

In[\*]:= { $\eta$ ,  $\xi$ ,  $y$ ,  $x$ }\*

Out[\*]:= { $y$ ,  $x$ ,  $\eta$ ,  $\xi$ }

In[\*]:=  $m_{i,j \rightarrow k}$

lhs =  $m_{1,2 \rightarrow 1}$  //  $m_{1,3 \rightarrow 1}$

rhs =  $m_{2,3 \rightarrow 2}$  //  $m_{1,2 \rightarrow 1}$

lhs  $\equiv$  rhs

Out[\*]:=  $E_{\{i,j\} \rightarrow \{k\}} [1, y_k (\eta_i + \eta_j) - \eta_j \xi_i + x_k (\xi_i + \xi_j), 0]$

Out[\*]:=  $E_{\{1,2,3\} \rightarrow \{1\}} [1, y_1 \eta_1 + y_1 \eta_2 + y_1 \eta_3 + x_1 \xi_1 - \eta_2 \xi_1 - \eta_3 \xi_1 + x_1 \xi_2 - \eta_3 \xi_2 + x_1 \xi_3, 0]$

Out[\*]:=  $E_{\{1,2,3\} \rightarrow \{1\}} [1, y_1 \eta_1 + y_1 \eta_2 + y_1 \eta_3 + x_1 \xi_1 - \eta_2 \xi_1 - \eta_3 \xi_1 + x_1 \xi_2 - \eta_3 \xi_2 + x_1 \xi_3, 0]$

Out[\*]:= True

In[\*]:=  $R_{1,2} \bar{R}_{3,4}$

Out[\*]:=  $E_{\{\} \rightarrow \{1,2,3,4\}} \left[ 1, (-1 + T) x_2 (y_1 - y_2) + \left( -1 + \frac{1}{T} \right) x_4 (y_3 - y_4), \right.$   
 $\left. \left( -\frac{1}{2} (1 - T) x_2^2 y_1^2 + x_1 x_2 y_1 y_2 + \frac{1}{2} (1 - 3T) x_2^2 y_1 y_2 - \right. \right.$   
 $\left. \left. \frac{(-1 + T) x_3 x_4 y_3^2}{T^2} - \frac{(1 - T) x_4^2 y_3^2}{2 T^3} - \frac{x_3 x_4 y_3 y_4}{T^2} - \frac{(-1 - T) x_4^2 y_3 y_4}{2 T^3} \right) \epsilon + O[\epsilon]^2 \right]$

In[\*]:=  $(R_{1,2} \bar{R}_{3,4})$  //  $m_{1,3 \rightarrow 1}$

Out[\*]:=  $E_{\{\} \rightarrow \{1,2,4\}} \left[ 1, (-1 + T) x_2 y_1 + \frac{(1 - T) x_4 y_1}{T} + (1 - T) x_2 y_2 + \frac{(-1 + T) x_4 y_4}{T}, \right.$   
 $\left( \frac{1}{2} (-1 + T) x_2^2 y_1^2 + \frac{(1 - T) x_1 x_4 y_1^2}{T^2} + \frac{(-1 + T) x_4^2 y_1^2}{2 T^3} + x_1 x_2 y_1 y_2 + \right.$   
 $\left. \frac{1}{2} (1 - 3T) x_2^2 y_1 y_2 + \frac{(-1 + T) x_2 x_4 y_1 y_2}{T} - \frac{x_1 x_4 y_1 y_4}{T^2} + \frac{(1 + T) x_4^2 y_1 y_4}{2 T^3} \right) \epsilon + O[\epsilon]^2 \right]$

