Title. Knot Theory as an Excuse.

With ωεβ := http://drorbn.net/d23, this Notice of Intent is also at ωεβ/NOI.

I'm considered an expert on Knot Theory, yet I don't understand knot theory at all. From a certain perspective, Knot Theory is the study of some silly combinatorial objects, that are considered modulo equally silly relations. My intuition as a student told me it must be a shallow topic, and there's still a remnant of that intuition in me.

Yet time after time this intuition is proven wrong and instead of shallow, Knot Theory is very deep. So much so, that Knot Theory sometimes serves to validate that other topics are interesting: **if it has applications to Knot Theory, it must be good**. (Historically, number theory's raison d'être had been similar; recently cryptography became a further bonus).

My plan over the grant period would be to continue to use knot theory as an excuse and as a benchmark to study several other topics, mostly in algebra:

1. I plan to continue to study, along with Roland van der Veen and others, how "solvable approximation" of semisimple Lie algebras (Inonu-Wigner contractions of their lower Borel subalgebras) leads via perturbed Gaussian formulas (in spirit, QFT) to poly-time computable knot invariants that "behave well" under useful knot theoretic operations. I hope this sounds powerful; it certainly sounds highly technical. Can we make it less technical? Can we rely less on Lie algebra and quantum algebra techniques and instead make the topic intrinsic to knot theory? See ωεβ/SolvApp, ωεβ/PG, ωεβ/DaNang.
2. The simplest of these invariants, ρ_1, is ridiculously simple to define (ωεβ/APAI, ωεβ/Cars) and it is perhaps even more ridiculous how much we fail to understand it. In short, ρ_1 is some quadratic expression in the entries of A^{-1}, where A is one of the standard matrices whose determinant is the Alexander polynomial Δ. Could we start from other matrices B whose determinants are Δ? Can we prove Alexander-like properties of ρ_1 using its similarity with Δ? By direct computations we observe many such properties, yet we still don't know how to prove them. And the $1M question: does ρ_1 have special properties on ribbon knots, similar to the Fox-Milnor property of Δ? If it does, it may lead to a new criteria to detect non-ribbon knots. Such criteria are in high demand for they may lead to the detection of counterexamples to the ribbon-slice conjecture, one of the greatest outstanding problems in knot theory.
3. Along with Zsuzsanna Dancso, Tamara Hogan, Jessica Liu, and Nancy Scherich (ωεβ/PDS), I plan to continue to study knots and tangles in a "pole dancing studio" (PDS, a cylinder with a few vertical lines removed) and their relationship with the Goldman-Turaev Lie bialgebra and Kashiwara-Vergne (KV) equations (ωεβ/AKKN). Are solutions of the KV equations sufficient to construct a homomorphic expansion of tangles in a PDS up to strand-strand degree 1? How is this related to my earlier work with Dancso (ωεβ/WKO1, ωεβ/WKO2) on welded knots? The subject is beautiful, yet it is a hard-to-penetrate patchwork of results and techniques and papers by different authors. In the past, this feeling that a subject's beauty is incongruous with its complexity had been a great motivator for me, often leading to deeper understanding. I have high hopes for this topic too.
4. Recently (ωεβ/PQ), along with Jessica Liu, we've found a truly elegant "signatures for tangles" invariant (sorry for complimenting ourselves, yet hey, it really is elegant). There is more to do before we can claim to fully understand these signatures. Will we be able to use our formalism to prove Kashaev's signatures conjecture (ωεβ/Kashaev)?