728\,T^4-4056\,T^5+7708\,T^6-12\,072\,T^7+15\,744\,T^8-17\,194\,T^9+15\,744\,T^{10}-12\,072\,T^{11}+7708\,T^{12}-4056\,T^{13}+1728\,T^{14}-578\,T^{15}+144\,T^{16}-24\,T^{17}+2\,T^{18}\right)+$$

$$\left(a\,x\,y\left(28\,T\,\hbar^3-168\,T^2\,\hbar^3+544\,T^3\,\hbar^3-1000\,T^4\,\hbar^3+1248\,T^5\,\hbar^3-1096\,T^6\,\hbar^3+656\,T^7\,\hbar^3-216\,T^8\,\hbar^3-20\,T^9\,\hbar^3+40\,T^{10}\,\hbar^3-12\,T^{11}\,\hbar^3\right)\right)\Big/\left(1-8\,T+32\,T^2-86\,T^3+168\,T^4-248\,T^5+283\,T^6-248\,T^7+168\,T^8-86\,T^9+32\,T^{10}-8\,T^{11}+T^{12}\right)+$$

$$\left(x\,y\left(-18\,\hbar^3+78\,T\,\hbar^3-146\,T^2\,\hbar^3+110\,T^3\,\hbar^3+78\,T^4\,\hbar^3-274\,T^5\,\hbar^3+274\,T^6\,\hbar^3-78\,T^7\,\hbar^3-110\,T^8\,\hbar^3+146\,T^9\,\hbar^3-78\,T^{10}\,\hbar^3+18\,T^{11}\,\hbar^3\right)\right)\Big/\left(1-8\,T+32\,T^2-86\,T^3+168\,T^4-248\,T^5+283\,T^6-248\,T^7+168\,T^8-86\,T^9+32\,T^{10}-8\,T^{11}+T^{12}\right)+$$

$$\left(x^2\,y^2\left(3\,\hbar^4-37\,T^2\,\hbar^4+153\,T^3\,\hbar^4-261\,T^4\,\hbar^4+325\,T^5\,\hbar^4-261\,T^6\,\hbar^4+153\,T^7\,\hbar^4-37\,T^8\,\hbar^4+3\,T^{10}\,\hbar^4\right)\right)\Big/$$

$$\left.\left.\left.\left(1-8\,T+32\,T^2-86\,T^3+168\,T^4-248\,T^5+283\,T^6-248\,T^7+168\,T^8-86\,T^9+32\,T^{10}-8\,T^{11}+T^{12}\right)\right]\right\}$$

In[•]:=
```
PrintProfile[]
```

Out[•]=
```
ProfileRoot is root. Profiled time: 3905.91
  (     4)     1.391/ 3905.905 above Z
  (     4)        0/       0 above RVK
CF: called 56217 times, time in 1927.4/3828.7
  (   745)     7.394/   15.875 under Z
  (   220)     0.281/    0.501 under Boot
  (   424)     0.660/    1.526 under Zip1
  (  1484)   315.690/  775.411 under Zip2
  ( 53344)  1603.374/ 3035.388 under Zip3
  ( 126292)  1901.302/ 1901.302 above CCF
CCF: called 126292 times, time in 1901.3/1901.3
  ( 126292)  1901.302/ 1901.302 under CF
Zip3: called 424 times, time in 66.68/3102.07
  (   316)    63.463/ 3094.009 under Z
  (   108)     3.217/    8.059 under Boot
  ( 53344)  1603.374/ 3035.388 above CF
Zip2: called 424 times, time in 4.767/780.178
  (   316)     3.800/  777.631 under Z
  (   108)     0.967/    2.547 under Boot
  (  1484)   315.690/  775.411 above CF
Zip1: called 212 times, time in 3.978/5.504
  (   158)     2.554/    3.749 under Z
  (    54)     1.424/    1.755 under Boot
  (   424)     0.660/    1.526 above CF
Z: called 4 times, time in 1.391/3905.91
  (     4)     1.391/ 3905.905 under ProfileRoot
  (    14)     0.077/   13.250 above Boot
  (   745)     7.394/   15.875 above CF
  (   158)     2.554/    3.749 above Zip1
  (   316)     3.800/  777.631 above Zip2
  (   316)    63.463/ 3094.009 above Zip3
Boot: called 51 times, time in 0.388/33.544
  (    14)     0.077/   13.250 under Z
  (    37)     0.311/   20.294 under Boot
  (    37)     0.311/   20.294 above Boot
  (   220)     0.281/    0.501 above CF
  (    54)     1.424/    1.755 above Zip1
  (   108)     0.967/    2.547 above Zip2
  (   108)     3.217/    8.059 above Zip3
RVK: called 4 times, time in 0./0.
  (     4)        0/       0 under ProfileRoot
```

In[•]:=
```
Timing@Block[{$k = 3}, Z[Knot[3, 1]]]
```

Out[•]=
```
$Aborted
```

In[•]:=
```
PrintProfile[]
```

```
Timing@Block[{$k = 3}, Z[Knot[8, 17]]]
```

In[•]:=
```
PrintProfile[]
```