Degree by degree constructions

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2.3. Degree by Degree Constructions.

MORE: to a thm of the form $H^2 = 0 \Rightarrow Z$, in both a plain and a group-like variant.

$$C, E \qquad C \in C^1 \qquad E \in C^2$$

$$E(g_1, g_2) = Z(g_1) Z(g_2) Z(g_1 g_2)^{-1} - 1$$

claim

$$0 = E(g_2, g_3) - E(g_1 g_2, g_3) + E(g_1, g_2 g_3) - E(g_1, g_2)$$

$$\underset{1}{\underbrace{z_2 z_3 z_{23}^{-1} - 1}} \qquad \underset{2}{\underbrace{z_{12} z_3 z_{123}^{-1} - 1}} \qquad \underset{3}{\underbrace{z_1 z_{23} z_{123}^{-1} - 1}} \qquad \underset{}{\underbrace{z_1 z_2 z_{12}^{-1} - 1}}$$

PF

$$E(g_1, g_2) = z_1 z_2 z_{12}^{-1} - 1 = z_{123} z_{23}^{-1} z_2 z_{12}^{-1} + E(1, 23)$$

$$= \underset{3}{\underbrace{z_{123} z_{13}^{-1}}} \underset{1}{\underbrace{z_{123} z_{13}^{-1}}} \underset{2}{\underbrace{z_3 z_{123}^{-1}}} - 1 + E(1, 23) + E(23) - E(12, 3) \quad \checkmark$$

Better derive from associativity:

$$z_{12} z_3$$
$$z_{123} \qquad z_1 z_2 z_3$$
$$z_1 z_{23}$$