

Pensieve header: A fresh implementation of baby DoPeGDO. Continues pensieve://2020-09/, pensieve://2020-03/Testing123.nb, and pensieve://People/VanDerVeen/TimidHeisenbergRGeneralForm@.nb.

$E[\omega, Q, P_\epsilon \text{Series}]$ represents ωe^{Q+P} , where ω is a scalar, Q is an ϵ -free quadratic, and $P = \sum_{k=0}^k P[k] \epsilon^k$ is a perturbation (it is ill-advised to include ω in P because then it will have log terms).

Scheme: $E[_] // E[_]$ calls FZip or Zip, which are functionally the same. Zip works by handling the quadratic part and calling PZip for the perturbation-only part. PZip works by iteratively solving the synthesis equation. FZip works by encapsulating coefficients, calling Zip, and back-substituting.

Initialization, minor utilities, and “Define” Code

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@".. /Profile/Profile.m"];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

(Alt) In[]:=

```
$k=1;
```

```
CCF[ε_] := PPCF@ExpandDenominator@ExpandNumerator@Together[ε];
CF[ε_List] := CF /@ ε;
CF[ε_εSeries] := CF /@ ε;
CF[ε_] := PPCF@Module[
  {vs = Cases[ε, (y | x | η | ξ)_ , ∞] ∪ {y | x | η | ξ}},
  Total[CoefficientRules[Expand[ε], vs] /. (ps_ -> c_) -> CCF[c] (Times@@vs^ps)]
];
(*CF[ε_] := PPCF@CCF[ε];*)
CF[ε_E] := CF /@ ε;
CF[E_sp___[εS___]] := CF /@ E_sp[εS];
```

(Alt) In[]:=

```
εSeries /: S1_εSeries ≡ S2_εSeries :=
  Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
εSeries[0] := εSeries @@ Table[0, $k + 1];
εSeries /: S1_εSeries + S2_εSeries :=
  εSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
εSeries /: S1_εSeries * S2_εSeries := εSeries @@
  Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
εSeries /: c_ * S_εSeries := (c #) & /@ S;
εSeries /: ∂_vs___ S_εSeries := (s ↦ ∂_vs s) /@ S;
```

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of

\$.k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

(Alt) In[]:=

```

SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs]);
Define[op_is__ =  $\varepsilon$ _] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
  ReleaseHold[Hold[
    SD[op_nisp, $k_Integer, Block[{i, j, k}, op_isp, $k =  $\varepsilon$ ; op_nis, $k]];
    SD[op_isp, op_{is}, $k]; SD[op_sis__, op_{sis}];
  ] /. {SD -> SetDelayed,
    isp -> {is} /. {i -> i_, j -> j_, k -> k_},
    nis -> {is} /. {i -> ii, j -> jj, k -> kk},
    nisp -> {is} /. {i -> ii_, j -> jj_, k -> kk_}
  } ] ]

```

The Main Program

Variables and their duals:

(Alt) In[]:=

```

{y*, x*,  $\eta^*$ ,  $\xi^*$ } = { $\eta$ ,  $\xi$ , y, x};
(vs_List)* := (v -> v*) /@ vs;
(u_i)* := (u*)_i;

```

E operations:

(Alt) In[]:=

```

E /: E[ $\omega_1$ _, Q1_, P1_] == E[ $\omega_2$ _, Q2_, P2_] := CF[ $\omega_1$  ==  $\omega_2$ ] ^ CF[Q1 == Q2] ^ (P1 == P2);
E /: E[ $\omega_1$ _, Q1_, P1_] E[ $\omega_2$ _, Q2_, P2_] := E[ $\omega_1$   $\omega_2$ , Q1 + Q2, P1 + P2];
E_{d1 -> r1}[ $\mathcal{E}1s$ ___] == E_{d2 -> r2}[ $\mathcal{E}2s$ ___] ^:= (d1 == d2) ^ (r1 == r2) ^ (E[ $\mathcal{E}1s$ ] == E[ $\mathcal{E}2s$ ]);
E_{d1 -> r1}[ $\mathcal{E}1s$ ___] E_{d2 -> r2}[ $\mathcal{E}2s$ ___] ^:= E_{(d1|d2) -> (r1|_r2)} @@ (E[ $\mathcal{E}1s$ ] E[ $\mathcal{E}2s$ ]);
E_{dr}[ $\mathcal{E}S$ ___]_{ $k$ _} := E_{dr} @@ E[ $\mathcal{E}S$ ]_{ $k$ };

```

(Alt) In[]:=

```

E_{d1 -> r1}[ $\mathcal{E}1s$ ___] // E_{d2 -> r2}[ $\mathcal{E}2s$ ___] := Module[{is = r1 | d2, lvs},
  lvs = Flatten@Table[{x_{ $\phi$ i}, y_{ $\phi$ i}}, {i, is}];
  E_{(d1|Complement[d2, is]) -> (r2|Complement[r1, is])} @@ (Zip_{lvs|lvs}*[lvs*.lvs, Times[
    E[ $\mathcal{E}1s$ ] /. Table[(v : x | y)_i -> v_{ $\phi$ i}, {i, is}],
    E[ $\mathcal{E}2s$ ] /. Table[(v :  $\xi$  |  $\eta$ )_i -> v_{ $\phi$ i}, {i, is}]
  ]])
]

```

$[F : \mathcal{E}]_B := \mathbb{e}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} \partial_{z_i} \partial_{z_j}} \mathcal{E}$ and $\langle F : \mathcal{E} \rangle_B := [F : \mathcal{E}]_B|_{z_B \rightarrow 0}$,
 where \mathcal{E} is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where \mathcal{E} has no B - B quadratic part:

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} \mathbb{e}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B.$$

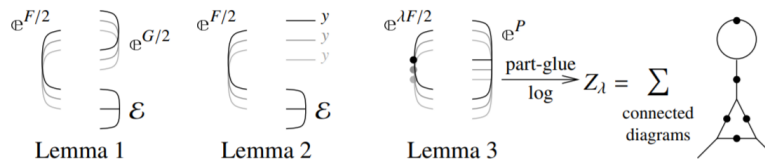
The next lemma dispatches the case where \mathcal{E} has a B -linear part:

Lemma 2. $\left\langle F : \mathcal{E} \mathbb{e}^{\sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{e}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{e}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} \left(\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda) \right).$$



(Alt) In[]:=

```
Zipvs[_F_, _E_] := <_F_, _E_> // Zip1vs // Zip2vs // Zip3vs
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} \mathbb{e}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

(Alt) In[]:=

```
Zip1{} = Identity;
Zip1vs[_F_, _E_, _Q_, _P_] := PPZip1@Module[{_I_, _F_, _G_, _u_, _v_},
  _I_ = IdentityMatrix@Length@vs;
  _F_ = Table[_D_<_u_, _v_>[_F_, {u, vs*}], {v, vs*}];
  _G_ = Table[_D_<_u_, _v_>[_Q_, {u, vs}], {v, vs}];
  CF /@ {vs*.F.Inverse[_I_ - G.F].vs* / 2,
    E[PowerExpand@Factor[_omega_ Det[_I_ - G.F]^-1/2], Q - vs.G.vs / 2, P]}
]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F : \mathcal{E} \mathbb{e}^{\sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{e}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

(Alt) In[]:=

```
Zip2{} = Identity;
Zip2vs[_F_, _E_, _Q_, _P_] := PPZip2@Module[{_F_, _Y_, _u_, _v_},
  _F_ = Table[_D_<_u_, _v_>[_F_, {u, vs*}], {v, vs*}];
  _Y_ = Table[_D_<_v_>[_Q_, {v, vs}];
  CF /@ {_F_, E[_omega_, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs -> vs + F.Y]]}
]
```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{E}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} \left(\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda) \right).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

(Alt) In[*]:=

```

Zip3vs_@<F_, E[ω_, Q_, P_]> := PPZip3@Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[
      1
      / (2 (m + 1))
      Sum[∂u,v*F (∂u,vZ[m] + Sum[(∂uZ[j]) (∂vZ[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]
    ];
  E[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v → 0, {v, vs}]]]

```

Solving for R, CC, \$k = 1

(Alt) In[*]:=

```

$k = 1;
{R1,2, CC1}
unknowns = Cases[{R1,2, R̄1,2, CC1, C̄C1}, (c | d | e | f)$k, ∞] // Union

```

(Alt) Out[*]=

```

{E_{1,2} [1, (-1 + T) x2 (y1 - y2),
  ∈Series[0, c1,1 + x1 y1 c1,2 + x2 y1 c1,3 + x1 y2 c1,4 + x2 y2 c1,5 + x12 y12 c1,6 + x1 x2 y12 c1,7 + x22 y12 c1,8 +
  x12 y1 y2 c1,9 + x1 x2 y1 y2 c1,10 + x22 y1 y2 c1,11 + x12 y22 c1,12 + x1 x2 y22 c1,13 + x22 y22 c1,14],
  E_{1,1} [√T, 0, ∈Series[0, e1,1 + x1 y1 e1,2 + x12 y12 e1,3]]]

