

Pensieve header: Time 9013.55. "ReplaceAll" removed from CF.

```
In[*]:= Date []
```

```
Out[*]:= {2020, 12, 24, 12, 35, 59.8203530}
```

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

Engine

```
In[*]:= CCF[ $\mathcal{E}$ ] := PPCCF@ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ];
(*CoefficientCanonical Form *)
CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _eSeries] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ ] := PPCF@Module[
  {vs = Cases[ $\mathcal{E}$ , (y | x |  $\eta$  |  $\xi$ )_,  $\infty$ ]  $\cup$  {y | x |  $\eta$  |  $\xi$ }},
  Total[(CCF[#][2]) (Times@@vs#[1]) & /@ CoefficientRules[Expand[ $\mathcal{E}$ ], vs]]
];
(*CF[ $\mathcal{E}$ ] := PPCF@CCF[ $\mathcal{E}$ ];*)
CF[ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF[Esp[_] [ES_]] := CF /@ Esp[ES];
```

```
In[*]:= eSeries /: S1_eSeries  $\equiv$  S2_eSeries :=
  Length[S1] == Length[S2]  $\wedge$  Inner[CF[#1] == CF[#2] &, S1, S2, And];
eSeries[0] := eSeries@@Table[0, $k+1];
eSeries /: S1_eSeries + S2_eSeries :=
  eSeries@@Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
eSeries /: S1_eSeries * S2_eSeries := eSeries@@
  Table[Sum[S1[[j+1]] * S2[[k-j+1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
eSeries /: c_*S_eSeries := (c#) & /@ S;
eSeries /:  $\partial_{vs}$  S_eSeries := (s  $\mapsto$   $\partial_{vs}$  s) /@ S;
```

The Main Program

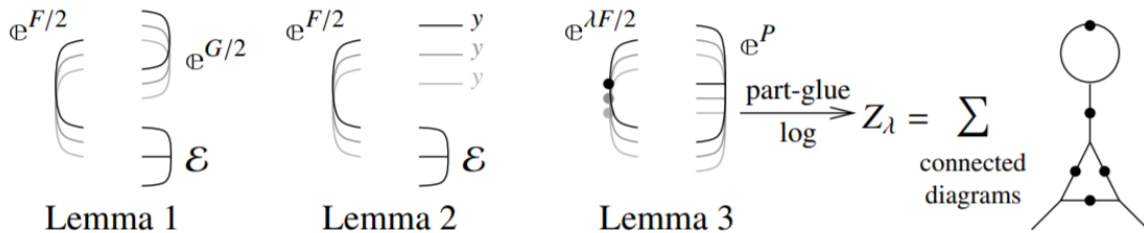
Variables and their duals:

```
In[*]:= {y*, x*, η*, ξ*} = {η, ξ, y, x};
(vs_List)* := (v ↦ v*) /@ vs;
(u_{i_})* := (u*)_i;
```

E operations:

```
In[*]:= E /: E[ω1_, Q1_, P1_] ≡ E[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 == P2);
E /: E[ω1_, Q1_, P1_] E[ω2_, Q2_, P2_] := E[ω1 ω2, Q1 + Q2, P1 + P2];
E_{d1 → r1}[ε1s___] ≡ E_{d2 → r2}[ε2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (E[ε1s] ≡ E[ε2s]);
E_{d1 → r1}[ε1s___] E_{d2 → r2}[ε2s___] ^:= E_{(d1 ∪ d2) → (r1 ∪ r2)} @@ (E[ε1s] E[ε2s]);
E_{dr_}[εs___]_{k_} := E_{dr} @@ E[εs]_{k_};
```

```
In[*]:= E_{d1 → r1}[ε1s___] // E_{d2 → r2}[ε2s___] := Module[{is = r1 ∩ d2, lvs},
  lvs = Flatten@Table[{x_{i}, y_{i}}, {i, is}];
  E_{(d1 ∪ Complement[d2, is]) → (r2 ∪ Complement[r1, is])} @@ (Zip_{lvs ∪ lvs*}[lvs*.lvs, Times[
    E[ε1s] /. Table[{v : x | y}_i → v_{i}, {i, is}],
    E[ε2s] /. Table[{v : ξ | η}_i → v_{i}, {i, is}]
  ]])
]
```



```
In[*]:= Zip_{vs}[F_, ε_] := <F, ε> // Zip1_{vs} // Zip2_{vs} // Zip3_{vs};
Zip_{vs}[F_, ε_] := <F, ε> // Zip1_{vs} // EZip23_{vs};
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} e^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[*]:= Zip1_{{} } = Identity;
Zip1_{vs_} @ <F_, E[ω_, Q_, P_] > := PP_{Zip1} @ Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[∂_{u,v} F, {u, vs*}, {v, vs*}];
  G = Table[∂_{u,v} Q, {u, vs}, {v, vs}];
  <CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF@PowerExpand@Factor[ω Det[I - G.F]^{-1/2}], CF[Q - vs.G.vs / 2], P] >
]
```

