Double Filtrations

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Def Doubly filtered $I_{n,m} \subset V$ s.t.

$\mathcal{F}^2$ $\qquad I_{n+1,m} \subset I_{n,m} \qquad I_{n,m+1} \subset I_{n,m}$

Def Doubly graded. $\mathcal{G}^2$

Def gr^2, a functor $\mathcal{F}^2 \to \mathcal{G}^2$ by

$$(I_{n,m}) \longmapsto \bigoplus \frac{I_{n,m}}{I_{n+1,m} + I_{n,m+1}}$$

Def $D: \mathcal{F}^2 \to \mathcal{F}^1$ by $I_k = \sum_{n+m=k} I_{n,m}$

$D: \mathcal{G}^2 \to \mathcal{G}^1$ by $V_k = \bigoplus_{n+m=k} V_{n,m}$

claim If V is doubly filtered,

$$D(gr^2 V) \cong gr^1(DV)$$