```

(Alt) Out[*]=

```

{c1,1, c1,2, c1,3, c1,4, c1,5, c1,6, c1,7, c1,8, c1,9, c1,10, c1,11, c1,12, c1,13, c1,14, d1,1, d1,2, d1,3,
  d1,4, d1,5, d1,6, d1,7, d1,8, d1,9, d1,10, d1,11, d1,12, d1,13, d1,14, e1,1, e1,2, e1,3, f1,1, f1,2, f1,3}

```

(Alt) In[]:= Short[errors = { (R_{1,2} R_{4,3} R_{5,6} // m_{1,4→1} // m_{2,5→2} // m_{3,6→3}) [[3, -1]] -
 (R_{2,3} R_{4,5} R_{1,6} // m_{1,4→1} // m_{2,5→2} // m_{3,6→3}) [[3, -1]],
 (R_{1,2} R_{3,4} // m_{1,3→1} // m_{2,4→2}) [[3, -1]],
 (CC₁ CC₂ // m_{1,2→1}) [[3, -1]],
 (CC₃ R_{1,2} // m_{2,3→2} // m_{2,1→1}) [[3, -1]] - (CC₃ R_{1,2} // m_{1,3→1} // m_{1,2→1}) [[3, -1]] },
 10]

(Alt) Out[]//Short=

$$\left\{ -x_3 y_1 c_{1,3} - x_2 y_1 (c_{1,2} - T c_{1,2} + c_{1,3}) + x_1 y_2 c_{1,4} + x_1 y_3 c_{1,4} - \right.$$

$$T x_1 y_3 c_{1,4} + T x_2 y_3 c_{1,4} + \langle\langle 99 \rangle\rangle + x_3^2 y_1 y_3 (T c_{1,11} + 2 T c_{1,14} - 2 T^2 c_{1,14}) -$$

$$x_3^2 y_2 y_3 (T^2 c_{1,11} + 2 T c_{1,14} - 2 T^2 c_{1,14}) - x_3^2 y_2^2 (T^2 c_{1,8} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}) +$$

$$x_3^2 y_1^2 (T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} + 2 c_{1,8} -$$

$$4 T c_{1,8} + 3 T^2 c_{1,8} + c_{1,11} - 2 T c_{1,11} + T^2 c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}) +$$

$$x_3^2 y_2^2 (T^2 c_{1,8} + T^2 c_{1,12} - 2 T^3 c_{1,12} + T^4 c_{1,12} + T c_{1,13} - 2 T^2 c_{1,13} + T^3 c_{1,13} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}),$$

$$c_{1,1} + \langle\langle 13 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle}, \langle\langle 1 \rangle\rangle, -c_{1,1} + \langle\langle 12 \rangle\rangle \left. \right\}$$

(Alt) In[]:= eqns =

Thread[0 == Union@@(CoefficientRules[#, {x₁, x₂, x₃, y₁, y₂, y₃}][[; ; , 2]] & /@ errors)]

(Alt) Out[]:=

$$\left\{ \begin{aligned} 0 &= c_{1,4} - T c_{1,4}, \quad \theta = -c_{1,4} + T c_{1,4}, \quad \theta = T c_{1,4} - T^2 c_{1,4}, \quad \theta = -c_{1,4} + 2 T c_{1,4} - T^2 c_{1,4}, \\ \theta &= -T c_{1,4} + T^2 c_{1,4}, \quad \theta = T c_{1,2} - T^2 c_{1,2} + c_{1,3} - T c_{1,3} + c_{1,5} - T c_{1,5}, \\ \theta &= -2 c_{1,6} + 2 T c_{1,6}, \quad \theta = 2 T c_{1,6} - 2 T^2 c_{1,6}, \quad \theta = c_{1,9} - T c_{1,9}, \\ \theta &= -c_{1,9} + T c_{1,9}, \quad \theta = 2 T c_{1,9} - 2 T^2 c_{1,9}, \quad \theta = -2 c_{1,9} + 4 T c_{1,9} - 2 T^2 c_{1,9}, \\ \theta &= -2 T c_{1,9} + 2 T^2 c_{1,9}, \quad \theta = 2 T c_{1,6} - 2 T^2 c_{1,6} - c_{1,9} + 4 T c_{1,9} - 4 T^2 c_{1,9} + T^3 c_{1,9}, \\ \theta &= 2 T c_{1,8} - 2 T^2 c_{1,8} + T^2 c_{1,9} - 2 T^3 c_{1,9} + T^4 c_{1,9} + T c_{1,10} - 2 T^2 c_{1,10} + T^3 c_{1,10}, \\ \theta &= 2 T c_{1,7} - 2 T^2 c_{1,7} - c_{1,10} + 4 T c_{1,10} - 3 T^2 c_{1,10} + 2 c_{1,11} - 2 T c_{1,11}, \\ \theta &= T^2 c_{1,9} - T^3 c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12}, \quad \theta = c_{1,12} - T^2 c_{1,12}, \quad \theta = -c_{1,12} + 2 T c_{1,12} - T^2 c_{1,12}, \\ \theta &= c_{1,9} - 2 T c_{1,9} + T^2 c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12}, \quad \theta = -2 T c_{1,12} + 2 T^2 c_{1,12}, \\ \theta &= -4 T c_{1,12} + 8 T^2 c_{1,12} - 4 T^3 c_{1,12}, \quad \theta = -2 c_{1,12} + 6 T c_{1,12} - 6 T^2 c_{1,12} + 2 T^3 c_{1,12}, \\ \theta &= -2 T^2 c_{1,12} + 2 T^3 c_{1,12}, \quad \theta = -T^2 c_{1,12} + 2 T^3 c_{1,12} - T^4 c_{1,12}, \\ \theta &= -c_{1,12} + 4 T c_{1,12} - 6 T^2 c_{1,12} + 4 T^3 c_{1,12} - T^4 c_{1,12}, \quad \theta = -2 T c_{1,12} + 6 T^2 c_{1,12} - 6 T^3 c_{1,12} + 2 T^4 c_{1,12}, \\ \theta &= 2 T c_{1,13} - 2 T^2 c_{1,13}, \quad \theta = T c_{1,13} - T^2 c_{1,13}, \quad \theta = 2 T c_{1,12} - 2 T^2 c_{1,12} + T c_{1,13} - T^2 c_{1,13}, \\ \theta &= 2 c_{1,8} - 2 T c_{1,8} + c_{1,10} - 2 T c_{1,10} + T^2 c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13}, \\ \theta &= -2 T c_{1,13} + 2 T^2 c_{1,13}, \quad \theta = -2 T c_{1,13} + 4 T^2 c_{1,13} - 2 T^3 c_{1,13}, \\ \theta &= T^2 c_{1,12} - 2 T^3 c_{1,12} + T^4 c_{1,12} + T c_{1,13} - 2 T^2 c_{1,13} + T^3 c_{1,13}, \quad \theta = -T^2 c_{1,13} + T^3 c_{1,13}, \\ \theta &= -c_{1,13} + 4 T c_{1,13} - 4 T^2 c_{1,13} + T^3 c_{1,13} + 2 c_{1,14} - 2 T c_{1,14}, \quad \theta = 2 T c_{1,14} - 2 T^2 c_{1,14}, \\ \theta &= T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} + c_{1,8} - 4 T c_{1,8} + 3 T^2 c_{1,8} + c_{1,11} - \\ & 2 T c_{1,11} + T^2 c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}, \quad \theta = -2 T c_{1,14} + 2 T^2 c_{1,14}, \quad \theta = c_{1,1} + d_{1,1}, \\ \theta &= c_{1,2} + d_{1,2} + d_{1,4} - T d_{1,4}, \quad \theta = c_{1,4} + T d_{1,4}, \quad \theta = c_{1,2} - \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T} + d_{1,3} + d_{1,5} - T d_{1,5}, \\ \theta &= c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} + T d_{1,5}, \quad \theta = c_{1,9} + T d_{1,9} + 2 T d_{1,12} - 2 T^2 d_{1,12}, \\ \theta &= c_{1,12} + T^2 d_{1,12}, \quad \theta = c_{1,6} + d_{1,6} + d_{1,9} - T d_{1,9} + d_{1,12} - 2 T d_{1,12} + T^2 d_{1,12}, \end{aligned} \right.$$

