Dear Reader,

While written words on old-style paper remain irreplaceable in many contexts, new modes of communication are now available, and much of my contribution is not in the form of published papers. The attached is a printout of a web page that contains links to 22 video clips containing about 30 hours of lectures explaining major parts of a paper being finished jointly with Zsuzsanna Dancso. The paper itself is also available at the same web page, http://drorbn.net/index.php?title=WKO.

Truly,

Dror Bar-Natan.

WKO

In Progress

Finite Type Invariants of W-Knotted Objects: From Alexander to Kashiwara and Vergne

Joint with Zsuzsanna Dancso

Download WKO.pdf: last updated ≥ March 3, 2012. first edition: not yet.

Abstract. w-Knots, and more generally, w-knotted objects (w-braids, w-tangles, etc.) make a class of knotted objects which is wider but weaker than their "usual" counterparts. To get (say) w-knots from u-knots, one has to allow non-planar "virtual" knot diagrams, hence enlarging the the base set of knots. But then one imposes a new relation, the "overcrossings commute" relation, further beyond the ordinary collection of Reidemeister moves, making w-knotted objects a bit weaker once again.

The group of w-braids was studied (under the name "welded braids") by Fenn, Rimanyi and Rourke [FRR] and was shown to be isomorphic to the McCool group [Mc] of "basis-conjugating" automorphisms of a free group F_n - the smallest subgroup of $\mathrm{Aut}(F_n)$ that contains both braids and permutations. Brendle and Hatcher [BH], in work that traces back to Goldsmith [Gol], have shown this group to be a group of movies of flying rings in $\mathbb{R}^3$. Satoh [Sa] studied several classes of w-knotted objects (under the name "weakly-virtual") and has shown them to be closely related to certain classes of knotted surfaces in $\mathbb{R}^4$. So w-knotted objects are algebraically and topologically interesting.

In this article we study finite type invariants of several classes of w-knotted objects. Following Berceanu and Papadima [BP], we construct a homomorphic universal finite type invariant of w-braids, and hence show that the McCool group of automorphisms is "1-formal". We also construct a homomorphic universal finite type invariant of w-tangles. We find that the universal finite type invariant of w-knots is more or less the Alexander polynomial (details inside).

Much as the spaces $\mathcal{A}$ of chord diagrams for ordinary knotted objects are related to metrized Lie algebras, we find that the spaces $\mathcal{A}^w$ of "arrow diagrams" for w-knotted objects are related to not-necessarily-metrized Lie algebras. Many questions concerning w-knotted objects turn out to be equivalent to questions about Lie algebras. Most notably we find that a homomorphic universal finite type invariant of w-knotted trivalent graphs is essentially the same as a solution of the Kashiwara-Vergne [KV] conjecture and much of the Alekseev-Torossian [AT] work on Drinfel'd associators and Kashiwara-Vergne can be re-interpreted as a study of w-knotted trivalent graphs.

The true value of w-knots, though, is likely to emerge later, for we expect them to serve as a warmup example for what we expect will be even more interesting - the study of virtual knots, or v-knots. We expect v-knotted objects to provide the global context whose projectivization (or "associated graded structure") will be the Etingof-Kazhdan theory of deformation quantization of Lie bialgebras [EK].

DBN: Publications: WKO / Navigation

Wideo Companion

The **wClips Seminar** is a series of weekly wideotaped meetings at the University of Toronto, systematically going over the content of the WKO paper section by section.

Next Meeting. Wednesday, July 25, 2012, 12-2, at Bahen 4010 on TBA.

Announcements. small circle, UofT, LDT Blog (also here). Email Dror to **join our mailing list!**

Resources. How to use this site, Dror's notebook, blackboard shots.

