

1. Most Significant Contributions to Research and/or to Practical Applications

- **My work on w-knotted objects.** "u-knotted objects" are the usual knots and links and braids and tangles. "v-knotted objects" are their "virtual" (in the sense of Kauffman) counterparts. "w-knotted objects" are a certain class of knotted 2-dimensional objects in 4-dimensional space. One reason for the name "w" is that these objects are a reduction of the "v" objects, hence they are "weakly virtual", another is their historical naming as "welded knots", and another is because they fit nicely within a sequence of maps $u \rightarrow v \rightarrow w$ which shows that these three classes of knotted objects are closely related to each other.

Over the last 4-5 years I have spent a great deal of effort studying "finite type invariants" and "expansions" for w-knotted objects, and how these invariants and expansions relate to the well-established theory of finite type invariants and the Kontsevich integral (which is an "expansion") of u-knotted objects and the still-very-tentative theory of finite type invariants and expansions of v-knotted objects.

In a few short words, the study of expansions for w-knotted objects leads directly to the study of the BF quantum field theory and of solutions of the Kashiwara-Vergne equations, which in themselves are related to the study of convolution algebras on Lie groups and Lie algebras. The relationship between "u" and "w" becomes the Alekseev-Torossian relationship between associators and the Kashiwara-Vergne equations, and the relationship between "v" and "w" leads to the not-yet-fully-understood relationship between the Kashiwara-Vergne equations and the Etingof-Kazhdan theory of quantization of Lie bialgebras.

This is a major project! Perhaps due to its magnitude, much of the work is not yet published in the classical sense. Yet much is already written (jointly with my former student **Zsuzsanna Dancso**, see <http://www.math.toronto.edu/~drorbn/papers/WKO>) and much had been described in a large number of videotaped lectures that I have given, and in at least two specialized workshops on these subject – a two-day workshop in Montpellier in June 2010 (<http://www.math.toronto.edu/~drorbn/Talks/Montpellier-1006/>) and a 9-day workshop in Caen in June 2012 (<http://www.math.toronto.edu/~drorbn/Talks/Caen-1206/>).

Sorry for giving you, my reader, a hard time evaluating my work. The true and tested method of counting publications in top journals will give you "0" in my case. It may be that my work isn't worth much more than that, yet "0" would still be a grossly unfair estimator. We live in the future, and I'm deep within the process of making the necessary adjustments. There remains a huge value in the classical mode of publication, and I remain committed to classically publishing my work. Yet new ways of assimilating knowledge are now available, and I am deeply committed to those [see my website, and the links throughout this document]. That, and the magnitude of my w-project (at least relative to my meagre abilities), are the reasons classical publication is yet to be achieved.

- **My work on the Alexander polynomial.** I have found, or participated in finding, several new frameworks for the Alexander polynomial. The first, with Jana Archibald (see her thesis and article at <http://www.math.toronto.edu/~jfa/> and my talk at <http://www.math.toronto.edu/~drorbn/Talks/Sandbjerg-0810/>) is a very clean and well behaved extension of the multi-variable Alexander polynomial to tangles and virtual tangles, and it allows for a simple and unified derivation of a large number of Alexander skein relations that previously did not exist or had to be derived separately and tediously. Another (at <http://www.math.toronto.edu/~drorbn/papers/WKO>) highlights the relationship between the Alexander polynomial and w-knotted objects. Another (at <http://www.math.toronto.edu/~drorbn/Talks/Chicago-1009/>) shows that as a quantum invariant, the Alexander invariant comes from the minimally-non-commutative two-dimensional Lie algebra. The last and the one I am the most proud of is my "meta-group" approach (<http://www.math.toronto.edu/~drorbn/Talks/Regina-1206/>), which again is a well-behaved extension of the Alexander polynomial to tangles and virtual tangles, except with the added benefit that everything along the way is of polynomial size, has topological meaning, and seems ripe for categorification.
- **My work on the Knot Atlas and on the program KnotTheory`.** I have participated in the continued development of the "Knot Atlas" (<http://katlas.org>), a wiki-style comprehensive data base of information about knots and especially their invariants, and the computer program "KnotTheory", which I believe is the most comprehensive computer package presently available to deal with knots and their invariants. Both projects started as my own initiatives about 10 years ago, and both received major contributions by others and especially by Scott Morrison of the Australian National University.
- **Some 81 talks that I have given (and two classes / lecture series).** I consider exposition to be a significant part of my work and indeed of my research. Over the last 6 years I gave about 81 lectures all over the world and on a wide range of subjects. My talks are almost always accompanied by written documentation and this documentation is always available on my web site (at <http://www.math.toronto.edu/~drorbn/Talks>), and as of about 3 years ago, almost all of my talks are also available as videos viewable on my web site. In addition I gave two classes / lecture series on research topics that are also available on video (at <http://www.math.toronto.edu/~drorbn/classes/0910/AKT> and at <http://www.math.toronto.edu/~drorbn/papers/WKO>). So my "Talks" and "Classes" web pages in themselves serve as significant repository of mathematical knowledge.
- **The retention of a lively research group.** I have several (about 10) post-doctoral fellows, Ph.D. students, master's students and other group members; I meet with them regularly and the group as a whole meets for weekly lunches and mathematical discussions.

2. Research Contributions and Practical Applications

Articles in refereed publications:

1. With **Z. Dancso**, *Homomorphic Expansions for Knotted Trivalent Graphs*, accepted to the Journal of Knot Theory and its Ramifications, arXiv:1103:1896.