$$\begin{aligned}
\theta &= 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + T d_{1,10} + 2 T d_{1,13} - 2 T^2 d_{1,13}, \quad \theta = 2 c_{1,12} - \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T} + T^2 d_{1,13}, \\
\theta &= 2 c_{1,6} - \frac{2 c_{1,6}}{T} + \frac{c_{1,7}}{T} + d_{1,7} + d_{1,10} - T d_{1,10} + d_{1,13} - 2 T d_{1,13} + T^2 d_{1,13}, \\
\theta &= c_{1,9} + \frac{c_{1,9}}{T^2} - \frac{2 c_{1,9}}{T} - \frac{c_{1,10}}{T^2} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + T d_{1,11} + 2 T d_{1,14} - 2 T^2 d_{1,14}, \\
\theta &= c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} - \frac{c_{1,13}}{T^2} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + T^2 d_{1,14}, \\
\theta &= c_{1,6} + \frac{c_{1,6}}{T^2} - \frac{2 c_{1,6}}{T} - \frac{c_{1,7}}{T^2} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + d_{1,8} + d_{1,11} - T d_{1,11} + d_{1,14} - 2 T d_{1,14} + T^2 d_{1,14}, \\
\theta &= -\frac{c_{1,3}}{T} + c_{1,4} + \frac{2 c_{1,8}}{T^2} - 2 c_{1,12} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2 e_{1,3} + \frac{2 e_{1,3}}{T^2} - \frac{4 e_{1,3}}{T} - f_{1,1}, \\
\theta &= e_{1,1} + f_{1,1}, \quad \theta = e_{1,2} + f_{1,2}, \quad \theta = c_{1,2} - T c_{1,2} - c_{1,3} + \frac{c_{1,3}}{T} + c_{1,4} - T c_{1,4} - c_{1,5} + \frac{c_{1,5}}{T} - \frac{2 c_{1,7}}{T} - \\
&\quad \frac{4 c_{1,8}}{T^2} + 2 T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} - \frac{2 c_{1,11}}{T^2} + 4 T c_{1,12} + 2 c_{1,13} + \frac{e_{1,2}}{T} - \frac{4 e_{1,3}}{T^2} + \frac{4 e_{1,3}}{T} - T f_{1,2}, \\
\theta &= e_{1,3} + f_{1,3}, \quad \theta = c_{1,6} - T^2 c_{1,6} + \frac{c_{1,7}}{T} - T c_{1,7} - c_{1,8} + \frac{c_{1,8}}{T^2} + c_{1,9} - T^2 c_{1,9} + \frac{c_{1,10}}{T} - \\
&\quad T c_{1,10} - c_{1,11} + \frac{c_{1,11}}{T^2} + c_{1,12} - T^2 c_{1,12} + \frac{c_{1,13}}{T} - T c_{1,13} - c_{1,14} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2} - T^2 f_{1,3} \}
\end{aligned}$$

(Alt) In[]:= **{sol} = Solve[eqns, unknowns]**

Solve: Equations may not give solutions for all "solve" variables.

$$\begin{aligned}
\text{(Alt) Out[]} &= \left\{ \left\{ c_{1,4} \rightarrow \theta, c_{1,5} \rightarrow -T c_{1,2} - c_{1,3}, c_{1,6} \rightarrow \theta, c_{1,8} \rightarrow -\frac{1}{2} (1 - T) c_{1,10}, c_{1,9} \rightarrow \theta, \right. \right. \\
&\quad c_{1,11} \rightarrow -T c_{1,7} - \frac{1}{2} (-1 + 3 T) c_{1,10}, c_{1,12} \rightarrow \theta, c_{1,13} \rightarrow \theta, c_{1,14} \rightarrow \theta, d_{1,1} \rightarrow -c_{1,1}, d_{1,2} \rightarrow -c_{1,2}, \\
&\quad d_{1,3} \rightarrow -\frac{c_{1,3}}{T^2}, d_{1,4} \rightarrow \theta, d_{1,5} \rightarrow \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T^2}, d_{1,6} \rightarrow \theta, d_{1,7} \rightarrow -\frac{c_{1,7}}{T} - \frac{(-1 + T) c_{1,10}}{T^2}, \\
&\quad d_{1,8} \rightarrow -\frac{(1 - T) c_{1,10}}{2 T^3}, d_{1,9} \rightarrow \theta, d_{1,10} \rightarrow -\frac{c_{1,10}}{T^2}, d_{1,11} \rightarrow \frac{c_{1,7}}{T^2} - \frac{(-1 - T) c_{1,10}}{2 T^3}, d_{1,12} \rightarrow \theta, \\
&\quad \left. \left. d_{1,13} \rightarrow \theta, d_{1,14} \rightarrow \theta, e_{1,1} \rightarrow \frac{c_{1,3}}{2 T}, e_{1,2} \rightarrow -\frac{c_{1,10}}{T}, e_{1,3} \rightarrow \theta, f_{1,1} \rightarrow -\frac{c_{1,3}}{2 T}, f_{1,2} \rightarrow \frac{c_{1,10}}{T}, f_{1,3} \rightarrow \theta \right\} \right\}
\end{aligned}$$

(Alt) In[]:= **sol /. (a_ -> b_) :-> (a = b)**

$$\begin{aligned}
\text{(Alt) Out[]} &= \left\{ \theta, -T c_{1,2} - c_{1,3}, \theta, -\frac{1}{2} (1 - T) c_{1,10}, \theta, -T c_{1,7} - \frac{1}{2} (-1 + 3 T) c_{1,10}, \theta, \theta, \right. \\
&\quad \theta, -c_{1,1}, -c_{1,2}, -\frac{c_{1,3}}{T^2}, \theta, \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T^2}, \theta, -\frac{c_{1,7}}{T} - \frac{(-1 + T) c_{1,10}}{T^2}, -\frac{(1 - T) c_{1,10}}{2 T^3}, \\
&\quad \left. \theta, -\frac{c_{1,10}}{T^2}, \frac{c_{1,7}}{T^2} - \frac{(-1 - T) c_{1,10}}{2 T^3}, \theta, \theta, \theta, \frac{c_{1,3}}{2 T}, -\frac{c_{1,10}}{T}, \theta, -\frac{c_{1,3}}{2 T}, \frac{c_{1,10}}{T}, \theta \right\}
\end{aligned}$$

$$(Alt) In[*]:= \{R_{1,2}, \bar{R}_{1,2}, CC_1, \overline{CC}_1\}$$

$$(Alt) Out[*]:= \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\mathbf{1}, (-1 + T) x_2 (y_1 - y_2), \right. \right. \\ \in Series \left[\theta, c_{1,1} + x_1 y_1 c_{1,2} + x_2 y_2 (-T c_{1,2} - c_{1,3}) + x_2 y_1 c_{1,3} + x_1 x_2 y_1^2 c_{1,7} - \frac{1}{2} (1 - T) x_2^2 y_1^2 c_{1,10} + \right. \\ \left. \left. x_1 x_2 y_1 y_2 c_{1,10} + x_2^2 y_1 y_2 \left(-T c_{1,7} - \frac{1}{2} (-1 + 3T) c_{1,10} \right) \right], \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\mathbf{1}, \left(-1 + \frac{1}{T} \right) x_2 (y_1 - y_2), \right. \right. \\ \in Series \left[\theta, -c_{1,1} - x_1 y_1 c_{1,2} - \frac{x_2 y_1 c_{1,3}}{T^2} + x_2 y_2 \left(\frac{c_{1,2}}{T} + \frac{c_{1,3}}{T^2} \right) - \frac{(1 - T) x_2^2 y_1^2 c_{1,10}}{2 T^3} - \right. \\ \left. \left. \frac{x_1 x_2 y_1 y_2 c_{1,10}}{T^2} + x_2^2 y_1 y_2 \left(\frac{c_{1,7}}{T^2} - \frac{(-1 - T) c_{1,10}}{2 T^3} \right) + x_1 x_2 y_1^2 \left(-\frac{c_{1,7}}{T} - \frac{(-1 + T) c_{1,10}}{T^2} \right) \right], \right. \\ \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in Series \left[\theta, \frac{c_{1,3}}{2 T} - \frac{x_1 y_1 c_{1,10}}{T} \right] \right], \right. \\ \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in Series \left[\theta, -\frac{c_{1,3}}{2 T} + \frac{x_1 y_1 c_{1,10}}{T} \right] \right] \right\}$$

$$(Alt) In[*]:= c_{1,1} = c_{1,2} = c_{1,3} = c_{1,7} = \theta; c_{1,10} = \mathbf{1}; \\ \{R_{1,2}, \bar{R}_{1,2}, CC_1, \overline{CC}_1\}$$

$$(Alt) Out[*]:= \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\mathbf{1}, (-1 + T) x_2 (y_1 - y_2), \in Series \left[\theta, \frac{1}{2} (-1 + T) x_2^2 y_1^2 + x_1 x_2 y_1 y_2 + \frac{1}{2} (1 - 3T) x_2^2 y_1 y_2 \right], \right. \right. \\ \left. \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\mathbf{1}, \left(-1 + \frac{1}{T} \right) x_2 (y_1 - y_2), \right. \right. \\ \left. \in Series \left[\theta, -\frac{(-1 + T) x_1 x_2 y_1^2}{T^2} - \frac{(1 - T) x_2^2 y_1^2}{2 T^3} - \frac{x_1 x_2 y_1 y_2}{T^2} - \frac{(-1 - T) x_2^2 y_1 y_2}{2 T^3} \right], \right. \\ \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in Series \left[\theta, -\frac{x_1 y_1}{T} \right] \right], \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in Series \left[\theta, \frac{x_1 y_1}{T} \right] \right] \right\}$$

