

Pensieve header: Time 6029.18. Zip3 computes most partial derivatives only once.

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 25, 21, 38, 52.4363146}
```

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"..\\Profile\\Profile.m"];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

Engine

```
In[ ]:= CCF[ $\mathcal{E}$ ] := PP_CCF@ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ];
(*CoefficientCanonical Form *)
CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _eSeries] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ ] := PP_CF@Module[
  {vs = Cases[ $\mathcal{E}$ , {y | x |  $\eta$  |  $\xi$ }_,  $\infty$ ]  $\cup$  {y | x |  $\eta$  |  $\xi$ }},
  Total[(CCF[#][2]] (Times@@vs#[1]) & /@ CoefficientRules[Expand[ $\mathcal{E}$ ], vs]]
];
(*CF[ $\mathcal{E}$ ] := PP_CF@CCF[ $\mathcal{E}$ ];*)
CF[ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF[E_sp___[ $\mathcal{E}$ S___]] := CF /@ E_sp[ $\mathcal{E}$ S];
```

```
In[ ]:= eSeries /: S1_eSeries  $\equiv$  S2_eSeries :=
  Length[S1] == Length[S2]  $\wedge$  Inner[CF[#1] == CF[#2] &, S1, S2, And];
eSeries[0] := eSeries@@Table[0, $k + 1];
eSeries /: S1_eSeries + S2_eSeries :=
  eSeries@@Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
eSeries /: S1_eSeries * S2_eSeries := eSeries@@
  Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
eSeries /: c_*S_eSeries := (c #) & /@ S;
eSeries /:  $\partial_{vs}$  S_eSeries := (s  $\mapsto$   $\partial_{vs}$  s) /@ S;
```

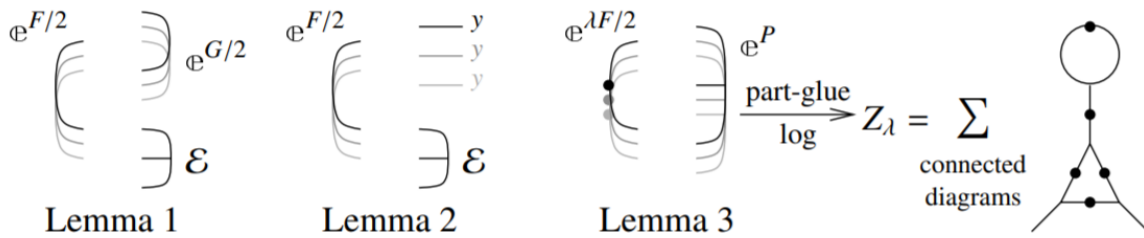
Variables and their duals:

```
In[ ]:= {y*, x*,  $\eta$ *,  $\xi$ *} = { $\eta$ ,  $\xi$ , y, x};
(vs_List)* := (v  $\mapsto$  v*) /@ vs;
(u_i)* := (u*)_i;
```

E operations:

```
In[*]:=
E /: E[\omega_1, Q1, P1] == E[\omega_2, Q2, P2] := CF[\omega1 == \omega2] \wedge CF[Q1 == Q2] \wedge (P1 == P2);
E /: E[\omega_1, Q1, P1] E[\omega_2, Q2, P2] := E[\omega1 \omega2, Q1 + Q2, P1 + P2];
E_{d1 \to r1}[\mathcal{E}1s] == E_{d2 \to r2}[\mathcal{E}2s] ^:= (d1 == d2) \wedge (r1 == r2) \wedge (E[\mathcal{E}1s] == E[\mathcal{E}2s]);
E_{d1 \to r1}[\mathcal{E}1s] E_{d2 \to r2}[\mathcal{E}2s] ^:= E_{(d1 \cup d2) \to (r1 \cup r2)} @ @ (E[\mathcal{E}1s] E[\mathcal{E}2s]);
E_{dr}[\mathcal{E}S]_{\$k} := E_{dr} @ @ E[\mathcal{E}S]_{\$k};
```

