Conversation with McCann, August 11, 2010

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9:56 PM

We want to transport $\mu = \sum_{i=1}^{M} m_i \delta_{x_i}$ to $\nu = \sum_{j=1}^{n} n_j \delta_{y_j}$.

$$\Gamma = \left\{ \gamma \in \mathcal{M}_{M \times N} : \sum_{i=1}^{M} \gamma_{ij} = n_j \; ; \sum_{j=1}^{N} \gamma_{ij} = m_i \; ; \; \gamma_{ij} \geq 0 \right\}$$

$$\min_{\gamma \in \Gamma} \sum_i \sum_j \gamma_{ij} |x_i - y_j|^2 \overset{THM}{=\!=} \max_{u_i + v_j \leq |x_i - y_j|^2} \sum_i m_i u_i + \sum_j n_j v_j$$