$$(Alt) In[*]:= \{ (R_{1,2} R_{4,3} R_{5,6} // m_{1,4 \rightarrow 1} // m_{2,5 \rightarrow 2} // m_{3,6 \rightarrow 3}) \equiv (R_{2,3} R_{4,5} R_{1,6} // m_{1,4 \rightarrow 1} // m_{2,5 \rightarrow 2} // m_{3,6 \rightarrow 3}), \\ (R_{1,2} \bar{R}_{3,4} // m_{1,3 \rightarrow 1} // m_{2,4 \rightarrow 2}) \equiv \mathbb{E}_{\{\} \rightarrow \{1,2\}} [\mathbf{1}, \theta, \in Series [\theta]], \\ (CC_1 \overline{CC}_2 // m_{1,2 \rightarrow 1}) \equiv \mathbb{E}_{\{\} \rightarrow \{1\}} [\mathbf{1}, \theta, \in Series [\theta]], \\ (CC_3 R_{1,2} // m_{2,3 \rightarrow 2} // m_{2,1 \rightarrow 1}) \equiv (\overline{CC}_3 R_{1,2} // m_{1,3 \rightarrow 1} // m_{1,2 \rightarrow 1}) \}$$

$$(Alt) Out[*]:= \{True, True, True, True\}$$

$$(Alt) In[*]:= \{c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6}, c_{1,7}, c_{1,8}, c_{1,9}, c_{1,10}, c_{1,11}, c_{1,12}, c_{1,13}, c_{1,14}, d_{1,1}, d_{1,2}, d_{1,3}, \\ d_{1,4}, d_{1,5}, d_{1,6}, d_{1,7}, d_{1,8}, d_{1,9}, d_{1,10}, d_{1,11}, d_{1,12}, d_{1,13}, d_{1,14}, e_{1,1}, e_{1,2}, e_{1,3}, f_{1,1}, f_{1,2}, f_{1,3}\}$$

$$(Alt) Out[*]:= \left\{ \theta, \theta, \theta, \theta, \theta, \theta, \theta, \frac{1}{2} (-1 + T), \theta, 1, \frac{1}{2} (1 - 3T), \theta, \theta, \theta, \theta, \theta, \right. \\ \left. \theta, \theta, \theta, \theta, -\frac{-1 + T}{T^2}, -\frac{1 - T}{2 T^3}, \theta, -\frac{1}{T^2}, -\frac{-1 - T}{2 T^3}, \theta, \theta, \theta, \theta, -\frac{1}{T}, \theta, \theta, \frac{1}{T}, \theta \right\}$$

Solving for R, CC, \$k = 2

$$(Alt) In[*]:= \$k = 2;$$

(Alt) In[*]:= **unknowns = Cases** [{ $R_{1,2}$, $\bar{R}_{1,2}$, CC_1 , \overline{CC}_1 }, (c | d | e | f)_{\$k, _}, ∞] // Union

(Alt) Out[*]:= { $C_{2,1}$, $C_{2,2}$, $C_{2,3}$, $C_{2,4}$, $C_{2,5}$, $C_{2,6}$, $C_{2,7}$, $C_{2,8}$, $C_{2,9}$, $C_{2,10}$, $C_{2,11}$, $C_{2,12}$, $C_{2,13}$, $C_{2,14}$, $C_{2,15}$, $C_{2,16}$, $C_{2,17}$, $C_{2,18}$, $C_{2,19}$, $C_{2,20}$, $C_{2,21}$, $C_{2,22}$, $C_{2,23}$, $C_{2,24}$, $C_{2,25}$, $C_{2,26}$, $C_{2,27}$, $C_{2,28}$, $C_{2,29}$, $C_{2,30}$, $d_{2,1}$, $d_{2,2}$, $d_{2,3}$, $d_{2,4}$, $d_{2,5}$, $d_{2,6}$, $d_{2,7}$, $d_{2,8}$, $d_{2,9}$, $d_{2,10}$, $d_{2,11}$, $d_{2,12}$, $d_{2,13}$, $d_{2,14}$, $d_{2,15}$, $d_{2,16}$, $d_{2,17}$, $d_{2,18}$, $d_{2,19}$, $d_{2,20}$, $d_{2,21}$, $d_{2,22}$, $d_{2,23}$, $d_{2,24}$, $d_{2,25}$, $d_{2,26}$, $d_{2,27}$, $d_{2,28}$, $d_{2,29}$, $d_{2,30}$, $e_{2,1}$, $e_{2,2}$, $e_{2,3}$, $e_{2,4}$, $f_{2,1}$, $f_{2,2}$, $f_{2,3}$, $f_{2,4}$ }

(Alt) In[*]:= **Short** [**errors = CF@** { ($R_{1,2}$ $R_{4,3}$ $R_{5,6}$ // $m_{1,4 \rightarrow 1}$ // $m_{2,5 \rightarrow 2}$ // $m_{3,6 \rightarrow 3}$) [[3, -1]] - ($R_{2,3}$ $R_{4,5}$ $R_{1,6}$ // $m_{1,4 \rightarrow 1}$ // $m_{2,5 \rightarrow 2}$ // $m_{3,6 \rightarrow 3}$) [[3, -1]] , ($R_{1,2}$ $\bar{R}_{3,4}$ // $m_{1,3 \rightarrow 1}$ // $m_{2,4 \rightarrow 2}$) [[3, -1]] , (CC_1 \overline{CC}_2 // $m_{1,2 \rightarrow 1}$) [[3, -1]] , (CC_3 $R_{1,2}$ // $m_{2,3 \rightarrow 2}$ // $m_{2,1 \rightarrow 1}$) [[3, -1]] - (\overline{CC}_3 $R_{1,2}$ // $m_{1,3 \rightarrow 1}$ // $m_{1,2 \rightarrow 1}$) [[3, -1]] } , **10**]

(Alt) Out[*]//Short= <<1>>

(Alt) In[*]:= **Short** [#, **10**] & [**eqns = Thread** [$\theta ==$ Union @@ (**CoefficientRules** [#, { x_1 , x_2 , x_3 , y_1 , y_2 , y_3 }] [[; ; , 2]] & /@ errors)]]

(Alt) Out[*]//Short= { $\theta == C_{2,4} - T C_{2,4}$, $\theta == -C_{2,4} + T C_{2,4}$, $\theta == T C_{2,4} - T^2 C_{2,4}$, <<168>>, $\theta == e_{2,4} + f_{2,4}$, $\theta == \frac{1}{2} - \frac{1}{2 T^3} + \frac{1}{2 T^2} - \frac{T}{2} + C_{2,15} - T^3 C_{2,15} + \frac{C_{2,16}}{T} - T^2 C_{2,16} + \frac{C_{2,17}}{T^2} - T C_{2,17} - C_{2,18} + \frac{C_{2,18}}{T^3} + C_{2,19} - T^3 C_{2,19} + \frac{C_{2,20}}{T} - T^2 C_{2,20} + \frac{C_{2,21}}{T^2} - T C_{2,21} - C_{2,22} + \frac{C_{2,22}}{T^3} + C_{2,23} - T^3 C_{2,23} + \frac{C_{2,24}}{T} - T^2 C_{2,24} + \frac{C_{2,25}}{T^2} - T C_{2,25} - C_{2,26} + \frac{C_{2,26}}{T^3} + C_{2,27} - T^3 C_{2,27} + \frac{C_{2,28}}{T} - T^2 C_{2,28} + \frac{C_{2,29}}{T^2} - T C_{2,29} - C_{2,30} + \frac{C_{2,30}}{T^3} + \frac{e_{2,4}}{T^3} - T^3 f_{2,4}$ }

(Alt) In[*]:= **{sol} = Solve[eqns, unknowns]**

Solve: Equations may not give solutions for all "solve" variables.

