

# RIBBON 2-KNOTS, $1 + 1 = 2$ , AND DUFLO'S THEOREM FOR ARBITRARY LIE ALGEBRAS

DROR BAR-NATAN AND ZSUZSANNA DANCOS

ABSTRACT. By performing the calculation “ $1 + 1 = 2$ ” on a 4D abacus, we explain in the most direct way we know how the study of “expansions”, or “universal finite type invariants”, for ribbon 2-knots leads to a proof of Duflo’s theorem for arbitrary finite-dimensional Lie algebras. This complements the results of B-N, Le, and Thurston [BLT] where a similar argument using a 3D abacus and the Kontsevich integral was used to deduce Duflo’s theorem yet only for metrized Lie algebras, and our results from [BND2] which also imply a relation of 2-knots with the full Duflo theorem, though via a lengthier path.

## CONTENTS

1. Introduction	1
References	3

## 1. INTRODUCTION

DRAFT! See <http://drorbn.net/AcademicPensieve/Projects/wDuflo/>

---

*Date:* First edition Not Yet, 2017, this edition Jan. 3, 2017. Electronic version and related files at <http://drorbn.net/AcademicPensieve/Projects/wDuflo/>.

2010 *Mathematics Subject Classification.* 57M25.

*Key words and phrases.* knots, 2-knots, tangles, expansions, finite type invariants, Lie algebras, Duflo’s theorem .

This work was partially supported by NSERC grant RGPIN 262178 and by ARC DECRA DE170101128.

DRAFT! See <http://drorbn.net/AcademicPensieve/Projects/wDuflo/>

## REFERENCES

- [KBH] [BN] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, Acta Mathematica Vietnamica **40-2** (2015) 271–329, [arXiv:1308.1721](https://arxiv.org/abs/1308.1721). Page(s) .
- [WK02] [BND2] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects II: Tangles and the Kashiwara-Vergne Problem*, <http://drorbn.net/AcademicPensieve/Projects/WK02>, [arXiv:1405.1955](https://arxiv.org/abs/1405.1955). Page(s) 1.
- [BLT] [applications] D. Bar-Natan, T. Q. T. Le, and D. P. Thurston, *Two applications of elementary knot theory to Lie algebras and Vassiliev invariants*, Geom. Topol. **7-1** (2003) 1–31, [arXiv:math.QA/0204311](https://arxiv.org/abs/math/0204311). Page(s) 1.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TORONTO, TORONTO ONTARIO M5S 2E4, CANADA  
*E-mail address:* [drorbn@math.toronto.edu](mailto:drorbn@math.toronto.edu)  
*URL:* <http://www.math.toronto.edu/~drorbn>

SCHOOL OF MATHEMATICS AND STATISTICS F07, UNIVERSITY OF SYDNEY NSW 2006, AUSTRALIA  
*E-mail address:* [zsuzsanna.dancso@anu.edu.au](mailto:zsuzsanna.dancso@anu.edu.au)  
*URL:* [www.math.toronto.edu/zsuzsi](http://www.math.toronto.edu/zsuzsi)