```
In[*]:=
E_{d1 \to r1}[\mathcal{E}1s] // E_{d2 \to r2}[\mathcal{E}2s] := Module[{is = r1 \cap d2, lvs},
  lvs = Flatten@Table[{x_{\$ei}, y_{\$ei}}, {i, is}];
  E_{(d1 \cup Complement[d2, is]) \to (r2 \cup Complement[r1, is])} @ @ (Zip_{lvs \cup lvs} [lvs*.lvs, Times[
    E[\mathcal{E}1s] /. Table[(v : x | y)_i \to v_{\$ei}, {i, is}],
    E[\mathcal{E}2s] /. Table[(v : \xi | \eta)_i \to v_{\$ei}, {i, is}]
  ]])
]
```



```
In[*]:=
Zip_{vs}[\mathcal{F}, \mathcal{E}] := \langle \mathcal{F}, \mathcal{E} \rangle // Zip1_{vs} // Zip2_{vs} // Zip3_{vs};
Zip_{vs}[\mathcal{F}, \mathcal{E}] := \langle \mathcal{F}, \mathcal{E} \rangle // Zip1_{vs} // EZip23_{vs};
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} e^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[*]:=
Zip1_{\{}} = Identity;
Zip1_{vs} @ \langle \mathcal{F}, E[\omega, Q, P] \rangle := PP_{Zip1} @ Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  G = Table[\partial_{u,v} Q, {u, vs}, {v, vs}];
  \langle CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF@PowerExpand@Factor[\omega Det[I - G.F]^{-1/2}, CF[Q - vs.G.vs / 2], P]]
]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F : \mathcal{E} e^{\sum_{i \in B} y_i z_i} \right\rangle_B = e^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E} |_{z_B \to z_B + F y_B} \right\rangle_B$.

```

In[ ]:= Zip2_{ } = Identity;
Zip2_{vs_} @ < F_, E[ω_, Q_, P_] > := PPZip2 @ Module[{F, Y, u, v},
  F = Table[∂_{u,v} F, {u, vs*}, {v, vs*}];
  Y = Table[∂_v Q, {v, vs}];
  CF /@ < F_, E[ω, Q - Y.vvs + Y.F.Y / 2, P /. Thread[v → vs + F.Y]] >
]

```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{C}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

```

In[ ]:= Zip3_{vs_} @ < F_, E[ω_, Q_, P_] > := PPZip3 @ Module[{F, Z, u, v, m, j},
  F[u_, v_] := F[u, v] = ∂_{u*,v*} F;
  Z[j_, v_] := Z[j, v] = ∂_v Z[j];
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[
      1 / (2 (m + 1))
      Sum[F[u, v] (∂_{u,v} Z[m] + Sum[Z[j, u] * Z[m - j, v], {j, 0, m}]), {u, vs}, {v, vs}]
    ];
  E[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v → 0, {v, vs}]]
]

```

```

In[ ]:= EZip23_{vs_} @ < F_, E[ω_, Q_, P_] > := PPEZip23 @ Module[
  {nP, nF, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total[
    CoefficientRules[#, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++] → CF@c]; t[j] (Times @@ vs^{ps}))
  ] & /@ P;
  nQ = Total[CoefficientRules[Q, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++] → CF@c]; t[j] (Times @@ vs^{ps}))];
  nF = Total[CoefficientRules[F, vs*] /. (ps_ → c_) ⇒
    (AppendTo[rr, t[++] → CF@c]; t[j] (Times @@ (vs*)^{ps}))];
  CF[Expand[<nF, E[ω, nQ, nP]> // Zip2_{vs_} // Zip3_{vs_} /. rr]
]

```

Profile

```
In[ ]:= BeginProfile[];
        PopupWindow[Button["Show Profile Monitor"],
                    Dynamic[PrintProfile[], UpdateInterval -> 3, TrackedSymbols -> {}]]
```

Out[]:=

\$k = 1

```
In[ ]:= NewBit[K_] := Module[{Alex = Alexander[K][T]},
    T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
In[ ]:= $k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[]:= $\left\{ 2 - T + T^2, (1 + T) (1 - 3 T + T^2), \frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}, 9 - 11 T + 7 T^2 - T^3 \right\}$