$$\begin{aligned}
 \text{(Alt) Out[*]} = & \left\{ \left\{ c_{2,4} \rightarrow 0, c_{2,5} \rightarrow -T c_{2,2} - c_{2,3}, c_{2,6} \rightarrow 0, c_{2,8} \rightarrow -\frac{1}{2} (1-T) c_{2,10}, c_{2,9} \rightarrow 0, \right. \right. \\
 & c_{2,11} \rightarrow -\frac{1}{2} - T c_{2,7} - \frac{1}{2} (-1+3T) c_{2,10}, c_{2,12} \rightarrow 0, c_{2,13} \rightarrow 0, c_{2,14} \rightarrow 0, c_{2,15} \rightarrow 0, \\
 & c_{2,17} \rightarrow -((-1+T) c_{2,16}), c_{2,18} \rightarrow -\frac{-1+4T-3T^2}{6T}, c_{2,19} \rightarrow 0, c_{2,20} \rightarrow -\frac{1}{2T}, \\
 & c_{2,21} \rightarrow -\frac{1-3T}{2T}, c_{2,22} \rightarrow -\frac{1-11T+16T^2}{6T} - (T-T^2) c_{2,16}, c_{2,23} \rightarrow 0, c_{2,24} \rightarrow 0, \\
 & c_{2,25} \rightarrow -\frac{1}{2}, c_{2,26} \rightarrow \frac{1}{6} (-1+7T) - T^2 c_{2,16}, c_{2,27} \rightarrow 0, c_{2,28} \rightarrow 0, c_{2,29} \rightarrow 0, c_{2,30} \rightarrow 0, \\
 & d_{2,1} \rightarrow -c_{2,1}, d_{2,2} \rightarrow -c_{2,2}, d_{2,3} \rightarrow -\frac{c_{2,3}}{T^2}, d_{2,4} \rightarrow 0, d_{2,5} \rightarrow \frac{c_{2,2}}{T} + \frac{c_{2,3}}{T^2}, d_{2,6} \rightarrow 0, \\
 & d_{2,7} \rightarrow -\frac{1-T}{T^3} - \frac{c_{2,7}}{T} - \frac{(-1+T) c_{2,10}}{T^2}, d_{2,8} \rightarrow -\frac{-1+T}{2T^4} - \frac{(1-T) c_{2,10}}{2T^3}, d_{2,9} \rightarrow 0, \\
 & d_{2,10} \rightarrow \frac{1}{T^3} - \frac{c_{2,10}}{T^2}, d_{2,11} \rightarrow -\frac{1}{2T^4} + \frac{c_{2,7}}{T^2} - \frac{(-1-T) c_{2,10}}{2T^3}, d_{2,12} \rightarrow 0, d_{2,13} \rightarrow 0, d_{2,14} \rightarrow 0, \\
 & d_{2,15} \rightarrow 0, d_{2,16} \rightarrow -\frac{-1+T}{2T^3} - \frac{c_{2,16}}{T}, d_{2,17} \rightarrow -\frac{3-4T+T^2}{2T^4} - \frac{(-1+T) c_{2,16}}{T^2}, d_{2,18} \rightarrow -\frac{-3+4T-T^2}{6T^5}, \\
 & d_{2,19} \rightarrow 0, d_{2,20} \rightarrow -\frac{1}{2T^3}, d_{2,21} \rightarrow \frac{2}{T^4}, d_{2,22} \rightarrow -\frac{4+T+T^2}{6T^5} - \frac{(1-T) c_{2,16}}{T^3}, d_{2,23} \rightarrow 0, d_{2,24} \rightarrow 0, \\
 & d_{2,25} \rightarrow -\frac{1}{2T^4}, d_{2,26} \rightarrow -\frac{-1+T}{6T^5} + \frac{c_{2,16}}{T^3}, d_{2,27} \rightarrow 0, d_{2,28} \rightarrow 0, d_{2,29} \rightarrow 0, d_{2,30} \rightarrow 0, e_{2,1} \rightarrow \frac{c_{2,3}}{2T}, \\
 & e_{2,2} \rightarrow -\frac{c_{2,10}}{T}, e_{2,3} \rightarrow 0, e_{2,4} \rightarrow 0, f_{2,1} \rightarrow -\frac{c_{2,3}}{2T}, f_{2,2} \rightarrow -\frac{1}{T^2} + \frac{c_{2,10}}{T}, f_{2,3} \rightarrow 0, f_{2,4} \rightarrow 0 \left. \right\} \}
 \end{aligned}$$

(Alt) In[*]:= sol /. (a_ -> b_) :-> (a = b)

$$\begin{aligned}
 \text{(Alt) Out[*]} = & \left\{ \theta, -T c_{2,2} - c_{2,3}, \theta, -\frac{1}{2} (1 - T) c_{2,10}, \theta, -\frac{1}{2} - T c_{2,7} - \frac{1}{2} (-1 + 3 T) c_{2,10}, \theta, \theta, \theta, \theta, \right. \\
 & - \left((-1 + T) c_{2,16} \right), -\frac{-1 + 4 T - 3 T^2}{6 T}, \theta, -\frac{1}{2 T}, -\frac{1 - 3 T}{2 T}, -\frac{1 - 11 T + 16 T^2}{6 T} - (T - T^2) c_{2,16}, \\
 & \theta, \theta, -\frac{1}{2}, \frac{1}{6} (-1 + 7 T) - T^2 c_{2,16}, \theta, \theta, \theta, \theta, -c_{2,1}, -c_{2,2}, -\frac{c_{2,3}}{T^2}, \theta, \frac{c_{2,2}}{T} + \frac{c_{2,3}}{T^2}, \\
 & \theta, -\frac{1 - T}{T^3} - \frac{c_{2,7}}{T} - \frac{(-1 + T) c_{2,10}}{T^2}, -\frac{-1 + T}{2 T^4} - \frac{(1 - T) c_{2,10}}{2 T^3}, \theta, \frac{1}{T^3} - \frac{c_{2,10}}{T^2}, \\
 & -\frac{1}{2 T^4} + \frac{c_{2,7}}{T^2} - \frac{(-1 - T) c_{2,10}}{2 T^3}, \theta, \theta, \theta, \theta, -\frac{-1 + T}{2 T^3} - \frac{c_{2,16}}{T}, -\frac{3 - 4 T + T^2}{2 T^4} - \frac{(-1 + T) c_{2,16}}{T^2}, \\
 & -\frac{-3 + 4 T - T^2}{6 T^5}, \theta, -\frac{1}{2 T^3}, \frac{2}{T^4}, -\frac{4 + T + T^2}{6 T^5} - \frac{(1 - T) c_{2,16}}{T^3}, \theta, \theta, -\frac{1}{2 T^4}, \\
 & \left. -\frac{-1 + T}{6 T^5} + \frac{c_{2,16}}{T^3}, \theta, \theta, \theta, \theta, \frac{c_{2,3}}{2 T}, -\frac{c_{2,10}}{T}, \theta, \theta, -\frac{c_{2,3}}{2 T}, -\frac{1}{T^2} + \frac{c_{2,10}}{T}, \theta, \theta \right\}
 \end{aligned}$$

(Alt) In[*]:= c_{2,1} = c_{2,2} = c_{2,3} = c_{2,7} = c_{2,10} = c_{2,16} = 0;
 {R_{1,2}, R̄_{1,2}, CC₁, C̄C₁}

$$\begin{aligned}
 \text{(Alt) Out[*]} = & \left\{ \mathbb{E}_{\{1\} \rightarrow \{1,2\}} \left[1, (-1 + T) x_2 (y_1 - y_2), \in \text{Series} \left[\theta, \frac{1}{2} (-1 + T) x_2^2 y_1^2 + x_1 x_2 y_1 y_2 + \frac{1}{2} (1 - 3 T) x_2^2 y_1 y_2, \right. \right. \right. \\
 & - \frac{(-1 + 4 T - 3 T^2) x_2^3 y_1^3}{6 T} - \frac{1}{2} x_2^2 y_1 y_2 - \frac{x_1^2 x_2 y_1^2 y_2}{2 T} - \frac{(1 - 3 T) x_1 x_2^2 y_1^2 y_2}{2 T} - \frac{(1 - 11 T + 16 T^2) x_2^3 y_1^2 y_2}{6 T} \\
 & \left. \left. \left. \frac{1}{2} x_1 x_2^2 y_1 y_2^2 + \frac{1}{6} (-1 + 7 T) x_2^3 y_1 y_2^2 \right] \right], \mathbb{E}_{\{1\} \rightarrow \{1,2\}} \left[1, \left(-1 + \frac{1}{T} \right) x_2 (y_1 - y_2), \right. \right. \\
 & \in \text{Series} \left[\theta, -\frac{(-1 + T) x_1 x_2 y_1^2}{T^2} - \frac{(1 - T) x_2^2 y_1^2}{2 T^3} - \frac{x_1 x_2 y_1 y_2}{T^2} - \frac{(-1 - T) x_2^2 y_1 y_2}{2 T^3}, -\frac{(1 - T) x_1 x_2 y_1^2}{T^3} \right. \\
 & - \frac{(-1 + T) x_2^2 y_1^2}{2 T^4} - \frac{(-1 + T) x_1^2 x_2 y_1^3}{2 T^3} - \frac{(3 - 4 T + T^2) x_1 x_2^2 y_1^3}{2 T^4} - \frac{(-3 + 4 T - T^2) x_2^3 y_1^3}{6 T^5} + \frac{x_1 x_2 y_1 y_2}{T^3} \\
 & \left. \left. \left. \frac{x_2^2 y_1 y_2}{2 T^4} - \frac{x_1^2 x_2 y_1^2 y_2}{2 T^3} + \frac{2 x_1 x_2^2 y_1^2 y_2}{T^4} - \frac{(4 + T + T^2) x_2^3 y_1^2 y_2}{6 T^5} - \frac{x_1 x_2^2 y_1 y_2^2}{2 T^4} - \frac{(-1 + T) x_2^3 y_1 y_2^2}{6 T^5} \right] \right], \\
 & \left. \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, -\frac{x_1 y_1}{T}, \theta \right] \right], \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, \frac{x_1 y_1}{T}, -\frac{x_1 y_1}{T^2} \right] \right] \right\}
 \end{aligned}$$

