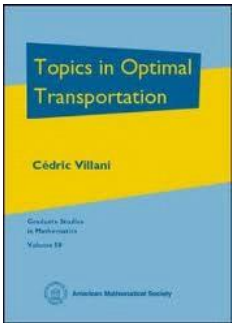
Conversation with McCann, August 3, 2010

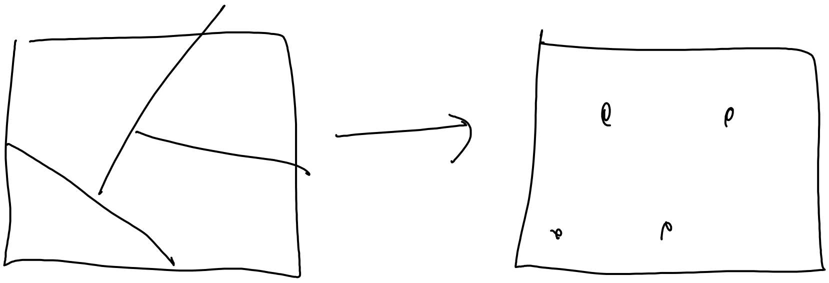
August-03-10
1:58 PM

Frisch, Sobolevski +3 in Nature 417 (2002)

F. + S. + Brenier + Loeper in Monthly of Royal Astronomical Soc.

Canonical refs: Books by Cédric Villani





Brenier's Thm,
chap 3 of Villani's first book.

$$u(x) = \max_{y_i \in S^2} \left\{ -\frac{d^2}{2}(x,y_i) + v_i \right\} = \max_{i=1..5} \{u_i(x)\}$$

$$\min_{0 \le \gamma \text{ on } S^2 \times S^2} \int_{S^2 \times S^2} \frac{d^2}{2}(x,y)\, d\gamma(x,y) = \max_{u(x)+v(y) \le \frac{d^2(x,y)}{2}} \int_{S^2} u\, d\mu + \int_{S^2} v\, d\nu$$

μ ν

BB

There are also McCann's own notes... at eprints/McCann.

Question: There's a map $\Phi : \{\text{convex functions}\} \to \{\text{measures}\}$ by $\Phi u = (\nabla u)_* \mu$, where μ is Lebesgue's measure. According to McCann, Φ is largely onto. Is

Φ^{-1} continuous? Does it make sense to find Φ^{-1} on discrete measures by repeatedly point-splitting and deforming?