```
In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
        Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
        $k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[]:= True

Out[]:= $\{19.2188, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3\}$

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 33.673
  ( 24) 0.016/ 0.032 above CF
  ( 237) 1.528/ 31.670 above EZip23
  ( 237) 0.811/ 1.973 above Zip1
Zip3: called 237 times, time in 13.292/14.515
  ( 237) 13.290/ 14.520 under EZip23
  ( 1422) 0.568/ 1.223 above CF
CCF: called 15895 times, time in 9.063/9.063
  ( 15895) 9.063/ 9.063 under CF
CF: called 8837 times, time in 8.557/17.62
  ( 5495) 6.627/ 12.450 under EZip23
  ( 24) 0.016/ 0.032 under ProfileRoot
  ( 711) 0.581/ 1.162 under Zip1
  ( 1185) 0.765/ 2.752 under Zip2
  ( 1422) 0.568/ 1.223 under Zip3
  ( 15895) 9.063/ 9.063 above CCF
EZip23: called 237 times, time in 1.528/31.668
  ( 237) 1.528/ 31.670 under ProfileRoot
  ( 5495) 6.627/ 12.450 above CF
  ( 237) 0.422/ 3.174 above Zip2
  ( 237) 13.290/ 14.520 above Zip3
Zip1: called 237 times, time in 0.811/1.973
  ( 237) 0.811/ 1.973 under ProfileRoot
  ( 711) 0.581/ 1.162 above CF
Zip2: called 237 times, time in 0.422/3.174
  ( 237) 0.422/ 3.174 under EZip23
  ( 1185) 0.765/ 2.752 above CF
```

```
In[ ]:= $k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Out[ ]:= 1
```

```
In[ ]:= $k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

```
Out[ ]:= 1
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 191.817
( 44) 0.016/ 0.048 above CF
( 652) 9.487/ 184.940 above EZip23
( 652) 2.809/ 6.829 above Zip1
CF: called 24383 times, time in 71.254/139.424
( 15211) 65.246/ 123.720 under EZip23
( 44) 0.016/ 0.048 under ProfileRoot
( 1956) 2.078/ 4.020 under Zip1
( 3260) 2.531/ 8.256 under Zip2
( 3912) 1.383/ 3.385 under Zip3
( 48358) 68.170/ 68.170 above CCF
CCF: called 48358 times, time in 68.17/68.17
( 48358) 68.170/ 68.170 under CF
Zip3: called 652 times, time in 38.903/42.288
( 652) 38.903/ 42.288 under EZip23
( 3912) 1.383/ 3.385 above CF
EZip23: called 652 times, time in 9.487/184.94
( 652) 9.487/ 184.940 under ProfileRoot
( 15211) 65.246/ 123.720 above CF
( 652) 1.194/ 9.450 above Zip2
( 652) 38.903/ 42.288 above Zip3
Zip1: called 652 times, time in 2.809/6.829
( 652) 2.809/ 6.829 under ProfileRoot
( 1956) 2.078/ 4.020 above CF
Zip2: called 652 times, time in 1.194/9.45
( 652) 1.194/ 9.450 under EZip23
( 3260) 2.531/ 8.256 above CF
```

\$k = 2

```
In[ ]:= $k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

```
Out[ ]:= 2
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 1385.55
( 54) 0.048/ 0.110 above CF
( 813) 33.488/ 1376.810 above EZip23
( 813) 3.466/ 8.627 above Zip1
CF: called 34431 times, time in 684.159/1270.53
( 21385) 667.252/ 1185.810 under EZip23
( 54) 0.048/ 0.110 under ProfileRoot
( 2439) 2.624/ 5.161 under Zip1
( 4226) 5.094/ 21.569 under Zip2
( 6327) 9.141/ 57.881 under Zip3
( 79352) 586.374/ 586.374 above CCF
CCF: called 79352 times, time in 586.374/586.374
( 79352) 586.374/ 586.374 under CF
Zip3: called 813 times, time in 76.353/134.234
( 813) 76.353/ 134.234 under EZip23
( 6327) 9.141/ 57.881 above CF
EZip23: called 813 times, time in 33.488/1376.81
( 813) 33.488/ 1376.810 under ProfileRoot
( 21385) 667.252/ 1185.810 above CF
( 813) 1.710/ 23.279 above Zip2
( 813) 76.353/ 134.234 above Zip3
Zip1: called 813 times, time in 3.466/8.627
( 813) 3.466/ 8.627 under ProfileRoot
( 2439) 2.624/ 5.161 above CF
Zip2: called 813 times, time in 1.71/23.279
( 813) 1.710/ 23.279 under EZip23
( 4226) 5.094/ 21.569 above CF
```

