The co-product on A(G)

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The co-product on $grQG = \hat{\bigoplus} I^n/I^{n+1}$

$\shortparallel$

$A(G)$

<u>claim 1</u> $A(G\times H) \cong A(G)\hat{\otimes} A(H)$

<u>claim 2</u> A is a functor

Hence the diagonal $G \longrightarrow G\times G$ induces a map $A(G) \longrightarrow A(G\times G) \cong A(G)\otimes A(G)$

<u>pf of claim 1</u>

$$(g,h)-(1,1) \longmapsto (g-1)\otimes 1 + g\otimes(h-1)$$

maps $I_{G\times H} \longmapsto I_G\otimes QH + QG\otimes I_H$

$$I^n_{G\times H} \longmapsto \sum_{n_1+n_2=n} I_G^{n_1}\otimes I_H^{n_2}$$

$$I^n_{G\times H}/I^{n+1}_{G\times H} \longmapsto \frac{\sum_{n_1+n_2=n} I^{n_1}\otimes I^{n_2}}{\sum_{n_1+n_2=n} I^{n_1+1}\otimes I^{n_2} + I^{n_1}\otimes I^{n_2+1}} = LHS$$

$$\underset{\text{Lemma 3}}{\overset{?}{=}} \bigoplus_{n_1+n_2=n} I^{n_1}/I^{n_1+1} \otimes I^{n_2}/I^{n_2+1} = RHS$$

<u>pf of Lemma 3.</u> The map

$$\frac{I^{n_1}}{I^{n_1+1}} \otimes \frac{I^{n_2}}{I^{n_2+1}} \longrightarrow \frac{I^{n_1}\otimes I^{n_2}}{I^{n_1+1}\otimes I^{n_2} + I^{n_1}\otimes I^{n_2+1}} \subset LHS$$

I^{n_2+1} $\qquad$ $I^{n_1+1}\otimes I^{n_2}+I^{n_1}\otimes I^{n_2+1}$

is obvious, and it obviously lead to a surjection

$$\bigoplus_{n_1+n_2=n}\frac{I^{n_1}}{I^{n_1+1}}\otimes\frac{I^{n_2}}{I^{n_2+1}}\xrightarrow{\ \phi\ }\frac{\sum_{n_1+n_2=n} I^{n_1}\otimes I^{n_2}}{\sum_{n_1+n_2=n} I^{n_1+1}\otimes I^{n_2}+I^{n_1}\otimes I^{n_2+1}}$$

Suppose for some $\gamma=(0,\dots,0,\gamma_{k_0},\gamma_{k_0+1},\dots)\in\bigoplus_{k=0}^{n} I^k\otimes I^{n-k}$,

$\phi(\bar\gamma)=0$, where $\bar\gamma$ is the image of γ in

$\bigoplus_{k=0}^{n}\frac{I^k}{I^{k+1}}\otimes\frac{I^{n-k}}{I^{n-k+1}}$. Write $\alpha=\gamma_{k_0}\in I^{k_0}\otimes I^{n-k_0}$

and $\beta=\sum_{j=1}\gamma_{k_0+j}\in I^{k_0+1}\otimes QH$. Then

$$\alpha+\beta\in I^{k_0+1}\otimes QH+QG\otimes I^{n-k_0+1}$$

Q. Is it true in general, that if $M'\subset M$ and $N'\subset N$ are modules, then

$$M\otimes N'\cap M'\otimes N=M'\otimes N' \quad ?$$