Getting rid of linear terms.

Lemma 2. $\langle F: \mathcal{E}_{\mathbb{E}^{\sum_{i \in B} Y_i Z_i}} \rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i, j \in B} F_{ij} Y_i Y_j} \langle F: \mathcal{E}_{|z_B \rightarrow z_B + F Y_B} \rangle_B$.

In[*]:=

```
Zip2_{ } = Identity;
Zip2_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := PP_{Zip2} @ Module[{F, Y, u, v},
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  Y = Table[\partial_v Q, {v, vs}];
  CF /@ < \mathcal{F}, \mathbb{E}[\omega, Q - Y.v s + Y.F.Y / 2, P /. Thread[vs \to vs + F.Y]] >
]
```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F: \mathbb{E}^P]_B$ satisfies

and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i, j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

In[*]:=

```
Zip3_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := PP_{Zip3} @ Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[
      \frac{1}{2 (m + 1)}
      Sum[\partial_{u*, v*} \mathcal{F} (\partial_{u,v} Z[m] + Sum[(\partial_u Z[j]) (\partial_v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]]
  ];
  \mathbb{E}[\omega, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v \to 0, {v, vs}]]]
]
```

```
EZip23_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := PEZip23 @ Module[
  {nP, n\mathcal{F}, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total[
    CoefficientRules[#, vs] /.
    (ps_ \to c_) \Rightarrow (AppendTo[rr, t[+ + j] \to CF@c]; t[j] (Times @@ vs^{ps}))
  ] & /@ P;
  nQ = Total[CoefficientRules[Q, vs] /.
    (ps_ \to c_) \Rightarrow (AppendTo[rr, t[+ + j] \to CF@c]; t[j] (Times @@ vs^{ps}))];
  n\mathcal{F} = Total[CoefficientRules[\mathcal{F}, vs*] /. (ps_ \to c_) \Rightarrow
    (AppendTo[rr, t[+ + j] \to CF@c]; t[j] (Times @@ (vs*)^{ps}))];
  CF[Expand[<n\mathcal{F}, \mathbb{E}[\omega, nQ, nP]> // Zip2_{vs} // Zip3_{vs}] /. rr]
]
```

Profile

```
In[ ]:= BeginProfile[];
        PopupWindow[Button["Show Profile Monitor"],
                    Dynamic[PrintProfile[], UpdateInterval -> 3, TrackedSymbols -> {}]]
```

```
Out[ ]:= 
```

\$k = 1

```
In[ ]:= NewBit[K_] := Module[{Alex = Alexander[K][T]},
    T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
In[ ]:= $k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[ ]:= {2 - T + T^2, (1 + T) (1 - 3 T + T^2),  $\frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}$ , 9 - 11 T + 7 T^2 - T^3}
```

```
In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
        Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
        $k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