The wClips

Date	Links
Jan 11, 2012	120111-1: Introduction. 120111-2: Section 2.1 - v-Braids.
Jan 18, 2012	120118-1: An introduction to this web site. 120118-2: Section 2.2 - w-Braids by generators and relations and as flying rings. 120118-3: Section 2.2 - w-Braids - other drawing conventions, "wens".
Jan 25, 2012	120125-1: Section 2.2.3 - basis conjugating automorphisms of F_n. 120125-2: A very quick introduction to finite type invariants in the "u" case.
Feb 1, 2012	120201: Section 2.3 - finite type invariants of v- and w-braids, arrow diagrams, 6T, TC and 4T relations, expansions / universal finite type invariants.
Feb 8, 2012	120208: Review of u,v, and w braids and of Section 2.3.
Feb 15, 2012	120215: Section 2.5 - mostly compatibilities of Z^w, also injectivity and uniqueness of Z^w.
Feb 22, 2012	120222: Section 2.5.5, $\alpha: \mathcal{A}^u \to \mathcal{A}^v$, and Section 3.1 (partially), the definition of v- and w-knots.
Feb 29, 2012	120229: Sections 3.1-3.4: v-Knots and w-Knots: Definitions, framings, finite type invariants, dimensions, and the expansion in the w case.
Mar 7, 2012	120307: Section 3.5: Jacobi diagrams and the bracket-rise theorem.
Mar 14, 2012	120314: Section 3.6 - the relation with Lie algebras.
Mar 21, 2012	120321: Section 4 - Algebraic Structures.
Mar 28, 2012	Out-of-sequence not-on-tape we watched the video of Talks: GWU-1203.
Apr 4, 2012	120404: Section 3.7 - The Alexander Theorem (statement).
Apr 18, 2012	120418: Aside on the Euler trick, the differential of exp, and the BCH formula

Kazhdan theory of deformation quantization of Lie bialgebras [EK].

The paper. WKO.pdf, WKO.zip. (Zsuzsi's version: WKO.pdf, WKO.zip).

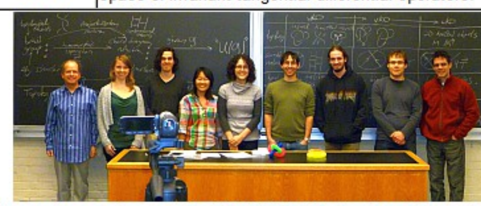
Related Mathematica Notebooks. "The Kishino Braid" (Source, PDF), "Dimensions" (Source, PDF), "wA" (Source, PDF), "InfinitesimalAlexanderModules" (Source, PDF).

Related talks. Oberwolfach-0805, MSRI-0808, Northeastern-081028, Trieste-0905, Bonn-0908, Caen-1206.

Links. SandersonsGarden.html.

Related Scratch Work is under Pensieve: WKO and Pensieve: Arrow_Diagrams_and_gl(N).

Apr 18, 2012	120418: Aside on the Euler trick, the differential of $\exp$, and the BCH formula.
Apr 25, 2012	120425: Section 3.8, a disorganized lecture towards the proof of the Alexander theorem.
May 2, 2012	120502: Section 4: Algebraic structures (review), circuit algebras, v- and w-tangles.
May 10, 2012	120510: Sections 5.1 and 5.2: tangles, their projectivization and its relationship with Alekseev-Torossian spaces.
May 23, 2012	120523: Section 5.2: Proof of the relationship with A-T spaces.
May 30, 2012	120530: Interpreting $\mathcal{A}^w(\uparrow_n)$ as a universal space of invariant tangential differential operators.



Group photo on January 11, 2012: DBN, ZD, Stephen Morgan, Lucy Zhang, Iva Halacheva, David Li-Bland, Sam Selmani, Oleg Chterental, Peter Lee.

References.

[AT] ^ A. Alekseev and C. Torossian, The Kashiwara-Vergne conjecture and Drinfeld's associators, arXiv:0802.4300.

[BP] ^ B. Berceanu and S. Papadima, Universal Representations of Braid and Braid-Permutation Groups, arXiv:0708.0634.

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[Mc] ^ J. McCool, On Basis-Conjugating Automorphisms of Free Groups, Can. J. Math. 38-6(1986) 1525-1529.

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