

Gr(PBn) from generators and relations

January-25-15 2:27 PM

From the Kassel-Turaev book:

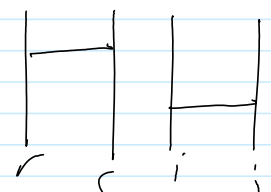
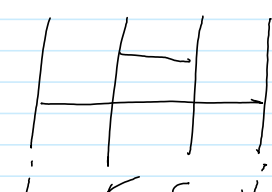
1.3 Pure braid groups 21

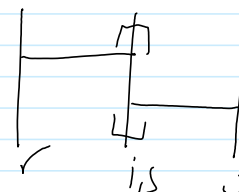
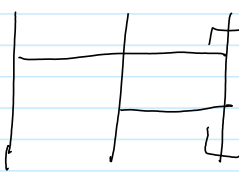
Corollary 1.19. P_n is generated by the $n(n-1)/2$ elements $\{A_{i,j}\}_{1 \leq i < j \leq n}$.

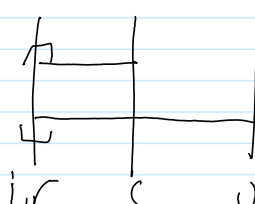
This directly follows from formula (1.6) and Theorem 1.16.

Here is a list of defining relations for the generators $\{A_{i,j}\}_{1 \leq i < j \leq n}$ of P_n :

$$A_{r,s}^{-1} A_{i,j} A_{r,s} = \begin{cases} 1 & \text{if } s < i \text{ or } i < r < s < j, \\ 2 & \left\{ \begin{array}{l} A_{i,j} \\ A_{r,j} A_{i,j} A_{r,j}^{-1} \end{array} \right. & \text{if } s = i, \\ 3 & \left\{ \begin{array}{l} A_{i,j} A_{s,j} A_{i,j} A_{s,j}^{-1} A_{r,j}^{-1} \\ A_{r,j} A_{s,j} A_{i,j} A_{s,j}^{-1} A_{r,j}^{-1} \end{array} \right. & \text{if } i = r < s < j, \\ 4 & \left\{ \begin{array}{l} A_{r,j} A_{s,j} A_{r,j}^{-1} A_{s,j}^{-1} A_{i,j} A_{s,j} A_{r,j} A_{s,j}^{-1} A_{r,j}^{-1} \\ A_{r,j} A_{s,j} A_{r,j}^{-1} A_{s,j}^{-1} A_{i,j} A_{s,j} A_{r,j} A_{s,j}^{-1} A_{r,j}^{-1} \end{array} \right. & \text{if } r < i < s < j. \end{cases} \quad (1.7)$$

1. Commutativity of  and of 

2.  = - 

3.  = -  - 

4.  = 0