2. With **Z. Dancso**, *Pentagon and Hexagon Equations Following Furusho*, Proceedings of the American Mathematical Society **140-4** (2012) 1243-1250.
3. With **I. Halacheva**, **L. Leung** and **F. Roukema**, *Some Dimensions of Spaces of Finite Type Invariants of Virtual Knots*, Experimental Mathematics **20-3** (2011) 282-287.
4. With S. Morrison, *The Karoubi Envelope and Lee's Degeneration of Khovanov Homology*, Algebraic and Geometric Topology **6** (2006) 1459-1469.
5. *Fast Khovanov Homology Computations*, Journal of Knot Theory and its Ramifications **16-3** (2007) 243-255.
6. *Finite Type Invariants*, Encyclopedia of Mathematical Physics (eds. J.-P. Francoise, G. L. Naber and Tsou S. T.) vol. **2** pp. 340, Elsevier, Oxford 2006.

Non-refereed contributions: Several thousand individual web pages ranging in value from the very dilute to the extremely concentrated (e.g., the nearly 60 "handouts" linked at <http://www.math.toronto.edu/~drorbn/Talks/Portfolio/>).

Contributions by students directly supervised by me:

1. **Karene Chu**, *Flat Virtual Pure Tangles*, University of Toronto PhD thesis, 2012.
2. **Peter Lee**, *The Pure Virtual Braid Group is Quadratic*, accepted to Selecta Mathematica, 2011, arXiv:1110.2356.
3. **Peter Lee**, *The Pure Virtual Braid Group is Quadratic*, University of Toronto PhD thesis, 2011.
4. **Zsuzsanna Dancso**, *On a Universal Finite Type Invariant of Knotted Trivalent Graphs*, University of Toronto PhD thesis, 2011.
5. **Louis Leung**, *Classical Lie Algebra Weight Systems of Arrow Diagrams*, University of Toronto PhD thesis, 2010.
6. **Jana Archibald**, *The Multivariable Alexander Polynomial on Tangles*, University of Toronto PhD thesis, 2010.
7. **Hernando Burgos Soto**, *Khovanov Homology for Alternating Tangles*, arXiv:1003.4766.
8. **Hernando Burgos Soto**, *The Jones Polynomial and the Planar algebra of Alternating Links*, Universidad Nacional de Colombia PhD thesis, 2009, Journal of Knot Theory and its Ramifications 19-11 (2010), 1487-1505, arXiv:math.GT/0807.2600.
9. **Zsuzsanna Dancso**, *On the Kontsevich Integral for Knotted Trivalent Graphs*, Algebraic and Geometric Topology 10 (2010) 1317-1365, arXiv:0811.4615.
10. **Louis Leung**, *Combinatorial Formulas for Classical Lie Weight Systems on Arrow Diagrams*, arXiv:0812.2342.
11. **Fionntan Roukema**, *Goussarov-Polyak-Viro Combinatorial Formulas for Finite Type Invariants*, arXiv:0711.4001.
12. **Jana Archibald**, *The Weight System of the Multivariable Alexander Polynomial*, Proceedings of The International Conference on Quantum Topology, Hanoi 2007, arXiv:0710.4885.
13. **Gad Naot**, *The Universal sl_2 Link Homology Theory*, University of Toronto PhD thesis, 2007, arXiv:math.GT/0706.3680.

3. Other Evidence of Impact and Contributions

Over the last 6 years I have given about 81 lectures, at **locations** including Hamburg, Caen, Regina, Binghamton, George Washington University, Buffalo, Toronto, St. Bonaventure University, Strasbourg, Oregon, Villa de Leyva, Geneva, Thun, Tennessee, Chicago, Montpellier, Gottingen, University of Western Ontario, the Fields Institute, Bonn, the Hebrew University, Tacoma, Paris, Trieste, Kansas, Bogota, Penn State, Northeastern, Copenhagen, Sandbjerg, MSRI, Zurich, Oberwolfach, Brown, Hanoi, Tianjin, Århus, Kyoto, Tokyo, Columbia University, and Uppsala, and on **subjects** including

- "A Quick Introduction to Khovanov Homology".
- "Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant".
- "Workshop on v- and w-Knotted Objects".
- "Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial".
- "The Pure Virtual Braid Group is Quadratic".
- "Expansions: A Loosely Tied Traverse from Feynman Diagrams to Quantum Algebra".
- "Facts and Dreams About v-Knots and Etingof-Kazhdan".
- "Cosmic Coincidences and Several Other Stories".
- "The Hardest Math I've Ever Really Used".
- "Braids and the Grothendieck-Teichmuller Group".
- "From the $ax+b$ Lie Algebra to the Alexander Polynomial and Beyond".
- "18 Conjectures".
- "I understand Drinfel'd and Alekseev-Torossian, I don't understand Etingof-Kazhdan yet, and I'm clueless about Kontsevich".
- "u, v, and w-Knots: Topology, Combinatorics and Low and High Algebra".
- "Dessert: Hilbert's 13th Problem, in Full Colour".
- "Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots".
- "The Penultimate Alexander Invariant".
- "Projectivization, W-Knots, Kashiwara-Vergne and Alekseev-Torossian".
- "Local Khovanov Homology".
- "Non-Commutative Gaussian Elimination and Rubik's Cube".
- "A Very Non-Planar Very Planar Algebra".
- "Following Lin: Expansions for Groups".
- "Algebraic Knot Theory".
- "The Virtues of Being an Isomorphism".
- "A Homological Construction of the Exponential Function".

See more at <http://www.math.toronto.edu/~drorbn/Talks/>.

MathSciNet thinks my citation count is 829. Google scholar thinks my h -index is 24 (meaning that 24 of my publications were cited at least 24 times each).