Pensieve header: Exponentiation in ybax algebras.

Startup

In[∘]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
BeginProfile[];
$k = 1;
<< Engine.m
<< Objects.m
<< KT.m
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, , ]];
```

Out[∘]=
```
{2021, 8, 10, 9, 39, 22.6525128}
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

Exponentials

Task. Define $\text{Exp}_m[\mathbb{U}_{\{\_\_\_\}\to\{i\_\}}[U\_\_]]$ to compute $e^{\mathbb{O}(U)}$ to order $\epsilon^{\text{Length@}\{U\}-1}$ using the $m_{i,i\to i}$ multiplication, where U is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.
Example: $\text{Exp}_{\text{dm},1}[\mathbb{U}_{\{\}\to\{2\}}[b_2\, a_2 + y_2\, x_2, 0]]$ is the exponential of the arrow on strand 2, computed to degree 1.

In[∘]:=
```
m = dm; i = 2; U = Sequence[b₂ a₂ + y₂ x₂, 0]
```

Out[∘]=
```
Sequence[a₂ b₂ + x₂ y₂, 0]
```

In[∘]:=
```
F[λ_, i_] := 𝔼{}→{i}[f₁[λ] + f₂[λ] aᵢ + f₃[λ] xᵢ + f₄[λ] yᵢ + f₅[λ] xᵢ yᵢ];
F[λ, i]
F[μ, j]
```

Out[∘]=
$\mathbb{E}_{\{\}\to\{2\}}[f_1[\lambda] + a_2 f_2[\lambda] + x_2 f_3[\lambda] + y_2 f_4[\lambda] + x_2 y_2 f_5[\lambda]]$

Out[∘]=
$\mathbb{E}_{\{\}\to\{j\}}[f_1[\mu] + a_j f_2[\mu] + x_j f_3[\mu] + y_j f_4[\mu] + x_j y_j f_5[\mu]]$

In[∘]:= **l1 = $\left(\partial_\mu \text{List} @@ \left(F[\lambda, i]\, F[\mu, j] \;//\; m_{i,j\to i}\right)\right)$ /. $\mu \to 0$**

Out[∘]= $$\Big\{a_2\, f_2'[0] + \frac{e^{-f_2[0]}\, x_2 \left(-e^{f_2[0]}\,\hbar\, f_3[0] - \hbar\, f_3[\lambda] - e^{f_2[0]}\, f_3[\lambda]\, f_5[0] + e^{f_2[0]}\, B_2\, f_3[\lambda]\, f_5[0]\right) f_2'[0]}{\hbar} + \frac{1}{\hbar} e^{-f_2[0]-f_2[\lambda]}\, x_2\, y_2 \left(-e^{f_2[0]}\,\hbar\, f_5[0] - e^{f_2[\lambda]}\,\hbar\, f_5[\lambda] - e^{f_2[0]+f_2[\lambda]}\, f_5[0]\, f_5[\lambda] + e^{f_2[0]+f_2[\lambda]}\, B_2\, f_5[0]\, f_5[\lambda]\right) f_2'[0] + \frac{\hbar\, f_1'[0] + f_3[\lambda]\, f_4'[0] - B_2\, f_3[\lambda]\, f_4'[0]}{\hbar} + \frac{e^{-f_2[\lambda]}\, y_2 \left(\hbar\, f_4'[0] + e^{f_2[\lambda]}\, f_5[\lambda]\, f_4'[0] - e^{f_2[\lambda]}\, B_2\, f_5[\lambda]\, f_4'[0]\right)}{\hbar} - \frac{1}{\hbar}\, e^{-f_2[0]}\, x_2 \left(-e^{f_2[0]}\,\hbar\, f_3[0]\, f_2'[0] - e^{f_2[0]}\, f_3[\lambda]\, f_5[0]\, f_2'[0] + e^{f_2[0]}\, B_2\, f_3[\lambda]\, f_5[0]\, f_2'[0] - e^{f_2[0]}\,\hbar\, f_3'[0] - e^{f_2[0]}\, f_3[\lambda]\, f_5'[0] + e^{f_2[0]}\, B_2\, f_3[\lambda]\, f_5'[0]\right) - \frac{1}{\hbar}\, e^{-f_2[0]-f_2[\lambda]}\, x_2\, y_2 \left(-e^{f_2[0]}\,\hbar\, f_5[0]\, f_2'[0] - e^{f_2[0]+f_2[\lambda]}\, f_5[0]\, f_5[\lambda]\, f_2'[0] + e^{f_2[0]+f_2[\lambda]}\, B_2\, f_5[0]\, f_5[\lambda]\, f_2'[0] - e^{f_2[0]}\,\hbar\, f_5'[0] - e^{f_2[0]+f_2[\lambda]}\, f_5[\lambda]\, f_5'[0] + e^{f_2[0]+f_2[\lambda]}\, B_2\, f_5[\lambda]\, f_5'[0]\right)\Big\}$$

In[∘]:= **r1 = $(\partial_\mu \text{List} @@ F[\lambda + \mu, i])$ /. $\mu \to 0$**

