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RIBBON 2-KNOTS, $1 + 1 = 2$, AND DUFLO'S THEOREM FOR ARBITRARY LIE ALGEBRAS

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ABSTRACT. By performing the calculation “ $1 + 1 = 2$ ” on a 4D abacus, we explain in the most direct way we know how the study of “expansions”, or “universal finite type invariants”, for ribbon 2-knots leads to a proof of Duflo’s theorem for arbitrary finite-dimensional Lie algebras. This complements the results of B-N, Le, and Thurston [BLT] where a similar argument using a 3D abacus and the Kontsevich integral was used to deduce Duflo’s theorem yet only for metrized Lie algebras, and our results from [BND2] which also imply a relation of 2-knots with the full Duflo theorem, though via a lengthier path.

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1. INTRODUCTION

DRAFT! See <http://drorbn.net/AcademicPensieve/Projects/wDuflo/>

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