In[\*]:= {Z[0], Z[1], Z[2]} // Column

$$\left( \frac{1}{2} (-1 + T) x_2^2 y_1^2 + \frac{(1-T) x_1 x_4 y_1^2}{T^2} + \frac{(-1+T) x_4^2 y_1^2}{2 T^3} + \frac{(1-T) x_4 x_{\$[3]} y_1^2}{T^2} + x_1 x_2 y_1 y_2 + \frac{1}{2} (1 - 3 T) x_2^2 y_1 y_2 + \frac{(-1+T) x_2 x_4 y_1 y_2}{T} + x_2 x_{\$[1]} y_1 y_2 - \frac{x_1 x_4 y_1 y_4}{T^2} + \frac{(1+T) x_4^2 y_1 y_4}{2 T^3} - \frac{x_4 x_{\$[3]} y_1 y_4}{T^2} + (-1 + T) x_2^2 y_1 y_{\$[1]} + x_1 x_2 y_2 y_{\$[1]} + \frac{1}{2} (1 - 3 T) x_2^2 y_2 y_{\$[1]} + \frac{(-1+T) x_2 x_4 y_2 y_{\$[1]}}{T} + x_2 x_{\$[1]} y_2 y_{\$[1]} + \frac{1}{2} (-1 + T) x_2^2 y_{\$[1]}^2 + \frac{(2-2 T) x_1 x_4 y_1 y_{\$[3]}}{T^2} + \frac{(-1+T) x_4^2 y_1 y_{\$[3]}}{T^3} + \frac{(2-2 T) x_4 x_{\$[3]} y_1 y_{\$[3]}}{T^2} - \frac{x_1 x_4 y_4 y_{\$[3]}}{T^2} + \frac{(1+T) x_4^2 y_4 y_{\$[3]}}{2 T^3} - \frac{x_4 x_{\$[3]} y_4 y_{\$[3]}}{T^2} + \frac{(1-T) x_1 x_4 y_{\$[3]}^2}{T^2} + \frac{(-1+T) x_4^2 y_{\$[3]}^2}{2 T^3} + \frac{(1-T) x_4 x_{\$[3]} y_{\$[3]}^2}{T^2} \right) \in + O[\epsilon]^2$$

$$\text{Out[*]} = \left( \frac{(-2+2 T) x_1 x_2 x_4 y_1^2 y_2}{T^2} + \frac{(1-T) x_2 x_4^2 y_1^2 y_2}{T^3} + \frac{(-2+2 T) x_2 x_4 x_{\$[3]} y_1^2 y_2}{T^2} + \frac{x_1 x_2 x_4 y_1 y_2 y_4}{T^2} + \frac{(-1-T) x_2 x_4^2 y_1 y_2 y_4}{2 T^3} + \frac{x_2 x_4 x_{\$[3]} y_1 y_2 y_4}{T^2} + \frac{(-2+2 T) x_1 x_2 x_4 y_1 y_2 y_{\$[1]}}{T^2} + \frac{(1-T) x_2 x_4^2 y_1 y_2 y_{\$[1]}}{T^3} + \frac{(-2+2 T) x_2 x_4 x_{\$[3]} y_1 y_2 y_{\$[1]}}{T^2} + \frac{x_1 x_2 x_4 y_2 y_4 y_{\$[1]}}{T^2} + \frac{(-1-T) x_2 x_4^2 y_2 y_4 y_{\$[1]}}{2 T^3} + \frac{x_2 x_4 x_{\$[3]} y_2 y_4 y_{\$[1]}}{T^2} + \frac{(-2+2 T) x_1 x_2 x_4 y_1 y_2 y_{\$[3]}}{T^2} + \frac{(1-T) x_2 x_4^2 y_1 y_2 y_{\$[3]}}{T^3} + \frac{(-2+2 T) x_2 x_4 x_{\$[3]} y_1 y_2 y_{\$[3]}}{T^2} + \frac{x_1 x_2 x_4 y_1 y_2 y_{\$[3]}}{T^2} + \frac{(-2+2 T) x_1 x_2 x_4 y_2 y_{\$[1]} y_{\$[3]}}{T^2} + \frac{(1-T) x_2 x_4^2 y_2 y_{\$[1]} y_{\$[3]}}{T^3} + \frac{(-2+2 T) x_2 x_4 x_{\$[3]} y_2 y_{\$[1]} y_{\$[3]}}{T^2} \right) \in^2 + O[\epsilon]^3$$