```
Out[ ]:= True
```

```
Out[ ]:= {28.5469, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3}
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 54.269
( 24) 0.047/ 0.047 above CF
( 237) 2.178/ 51.381 above EZip23
( 237) 1.246/ 2.841 above Zip1
Zip3: called 237 times, time in 24.616/26.458
( 237) 24.616/ 26.458 under EZip23
( 1422) 0.783/ 1.842 above CF
CCF: called 15895 times, time in 13.235/13.235
( 15895) 13.235/ 13.235 under CF
CF: called 8837 times, time in 12.397/25.632
( 5495) 9.681/ 18.070 under EZip23
( 24) 0.047/ 0.047 under ProfileRoot
( 711) 0.763/ 1.595 under Zip1
( 1185) 1.123/ 4.078 under Zip2
( 1422) 0.783/ 1.842 under Zip3
( 15895) 13.235/ 13.235 above CCF
EZip23: called 237 times, time in 2.178/51.381
( 237) 2.178/ 51.381 under ProfileRoot
( 5495) 9.681/ 18.070 above CF
( 237) 0.597/ 4.675 above Zip2
( 237) 24.616/ 26.458 above Zip3
Zip1: called 237 times, time in 1.246/2.841
( 237) 1.246/ 2.841 under ProfileRoot
( 711) 0.763/ 1.595 above CF
Zip2: called 237 times, time in 0.597/4.675
( 237) 0.597/ 4.675 under EZip23
( 1185) 1.123/ 4.078 above CF
```

```
In[ ]:= $k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Out[ ]:= 1
```

```
In[ ]:= $k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

```
Out[ ]:= 1
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 281.771
( 44) 0.079/ 0.094 above CF
( 652) 13.403/ 271.790 above EZip23
( 652) 3.828/ 9.886 above Zip1
CF: called 24383 times, time in 100.447/195.397
( 15211) 91.824/ 172.240 under EZip23
( 44) 0.079/ 0.094 under ProfileRoot
( 1956) 2.990/ 6.058 under Zip1
( 3260) 3.574/ 12.004 under Zip2
( 3912) 1.980/ 5.003 under Zip3
( 48358) 94.950/ 94.950 above CCF
CCF: called 48358 times, time in 94.95/94.95
( 48358) 94.950/ 94.950 under CF
Zip3: called 652 times, time in 67.513/72.516
( 652) 67.513/ 72.516 under EZip23
( 3912) 1.980/ 5.003 above CF
EZip23: called 652 times, time in 13.403/271.791
( 652) 13.403/ 271.790 under ProfileRoot
( 15211) 91.824/ 172.240 above CF
( 652) 1.630/ 13.634 above Zip2
( 652) 67.513/ 72.516 above Zip3
Zip1: called 652 times, time in 3.828/9.886
( 652) 3.828/ 9.886 under ProfileRoot
( 1956) 2.990/ 6.058 above CF
Zip2: called 652 times, time in 1.63/13.634
( 652) 1.630/ 13.634 under EZip23
( 3260) 3.574/ 12.004 above CF
```

\$k = 2

```
In[ ]:= $k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 2
```

```
In[ ]:= PrintProfile[ ]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 2198.88
( 54) 0.110/ 0.204 above CF
( 813) 52.558/ 2186.060 above EZip23
( 813) 4.692/ 12.621 above Zip1
CF: called 34431 times, time in 1037.16/1943.37
( 21385) 1013.270/ 1818.150 under EZip23
( 54) 0.110/ 0.204 under ProfileRoot
( 2439) 3.955/ 7.929 under Zip1
( 4226) 7.267/ 31.647 under Zip2
( 6327) 12.559/ 85.439 under Zip3
( 79352) 906.204/ 906.204 above CCF
CCF: called 79352 times, time in 906.204/906.204
( 79352) 906.204/ 906.204 under CF
Zip3: called 813 times, time in 196.05/281.489
( 813) 196.050/ 281.489 under EZip23
( 6327) 12.559/ 85.439 above CF
EZip23: called 813 times, time in 52.558/2186.06
( 813) 52.558/ 2186.060 under ProfileRoot
( 21385) 1013.270/ 1818.150 above CF
( 813) 2.212/ 33.859 above Zip2
( 813) 196.050/ 281.489 above Zip3
Zip1: called 813 times, time in 4.692/12.621
( 813) 4.692/ 12.621 under ProfileRoot
( 2439) 3.955/ 7.929 above CF
Zip2: called 813 times, time in 2.212/33.859
( 813) 2.212/ 33.859 under EZip23
( 4226) 7.267/ 31.647 above CF
```

```
In[ ]:= $k = 2; equiv =
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@equiv]
```

```
Out[ ]:= 3
```