(Alt) In[]:= **Short** [# , 10] &[eqns = **Thread**[0 == **Union** @@ (**CoefficientRules** [# , {x₁, x₂, x₃, y₁, y₂, y₃}][[; ; , 2]] & /@ errors)]]

(Alt) Out[]:= Short=

$$\left\{ \begin{aligned} &0 = c_{3,4} - T c_{3,4}, \ll 418 \gg, \\ &0 = \frac{3}{4} + \frac{5}{12 T^5} - \frac{3}{4 T^4} - \frac{1}{6 T^3} - \frac{5}{12 T} + \frac{T}{6} + c_{3,31} - T^4 c_{3,31} + \frac{c_{3,32}}{T} - T^3 c_{3,32} + \frac{c_{3,33}}{T^2} - T^2 c_{3,33} + \frac{c_{3,34}}{T^3} - \\ &T c_{3,34} - c_{3,35} + \frac{c_{3,35}}{T^4} + c_{3,36} - T^4 c_{3,36} + \frac{c_{3,37}}{T} - T^3 c_{3,37} + \frac{c_{3,38}}{T^2} - T^2 c_{3,38} + \frac{c_{3,39}}{T^3} - T c_{3,39} - \\ &c_{3,40} + \frac{c_{3,40}}{T^4} + c_{3,41} - T^4 c_{3,41} + \frac{c_{3,42}}{T} - T^3 c_{3,42} + \frac{c_{3,43}}{T^2} - T^2 c_{3,43} + \frac{c_{3,44}}{T^3} - T c_{3,44} - c_{3,45} + \frac{c_{3,45}}{T^4} + \\ &c_{3,46} - T^4 c_{3,46} + \frac{c_{3,47}}{T} - T^3 c_{3,47} + \frac{c_{3,48}}{T^2} - T^2 c_{3,48} + \frac{c_{3,49}}{T^3} - T c_{3,49} - c_{3,50} + \frac{c_{3,50}}{T^4} + c_{3,51} - \\ &T^4 c_{3,51} + \frac{c_{3,52}}{T} - T^3 c_{3,52} + \frac{c_{3,53}}{T^2} - T^2 c_{3,53} + \frac{c_{3,54}}{T^3} - T c_{3,54} - c_{3,55} + \frac{c_{3,55}}{T^4} + \frac{e_{3,5}}{T^4} - T^4 f_{3,5} \end{aligned} \right\}$$

(Alt) In[]:= {sol} = **Solve**[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{ \left\{ \begin{aligned} &c_{3,4} \rightarrow 0, c_{3,5} \rightarrow -T c_{3,2} - c_{3,3}, c_{3,6} \rightarrow 0, c_{3,8} \rightarrow -\frac{1}{2} (1 - T) c_{3,10}, c_{3,9} \rightarrow 0, \\ &c_{3,11} \rightarrow -T c_{3,7} - \frac{1}{2} (-1 + 3 T) c_{3,10}, c_{3,12} \rightarrow 0, c_{3,13} \rightarrow 0, c_{3,14} \rightarrow 0, c_{3,15} \rightarrow 0, \\ &c_{3,17} \rightarrow -((-1 + T) c_{3,16}), c_{3,18} \rightarrow -\frac{1 - T}{6 T}, c_{3,19} \rightarrow 0, c_{3,20} \rightarrow 0, c_{3,21} \rightarrow \frac{1}{2 T}, \\ &c_{3,22} \rightarrow -\frac{-2 + 5 T}{2 T} - (T - T^2) c_{3,16}, c_{3,23} \rightarrow 0, c_{3,24} \rightarrow 0, c_{3,25} \rightarrow 0, c_{3,26} \rightarrow \frac{5}{6} - T^2 c_{3,16}, c_{3,27} \rightarrow 0, \\ &c_{3,28} \rightarrow 0, c_{3,29} \rightarrow 0, c_{3,30} \rightarrow 0, c_{3,31} \rightarrow 0, c_{3,33} \rightarrow -\frac{3}{2} (-1 + T) c_{3,32}, c_{3,34} \rightarrow -((-1 + 2 T - T^2) c_{3,32}), \\ &c_{3,35} \rightarrow -\frac{1 - 12 T + 27 T^2 - 16 T^3}{24 T^2}, c_{3,36} \rightarrow 0, c_{3,37} \rightarrow \frac{1}{6 T^2}, c_{3,38} \rightarrow -\frac{-1 + 3 T}{4 T^2}, \\ &c_{3,39} \rightarrow -\frac{-1 + 11 T - 16 T^2}{6 T^2}, c_{3,40} \rightarrow -\frac{-1 + 31 T - 131 T^2 + 125 T^3}{24 T^2} - (T - 2 T^2 + T^3) c_{3,32}, c_{3,41} \rightarrow 0, \\ &c_{3,42} \rightarrow 0, c_{3,43} \rightarrow \frac{1}{T}, c_{3,44} \rightarrow -\frac{-5 + 23 T}{6 T}, c_{3,45} \rightarrow -\frac{-5 + 69 T - 142 T^2}{24 T} + \frac{3}{2} (-1 + T) T^2 c_{3,32}, c_{3,46} \rightarrow 0, \\ &c_{3,47} \rightarrow 0, c_{3,48} \rightarrow 0, c_{3,49} \rightarrow \frac{1}{6}, c_{3,50} \rightarrow \frac{1}{24} (1 - 15 T) - T^3 c_{3,32}, c_{3,51} \rightarrow 0, c_{3,52} \rightarrow 0, c_{3,53} \rightarrow 0, \\ &c_{3,54} \rightarrow 0, c_{3,55} \rightarrow 0, d_{3,1} \rightarrow -c_{3,1}, d_{3,2} \rightarrow -c_{3,2}, d_{3,3} \rightarrow -\frac{c_{3,3}}{T^2}, d_{3,4} \rightarrow 0, d_{3,5} \rightarrow \frac{c_{3,2}}{T} + \frac{c_{3,3}}{T^2}, \\ &d_{3,6} \rightarrow 0, d_{3,7} \rightarrow -\frac{-1 + T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1 + T) c_{3,10}}{T^2}, d_{3,8} \rightarrow -\frac{1 - T}{2 T^5} - \frac{(1 - T) c_{3,10}}{2 T^3}, d_{3,9} \rightarrow 0, \\ &d_{3,10} \rightarrow -\frac{1}{T^4} - \frac{c_{3,10}}{T^2}, d_{3,11} \rightarrow \frac{1}{2 T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1 - T) c_{3,10}}{2 T^3}, d_{3,12} \rightarrow 0, d_{3,13} \rightarrow 0, d_{3,14} \rightarrow 0, d_{3,15} \rightarrow 0, \\ &d_{3,16} \rightarrow -\frac{1 - T}{T^4} - \frac{c_{3,16}}{T}, d_{3,17} \rightarrow -\frac{-7 + 9 T - 2 T^2}{2 T^5} - \frac{(-1 + T) c_{3,16}}{T^2}, d_{3,18} \rightarrow -\frac{7 - 9 T + 2 T^2}{6 T^6}, d_{3,19} \rightarrow 0, \end{aligned} \right\} \right\}$$