Out[∘]= $\{f_1'[\lambda] + a_2\, f_2'[\lambda] + x_2\, f_3'[\lambda] + y_2\, f_4'[\lambda] + x_2\, y_2\, f_5'[\lambda]\}$

In[∘]:= **eqs1 = And @@ ((# == 0) & /@ Flatten@CoefficientList[l1 − r1, {a_i, x_i, y_i}]) /. f_[0] → 0**

Out[∘]= $$-f_1'[\lambda] + \frac{\hbar\, f_1'[0] + f_3[\lambda]\, f_4'[0] - B_2\, f_3[\lambda]\, f_4'[0]}{\hbar} == 0\;\&\&\; \frac{e^{-f_2[\lambda]}\left(\hbar\, f_4'[0] + e^{f_2[\lambda]}\, f_5[\lambda]\, f_4'[0] - e^{f_2[\lambda]}\, B_2\, f_5[\lambda]\, f_4'[0]\right)}{\hbar} - f_4'[\lambda] == 0\;\&\&\; -f_3[\lambda]\, f_2'[0] - f_3'[\lambda] - \frac{-\hbar\, f_3'[0] - f_3[\lambda]\, f_5'[0] + B_2\, f_3[\lambda]\, f_5'[0]}{\hbar} == 0\;\&\&\; -f_5[\lambda]\, f_2'[0] - \frac{e^{-f_2[\lambda]}\left(-\hbar\, f_5'[0] - e^{f_2[\lambda]}\, f_5[\lambda]\, f_5'[0] + e^{f_2[\lambda]}\, B_2\, f_5[\lambda]\, f_5'[0]\right)}{\hbar} - f_5'[\lambda] == 0\;\&\&\; f_2'[0] - f_2'[\lambda] == 0$$

In[∘]:= **l2 = Take[{U}, 1]**

Out[∘]= $\{a_2\, b_2 + x_2\, y_2\}$

In[∘]:= **r2 = $(\partial_\mu \text{List} @@ F[\mu, i])$ /. $\mu \to 0$**

Out[∘]= $\{f_1'[0] + a_2\, f_2'[0] + x_2\, f_3'[0] + y_2\, f_4'[0] + x_2\, y_2\, f_5'[0]\}$

In[∘]:= **eqs2 = And @@ ((# == 0) & /@ Flatten@CoefficientList[l2 − r2, {a_i, x_i, y_i}])**

Out[∘]= $-f_1'[0] == 0\;\&\&\; -f_4'[0] == 0\;\&\&\; -f_3'[0] == 0\;\&\&\; 1 - f_5'[0] == 0\;\&\&\; b_2 - f_2'[0] == 0$

In[]:= eqs3 = eqs1 /. {f5′[0] → 1, f2′[0] → b2, f_′[0] → 0}

Out[]= $-f_1'[\lambda] == 0 \,\&\&\, -f_4'[\lambda] == 0 \,\&\&\, -b_2 f_3[\lambda] - \frac{-f_3[\lambda] + B_2 f_3[\lambda]}{\hbar} - f_3'[\lambda] == 0 \,\&\&$
$-b_2 f_5[\lambda] - \frac{e^{-f_2[\lambda]}\left(-\hbar - e^{f_2[\lambda]} f_5[\lambda] + e^{f_2[\lambda]} B_2 f_5[\lambda]\right)}{\hbar} - f_5'[\lambda] == 0 \,\&\&\, b_2 - f_2'[\lambda] == 0$

In[]:= DSolve[f1[0] == 0 ∧ f2[0] == 0 ∧ f3[0] == 0 ∧ f4[0] == 0 ∧ f5[0] == 0 ∧ eqs3,
{f1[λ], f2[λ], f3[λ], f4[λ], f5[λ]}, λ]

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $-\text{Log}[e^{c_4}] == 0$.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[]= $\left\{\left\{f_1[\lambda] \to 0, f_4[\lambda] \to 0, f_3[\lambda] \to 0, f_2[\lambda] \to \lambda b_2, f_5[\lambda] \to \frac{e^{-\frac{\lambda}{\hbar} - \frac{\lambda(-1+\hbar b_2 + B_2)}{\hbar}}\left(-e^{\lambda/\hbar} + e^{\frac{\lambda B_2}{\hbar}}\right)\hbar}{-1 + B_2}\right\}\right\}$

In[]:= ans = FullSimplify[f5[λ] /.
DSolve[$-b_2 f_5[\lambda] - \frac{e^{-\lambda b_2}\left(-\hbar - e^{\lambda b_2} f_5[\lambda] + e^{\lambda b_2} B_2 f_5[\lambda]\right)}{\hbar} - f_5'[\lambda] == 0$ ∧ f5[0] == 0, f5[λ], λ]]

Out[]= $\left\{\frac{e^{-\frac{\lambda(-1+\hbar b_2 + B_2)}{\hbar}}\left(-1 + e^{\frac{\lambda(-1+B_2)}{\hbar}}\right)\hbar}{-1 + B_2}\right\}$

In[]:= FullSimplify[ans /. b2 → 0]

Out[]= $\left\{\frac{\left(1 - e^{\frac{\lambda - \lambda B_2}{\hbar}}\right)\hbar}{-1 + B_2}\right\}$