$$\left( \frac{(1-T) x_1 x_2^2 x_4 y_1^2 y_2^2}{T^2} + \frac{(-1+T) x_2^2 x_4^2 y_1^2 y_2^2}{2 T^3} + \frac{(1-T) x_2^2 x_4 x_{\$[3]} y_1^2 y_2^2}{T^2} + \frac{(2-2 T) x_1 x_2^2 x_4 y_1 y_2^2 y_{\$[1]}}{T^2} + \frac{(-1+T) x_2^2 x_4^2 y_1 y_2^2 y_{\$[1]}}{T^3} + \frac{(2-2 T) x_2^2 x_4 x_{\$[3]} y_1 y_2^2 y_{\$[1]}}{T^2} + \frac{(1-T) x_1 x_2^2 x_4 y_2^2 y_{\$[1]}^2}{T^2} + \frac{(-1+T) x_2^2 x_4^2 y_2^2 y_{\$[1]}^2}{2 T^3} + \frac{(1-T) x_2^2 x_4 x_{\$[3]} y_2^2 y_{\$[1]}^2}{T^2} \right) \in^3 + O[\epsilon]^4$$

In[\*]:= m<sub>2,4→2</sub>

Out[\*]= E<sub>{2,4}→{2}</sub> [1, y<sub>2</sub> (η<sub>2</sub> + η<sub>4</sub>) - η<sub>4</sub> ξ<sub>2</sub> + x<sub>2</sub> (ξ<sub>2</sub> + ξ<sub>4</sub>), 0]

In[\*]:= R<sub>1,2</sub> R<sub>3,4</sub> // m<sub>1,3→1</sub> // m<sub>2,4→2</sub>

Out[\*]= E<sub>{}</sub>→{1,2} [1, 0, 0[ε]<sup>2</sup>]

In[\*]:= ? Z

Symbol	
Global`Z	
Definitions	
	$Z[0] = \left( \frac{(-1+T) x_2 x_{\$[2]} y_1^2}{T} + \frac{1}{2} (-1 + T) x_{\$[2]}^2 y_1^2 + \frac{(1-T) x_2 x_{\$[4]} y_1^2}{T^2} + \frac{(1-T) x_{\$[4]}^2 y_1^2}{2 T^2} + x_1 x_{\$[2]} y_1 y_2 - 2 x_2 x_{\$[2]} y_1 y_2 + \frac{1}{2} \right)$
Out[*]=	$Z[1] = \left( \frac{(-1+T) x_1 x_2^2 y_1^3}{T^3} + \frac{(1-T^2) x_2^2 y_1^3}{2 T^4} + \frac{(-1+T) x_1 x_2 x_{\$[2]} y_1^3}{T^2} + \frac{(1-T^2) x_2^2 x_{\$[2]} y_1^3}{2 T^3} + \frac{(-1+T) x_1 x_2 x_{\$[4]} y_1^3}{T^3} + \frac{(1-T^2) x_2^2 x_{\$[4]} y_1^3}{T^4} + \frac{(-1+T)}{T^4} \right)$
	$Z[2] = \left( \frac{(-1+T) x_1^2 x_2^2 y_1^4}{2 T^4} + \frac{(1-T^2) x_1 x_2^2 y_1^4}{2 T^5} + \frac{(-1-T+T^2+T^3) x_2^2 y_1^4}{8 T^6} + \frac{(-1+T) x_1^2 x_2 x_{\$[4]} y_1^4}{T^4} + \frac{(3-3 T^2) x_1 x_2^2 x_{\$[4]} y_1^4}{2 T^5} + \frac{(-1-T+T^2+T^3) x_2^2 x_{\$[4]} y_1^4}{2 T^6} \right)$
Full Name Global`Z	
^	