$$\begin{aligned}
d_{3,20} &\rightarrow \frac{1}{T^4}, d_{3,21} \rightarrow -\frac{9-T}{2T^5}, d_{3,22} \rightarrow \frac{3}{2T^6} - \frac{(1-T)c_{3,16}}{T^3}, d_{3,23} \rightarrow 0, d_{3,24} \rightarrow 0, d_{3,25} \rightarrow \frac{1}{T^5}, \\
d_{3,26} &\rightarrow -\frac{1}{3T^6} + \frac{c_{3,16}}{T^3}, d_{3,27} \rightarrow 0, d_{3,28} \rightarrow 0, d_{3,29} \rightarrow 0, d_{3,30} \rightarrow 0, d_{3,31} \rightarrow 0, d_{3,32} \rightarrow -\frac{-1+T}{6T^4} - \frac{c_{3,32}}{T}, \\
d_{3,33} &\rightarrow -\frac{2-3T+T^2}{T^5} - \frac{3(-1+T)c_{3,32}}{2T^2}, d_{3,34} \rightarrow -\frac{-16+27T-12T^2+T^3}{6T^6} - \frac{(1-2T+T^2)c_{3,32}}{T^3}, \\
d_{3,35} &\rightarrow -\frac{16-27T+12T^2-T^3}{24T^7}, d_{3,36} \rightarrow 0, d_{3,37} \rightarrow -\frac{1}{6T^4}, d_{3,38} \rightarrow -\frac{-3+T}{T^5}, \\
d_{3,39} &\rightarrow \frac{3(-3+T)}{2T^6}, d_{3,40} \rightarrow -\frac{-27+5T-T^2-T^3}{24T^7} - \frac{(-1+2T-T^2)c_{3,32}}{T^4}, d_{3,41} \rightarrow 0, \\
d_{3,42} &\rightarrow 0, d_{3,43} \rightarrow -\frac{1}{T^5}, d_{3,44} \rightarrow \frac{2}{T^6}, d_{3,45} \rightarrow -\frac{12-T-5T^2}{24T^7} + \frac{3(-1+T)c_{3,32}}{2T^4}, \\
d_{3,46} &\rightarrow 0, d_{3,47} \rightarrow 0, d_{3,48} \rightarrow 0, d_{3,49} \rightarrow -\frac{1}{6T^6}, d_{3,50} \rightarrow -\frac{-1-T}{24T^7} + \frac{c_{3,32}}{T^4}, d_{3,51} \rightarrow 0, \\
d_{3,52} &\rightarrow 0, d_{3,53} \rightarrow 0, d_{3,54} \rightarrow 0, d_{3,55} \rightarrow 0, e_{3,1} \rightarrow \frac{c_{3,3}}{2T}, e_{3,2} \rightarrow -\frac{c_{3,10}}{T}, e_{3,3} \rightarrow 0, \\
e_{3,4} &\rightarrow 0, e_{3,5} \rightarrow 0, f_{3,1} \rightarrow -\frac{c_{3,3}}{2T}, f_{3,2} \rightarrow \frac{1}{T^3} + \frac{c_{3,10}}{T}, f_{3,3} \rightarrow 0, f_{3,4} \rightarrow 0, f_{3,5} \rightarrow 0 \}
\end{aligned}$$

(Alt) In[]:= sol /. (a_ -> b_) :-> (a = b)

$$\begin{aligned}
 \text{(Alt) Out[*]} = & \left\{ \theta, -T c_{3,2} - c_{3,3}, \theta, -\frac{1}{2} (1 - T) c_{3,10}, \theta, -T c_{3,7} - \frac{1}{2} (-1 + 3 T) c_{3,10}, \theta, \theta, \theta, \theta, \right. \\
 & - ((-1 + T) c_{3,16}), -\frac{1 - T}{6 T}, \theta, \theta, \frac{1}{2 T}, -\frac{-2 + 5 T}{2 T} - (T - T^2) c_{3,16}, \theta, \theta, \theta, \frac{5}{6} - T^2 c_{3,16}, \theta, \\
 & \theta, \theta, \theta, \theta, -\frac{3}{2} (-1 + T) c_{3,32}, -(((-1 + 2 T - T^2) c_{3,32})), -\frac{1 - 12 T + 27 T^2 - 16 T^3}{24 T^2}, \theta, \frac{1}{6 T^2}, \\
 & -\frac{-1 + 3 T}{4 T^2}, -\frac{-1 + 11 T - 16 T^2}{6 T^2}, -\frac{-1 + 31 T - 131 T^2 + 125 T^3}{24 T^2} - (T - 2 T^2 + T^3) c_{3,32}, \theta, \theta, \frac{1}{T}, \\
 & -\frac{-5 + 23 T}{6 T}, -\frac{-5 + 69 T - 142 T^2}{24 T} + \frac{3}{2} (-1 + T) T^2 c_{3,32}, \theta, \theta, \theta, \frac{1}{6}, \frac{1}{24} (1 - 15 T) - T^3 c_{3,32}, \\
 & \theta, \theta, \theta, \theta, \theta, -c_{3,1}, -c_{3,2}, -\frac{c_{3,3}}{T^2}, \theta, \frac{c_{3,2}}{T} + \frac{c_{3,3}}{T^2}, \theta, -\frac{-1 + T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1 + T) c_{3,10}}{T^2}, \\
 & -\frac{1 - T}{2 T^5} - \frac{(1 - T) c_{3,10}}{2 T^3}, \theta, -\frac{1}{T^4} - \frac{c_{3,10}}{T^2}, \frac{1}{2 T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1 - T) c_{3,10}}{2 T^3}, \theta, \theta, \theta, \theta, \\
 & -\frac{1 - T}{T^4} - \frac{c_{3,16}}{T}, -\frac{-7 + 9 T - 2 T^2}{2 T^5} - \frac{(-1 + T) c_{3,16}}{T^2}, -\frac{7 - 9 T + 2 T^2}{6 T^6}, \theta, \frac{1}{T^4}, -\frac{9 - T}{2 T^5}, \\
 & \frac{3}{2 T^6} - \frac{(1 - T) c_{3,16}}{T^3}, \theta, \theta, \frac{1}{T^5}, -\frac{1}{3 T^6} + \frac{c_{3,16}}{T^3}, \theta, \theta, \theta, \theta, \theta, -\frac{-1 + T}{6 T^4} - \frac{c_{3,32}}{T}, \\
 & -\frac{2 - 3 T + T^2}{T^5} - \frac{3 (-1 + T) c_{3,32}}{2 T^2}, -\frac{-16 + 27 T - 12 T^2 + T^3}{6 T^6} - \frac{(1 - 2 T + T^2) c_{3,32}}{T^3}, \\
 & -\frac{16 - 27 T + 12 T^2 - T^3}{24 T^7}, \theta, -\frac{1}{6 T^4}, -\frac{-3 + T}{T^5}, \frac{3 (-3 + T)}{2 T^6}, -\frac{-27 + 5 T - T^2 - T^3}{24 T^7} - \frac{(-1 + 2 T - T^2) c_{3,32}}{T^4}, \\
 & \theta, \theta, -\frac{1}{T^5}, \frac{2}{T^6}, -\frac{12 - T - 5 T^2}{24 T^7} + \frac{3 (-1 + T) c_{3,32}}{2 T^4}, \theta, \theta, \theta, -\frac{1}{6 T^6}, -\frac{-1 - T}{24 T^7} + \frac{c_{3,32}}{T^4}, \\
 & \left. \theta, \theta, \theta, \theta, \theta, \frac{c_{3,3}}{2 T}, -\frac{c_{3,10}}{T}, \theta, \theta, \theta, -\frac{c_{3,3}}{2 T}, \frac{1}{T^3} + \frac{c_{3,10}}{T}, \theta, \theta, \theta \right\}
 \end{aligned}$$

(Alt) In[*]= $C_{3,1} = C_{3,2} = C_{3,3} = C_{3,7} = C_{3,10} = C_{3,16} = C_{3,32} = \mathbf{0};$

$\{R_{1,2}, \bar{R}_{1,2}, CC_1, \bar{CC}_1\}$

(Alt) Out[*]= $\{E_{\{\} \rightarrow \{1,2\}} \left[1, (-1 + T) x_2 (y_1 - y_2), \right.$

