

Quillen's theorem - degree 2 testing, 2

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Goal. Construct an expansion $Z: G \rightarrow U(L_*(G))$.

$$G_1 = G \ ; \ G_{n+1} = (G, G_n) \quad L_n := \mathbb{Q} \otimes (G_n / G_{n+1})$$

$$L_n \longrightarrow \mathcal{P}_n(A) =: \mathcal{L}_n \quad \text{by } C \mapsto \bar{C} = C - 1$$

There is $\pi: A = U(L_*) \rightarrow A$ surjective because it is surjective in degree 1.

Can I construct an A -expansion?

$$G_1/G_2 \begin{matrix} \xrightarrow{\pi_1} \\ \xleftarrow{\pi_1} \end{matrix} G_1 \qquad G_2/G_3 \begin{matrix} \xrightarrow{\pi_2} \\ \xleftarrow{\pi_2} \end{matrix} G_2$$

$$\zeta(g) = (0, \pi_0(g), \pi_1(g \sigma_1(\pi_0(g))^{-1}), \dots) \in L_*$$

so there is

And also

$$\zeta: G \rightarrow L_* \qquad \sigma: L_* \rightarrow G$$

claim $Z = e^{\zeta_1} e^{\zeta_2} \dots$ is an A -expansion.

check in degree 2:

Need to consider $Z(\pi(ab))$

$$Z(\pi(ab)) : \quad (\text{assume } a = \sigma_1(\alpha) \quad b = \sigma_1(\beta))$$

$$ab \xrightarrow{\pi} \pi(a)\pi(b) = (a-1)(b-1) = 1 - a - b + ab$$

$$\xrightarrow{Z} (1, 0, 0, \dots) - (1, \alpha, \frac{1}{2}\alpha\alpha) - (1, \beta, \frac{1}{2}\beta\beta)$$

$$\dots \dots (\alpha+\beta)(\alpha+\beta) \dots \dots$$

$$+ (1, \alpha + \beta, \frac{\alpha\beta + \beta\alpha}{2} + \pi_2(ab\sigma_1(\alpha + \beta)^{-1}))$$

$$= (0, 0, \frac{\alpha\beta + \beta\alpha}{2} + \pi_2(ab\sigma_1(\alpha + \beta)^{-1}))$$

$$= (0, 0, \alpha\beta + \frac{1}{2}[\beta, \alpha] + \pi_2(ab\sigma_1(\alpha + \beta)^{-1}))$$

... ?
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