In[]:= **PrintProfile[]**

```
Out[ ]:= ProfileRoot is root. Profiled time: 6710.66
( 69) 0.110/ 0.220 above CF
( 1071) 130.272/ 6693.897 above EZip23
( 1071) 6.162/ 16.543 above Zip1
CF: called 50814 times, time in 3476.1/6141.23
( 31561) 3420.967/ 5813.652 under EZip23
( 69) 0.110/ 0.220 under ProfileRoot
( 3213) 5.192/ 10.381 under Zip1
( 5774) 14.252/ 68.444 under Zip2
( 10197) 35.576/ 248.533 under Zip3
( 133087) 2665.133/ 2665.133 above CCF
CCF: called 133087 times, time in 2665.13/2665.13
( 133087) 2665.133/ 2665.133 under CF
Zip3: called 1071 times, time in 430.006/678.539
( 1071) 430.006/ 678.539 under EZip23
( 10197) 35.576/ 248.533 above CF
EZip23: called 1071 times, time in 130.272/6693.9
( 1071) 130.272/ 6693.897 under ProfileRoot
( 31561) 3420.967/ 5813.652 above CF
( 1071) 2.990/ 71.434 above Zip2
( 1071) 430.006/ 678.539 above Zip3
Zip1: called 1071 times, time in 6.162/16.543
( 1071) 6.162/ 16.543 under ProfileRoot
( 3213) 5.192/ 10.381 above CF
Zip2: called 1071 times, time in 2.99/71.434
( 1071) 2.990/ 71.434 under EZip23
( 5774) 14.252/ 68.444 above CF
```

In[]:= **Date[]**

```
Out[ ]:= {2020, 12, 24, 14, 40, 1.3207989}
```

```
In[ ]:= $k = 2; equiv = {Knot[12, NonAlternating, 60],
    Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};
    Length@Union[Z /@ equiv]
```

```
Out[ ]:= 1
```


In[]:= **PrintProfile** []

```
Out[ ]:= ProfileRoot is root. Profiled time: 9013.55
( 84) 0.187/ 0.345 above CF
( 1329) 189.815/ 8992.993 above EZip23
( 1329) 7.691/ 20.210 above Zip1
CF: called 67039 times, time in 4524.05/8159.68
( 41579) 4444.114/ 7671.017 under EZip23
( 84) 0.187/ 0.345 under ProfileRoot
( 3987) 6.205/ 12.519 under Zip1
( 7322) 20.233/ 99.396 under Zip2
( 14067) 53.313/ 376.406 under Zip3
( 177470) 3635.631/ 3635.631 above CCF
CCF: called 177470 times, time in 3635.63/3635.63
( 177470) 3635.631/ 3635.631 under CF
Zip3: called 1329 times, time in 652.514/1028.92
( 1329) 652.514/ 1028.920 under EZip23
( 14067) 53.313/ 376.406 above CF
EZip23: called 1329 times, time in 189.815/8992.99
( 1329) 189.815/ 8992.993 under ProfileRoot
( 41579) 4444.114/ 7671.017 above CF
( 1329) 3.845/ 103.241 above Zip2
( 1329) 652.514/ 1028.920 above Zip3
Zip1: called 1329 times, time in 7.691/20.21
( 1329) 7.691/ 20.210 under ProfileRoot
( 3987) 6.205/ 12.519 above CF
Zip2: called 1329 times, time in 3.845/103.241
( 1329) 3.845/ 103.241 under EZip23
( 7322) 20.233/ 99.396 above CF
```

In[]:= **Date** []

```
Out[ ]:= {2020, 12, 24, 15, 21, 37.1393371}
```