$$\begin{aligned} & \in \text{Series} \left[0, \frac{1}{2} (-1 + T) x_2^2 y_1^2 + x_1 x_2 y_1 y_2 + \frac{1}{2} (1 - 3 T) x_2^2 y_1 y_2, - \frac{(-1 + 4 T - 3 T^2) x_2^3 y_1^3}{6 T} - \frac{1}{2} x_2^2 y_1 y_2 - \right. \\ & \frac{x_1^2 x_2 y_1^2 y_2}{2 T} - \frac{(1 - 3 T) x_1 x_2^2 y_1^2 y_2}{2 T} - \frac{(1 - 11 T + 16 T^2) x_2^3 y_1^2 y_2}{6 T} - \frac{1}{2} x_1 x_2^2 y_1 y_2^2 + \frac{1}{6} (-1 + 7 T) x_2^3 y_1 y_2^2, \\ & - \frac{(1 - T) x_2^3 y_1^3}{6 T} - \frac{(1 - 12 T + 27 T^2 - 16 T^3) x_2^4 y_1^4}{24 T^2} + \frac{x_1 x_2^2 y_1^2 y_2}{2 T} - \frac{(-2 + 5 T) x_2^3 y_1^2 y_2}{2 T} + \frac{x_1^3 x_2 y_1^3 y_2}{6 T^2} - \\ & \frac{(-1 + 3 T) x_1^2 x_2^2 y_1^3 y_2}{4 T^2} - \frac{(-1 + 11 T - 16 T^2) x_1 x_2^3 y_1^3 y_2}{6 T^2} - \frac{(-1 + 31 T - 131 T^2 + 125 T^3) x_2^4 y_1^3 y_2}{24 T^2} + \\ & \frac{5}{6} x_2^3 y_1 y_2^2 + \frac{x_1^2 x_2^2 y_1^2 y_2^2}{T} - \frac{(-5 + 23 T) x_1 x_2^3 y_1^2 y_2^2}{6 T} - \frac{(-5 + 69 T - 142 T^2) x_2^4 y_1^2 y_2^2}{24 T} + \\ & \left. \frac{1}{6} x_1 x_2^3 y_1 y_2^2 + \frac{1}{24} (1 - 15 T) x_2^4 y_1 y_2^2 \right], E_{\{\} \rightarrow \{1,2\}} \left[1, \left(-1 + \frac{1}{T} \right) x_2 (y_1 - y_2), \right. \\ & \in \text{Series} \left[0, - \frac{(-1 + T) x_1 x_2 y_1^2}{T^2} - \frac{(1 - T) x_2^2 y_1^2}{2 T^3} - \frac{x_1 x_2 y_1 y_2}{T^2} - \frac{(-1 - T) x_2^2 y_1 y_2}{2 T^3}, \right. \\ & - \frac{(1 - T) x_1 x_2 y_1^2}{T^3} - \frac{(-1 + T) x_2^2 y_1^2}{2 T^4} - \frac{(-1 + T) x_1^2 x_2 y_1^3}{2 T^3} - \frac{(3 - 4 T + T^2) x_1 x_2^2 y_1^3}{2 T^4} - \\ & \frac{(-3 + 4 T - T^2) x_2^3 y_1^3}{6 T^5} + \frac{x_1 x_2 y_1 y_2}{T^3} - \frac{x_2^2 y_1 y_2}{2 T^4} - \frac{x_1^2 x_2 y_1^2 y_2}{2 T^3} + \frac{2 x_1 x_2^2 y_1^2 y_2}{T^4} - \frac{(4 + T + T^2) x_2^3 y_1^2 y_2}{6 T^5} - \\ & \frac{x_1 x_2^2 y_1 y_2^2}{2 T^4} - \frac{(-1 + T) x_2^3 y_1 y_2^2}{6 T^5}, - \frac{(-1 + T) x_1 x_2 y_1^2}{T^4} - \frac{(1 - T) x_2^2 y_1^2}{2 T^5} - \frac{(1 - T) x_1^2 x_2 y_1^3}{T^4} - \\ & \frac{(-7 + 9 T - 2 T^2) x_1 x_2^2 y_1^3}{2 T^5} - \frac{(7 - 9 T + 2 T^2) x_2^3 y_1^3}{6 T^6} - \frac{(-1 + T) x_1^3 x_2 y_1^4}{6 T^4} - \frac{(2 - 3 T + T^2) x_1^2 x_2^2 y_1^4}{T^5} - \\ & \frac{(-16 + 27 T - 12 T^2 + T^3) x_1 x_2^3 y_1^4}{6 T^6} - \frac{(16 - 27 T + 12 T^2 - T^3) x_2^4 y_1^4}{24 T^7} - \frac{x_1 x_2 y_1 y_2}{T^4} + \frac{x_2^2 y_1 y_2}{2 T^5} + \\ & \frac{x_1^2 x_2 y_1^2 y_2}{T^4} - \frac{(9 - T) x_1 x_2^2 y_1^2 y_2}{2 T^5} + \frac{3 x_2^3 y_1^2 y_2}{2 T^6} - \frac{x_1^3 x_2 y_1^3 y_2}{6 T^4} - \frac{(-3 + T) x_1^2 x_2^2 y_1^3 y_2}{T^5} + \\ & \frac{3 (-3 + T) x_1 x_2^3 y_1^3 y_2}{2 T^6} - \frac{(-27 + 5 T - T^2 - T^3) x_2^4 y_1^3 y_2}{24 T^7} + \frac{x_1 x_2^2 y_1 y_2^2}{T^5} - \frac{x_2^3 y_1 y_2^2}{3 T^6} - \\ & \left. \frac{x_1^2 x_2^2 y_1^2 y_2^2}{T^5} + \frac{2 x_1 x_2^3 y_1^2 y_2^2}{T^6} - \frac{(12 - T - 5 T^2) x_2^4 y_1^2 y_2^2}{24 T^7} - \frac{x_1 x_2^3 y_1 y_2^3}{6 T^6} - \frac{(-1 - T) x_2^4 y_1 y_2^3}{24 T^7} \right] \}, \end{aligned}$$

$E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \mathbf{0}, \in \text{Series} \left[\mathbf{0}, - \frac{x_1 y_1}{T}, \mathbf{0}, \mathbf{0} \right] \right],$

$E_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \mathbf{0}, \in \text{Series} \left[\mathbf{0}, \frac{x_1 y_1}{T}, - \frac{x_1 y_1}{T^2}, \frac{x_1 y_1}{T^3} \right] \right] \}$

(Alt) In[]:=

```

RVK[pd_PD] := Module[{n, xs, x, rots, front = {0}, k},
  n = Length@pd; rots = Table[0, {2 n}];
  xs = Cases[pd, x_X => { Xp[x[[4]], x[[1]] PositiveQ@x
                        | Xm[x[[2]], x[[1]] True };
  For[k = 0, k < 2 n, ++k, If[k == 0 ∨ FreeQ[front, -k],
    front = Flatten[front /. k → (xs /. {
      Xp[k + 1, L_] | Xm[L_, k + 1] => {L, k + 1, 1 - L},
      Xp[L_, k + 1] | Xm[k + 1, L_] => (++)rots[[L]; {1 - L, k + 1, L}
    }]],
    Cases[front, k | -k] /. {k, -k} => --rots[[k + 1]];
  ];
  RVK[xs, rots ] ];
RVK[K_] := RVK[PD[K]];

```

(Alt) In[]:=

```

rot[i_, 0] := E[{}->{i}][1, 0, eSeries@0];
rot[i_, n_] := Module[{j},
  rot[i, n] = If[n > 0, rot[i, n - 1] CCj, rot[i, n + 1] C̄Cj] // mi,j->i;

```

(Alt) In[]:=

```

Z[K_] := Z[RVK@K];
Z[rvk_RVK] := (*Z[rvk] =*)
Module[{todo, n, rots, ζ, done, st, cx, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[{}->{0}] [1, 0, eSeries@0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[{} != ($M = todo),
    {cx} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    {i, j} = List@@cx;
    ζ1 = Switch[Head[cx],
      Xp, (Ri,j  $\overline{\text{Kink}}_k$ ) // mj,k→j,
      Xm, ( $\overline{R}$ i,j Kinkk) // mj,k→j
    ];
    ζ1 = (rot[k, rots[[i]] ζ1) // mk,i→i; rots[[i]] = 0;
    ζ1 = (ζ1 rot[k, rots[[i + 1]]) // mi,k→i; rots[[i + 1]] = 0;
    ζ1 = (rot[k, rots[[j]] ζ1) // mk,j→j; rots[[j]] = 0;
    ζ1 = (ζ1 rot[k, rots[[j + 1]]) // mj,k→j; rots[[j + 1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1], ζ = ζ // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1], ζ = ζ // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]];
    done = done ∪ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, cx]
  ];
  CF /@ (ζ (* /. {x0→x, y0→y, a0→a}*))
]

```

(Alt) In[]:=

```

BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
  Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]

```

(Alt) Out[]:=

Show Profile Monitor

(Alt) In[]:= \$k = 1

(Alt) Out[]:= 1

(Alt) In[]:=

```

NewBit[K_] := Module[{Alex = Alexander[K][T]},
  T3  $\frac{\text{Alex}^2}{T - 1}$  Z[K][[3, 2]] // Factor]

```

```
(Alt) In[ ]:= NewBit /@ AllKnots[{3, 5}]
```

```
KnotTheory: Loading precomputed data in PD4Knots`.
```

```
(Alt) Out[ ]:= {2 - T + T^2, (1 + T) (1 - 3 T + T^2),  $\frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}$ , 9 - 11 T + 7 T^2 - T^3}
```

```
(Alt) In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
```

```
Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
```

```
Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

```
(Alt) Out[ ]:= True
```

```
(Alt) Out[ ]:= {58.25, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3}
```

```
(Alt) In[ ]:= PrintProfile[]
```

```
(Alt) Out[ ]:= ProfileRoot is root. Profiled time: 97.563
```

```
( 24) 0.031/ 0.031 above CF
```

```
( 237) 1.694/ 7.169 above Zip1
```

```
( 237) 0.921/ 47.090 above Zip2
```

```
( 237) 36.929/ 43.273 above Zip3
```

```
CF: called 3816 times, time in 58.019/58.019
```

```
( 24) 0.031/ 0.031 under ProfileRoot
```

```
( 1185) 5.475/ 5.475 under Zip1
```

```
( 1185) 46.169/ 46.169 under Zip2
```

```
( 1422) 6.344/ 6.344 under Zip3
```

```
Zip3: called 237 times, time in 36.929/43.273
```

```
( 237) 36.929/ 43.273 under ProfileRoot
```

```
( 1422) 6.344/ 6.344 above CF
```

```
Zip1: called 237 times, time in 1.694/7.169
```

```
( 237) 1.694/ 7.169 under ProfileRoot
```

```
( 1185) 5.475/ 5.475 above CF
```

```
Zip2: called 237 times, time in 0.921/47.09
```

```
( 237) 0.921/ 47.090 under ProfileRoot
```

```
( 1185) 46.169/ 46.169 above CF
```