```
In[ ]:= $k = 2; equiv =
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 3
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 4487.52
( 69) 0.048/ 0.141 above CF
( 1071) 88.732/ 4476.283 above EZip23
( 1071) 4.482/ 11.099 above Zip1
CF: called 50814 times, time in 2422.21/4253.04
( 31561) 2385.370/ 4027.910 under EZip23
( 69) 0.048/ 0.141 under ProfileRoot
( 3213) 3.294/ 6.617 under Zip1
( 5774) 9.922/ 48.337 under Zip2
( 10197) 23.581/ 170.038 under Zip3
( 133087) 1830.828/ 1830.828 above CCF
CCF: called 133087 times, time in 1830.83/1830.83
( 133087) 1830.828/ 1830.828 under CF
Zip3: called 1071 times, time in 139.043/309.081
( 1071) 139.043/ 309.081 under EZip23
( 10197) 23.581/ 170.038 above CF
EZip23: called 1071 times, time in 88.732/4476.28
( 1071) 88.732/ 4476.283 under ProfileRoot
( 31561) 2385.370/ 4027.910 above CF
( 1071) 2.223/ 50.560 above Zip2
( 1071) 139.043/ 309.081 above Zip3
Zip1: called 1071 times, time in 4.482/11.099
( 1071) 4.482/ 11.099 under ProfileRoot
( 3213) 3.294/ 6.617 above CF
Zip2: called 1071 times, time in 2.223/50.56
( 1071) 2.223/ 50.560 under EZip23
( 5774) 9.922/ 48.337 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 25, 22, 57, 10.5144898}
```

```
In[ ]:= $k = 2; equiv = {Knot[12, NonAlternating, 60],  

    Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};  

Length@Union[Z /@equiv]
```

```
Out[ ]:= 1
```


In[]:= **PrintProfile** []

```
Out[ ]:= ProfileRoot is root. Profiled time: 6029.18
( 84) 0.093/ 0.234 above CF
( 1329) 131.492/ 6014.676 above EZip23
( 1329) 6.030/ 14.273 above Zip1
CF: called 67039 times, time in 3163.13/5687.83
( 41579) 3107.804/ 5343.755 under EZip23
( 84) 0.093/ 0.234 under ProfileRoot
( 3987) 4.013/ 8.243 under Zip1
( 7322) 14.302/ 70.886 under Zip2
( 14067) 36.913/ 264.715 under Zip3
( 177470) 2524.708/ 2524.708 above CCF
CCF: called 177470 times, time in 2524.71/2524.71
( 177470) 2524.708/ 2524.708 under CF
Zip3: called 1329 times, time in 201.079/465.794
( 1329) 201.079/ 465.794 under EZip23
( 14067) 36.913/ 264.715 above CF
EZip23: called 1329 times, time in 131.492/6014.68
( 1329) 131.492/ 6014.676 under ProfileRoot
( 41579) 3107.804/ 5343.755 above CF
( 1329) 2.749/ 73.635 above Zip2
( 1329) 201.079/ 465.794 above Zip3
Zip1: called 1329 times, time in 6.03/14.273
( 1329) 6.030/ 14.273 under ProfileRoot
( 3987) 4.013/ 8.243 above CF
Zip2: called 1329 times, time in 2.749/73.635
( 1329) 2.749/ 73.635 under EZip23
( 7322) 14.302/ 70.886 above CF
```

In[]:= **Date** []

```
Out[ ]:= {2020, 12, 25, 23, 23, 53.6432916}
```