```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BBS"];
srcdir = "C:\\drorbn\\BBS\\shots\\"
```

```
C:\drorbn\BBS\shots\
```

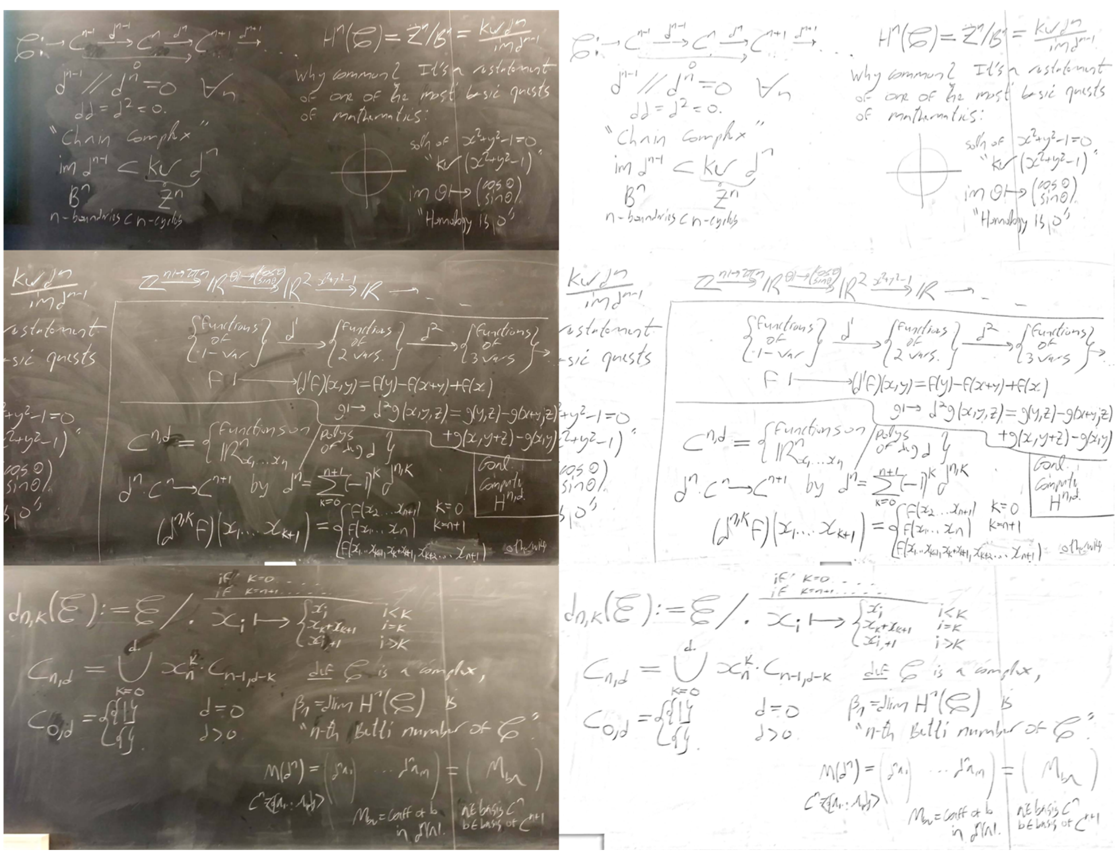
```
fnames = FileNames["17-1750-*", srcdir][[-3 ;; -1]]
```

```
{C:\drorbn\BBS\shots\17-1750-171020-123519.jpg,
 C:\drorbn\BBS\shots\17-1750-171020-123520.jpg,
 C:\drorbn\BBS\shots\17-1750-171020-123521.jpg}
```

```
BBImprove[shot_] := Module[{diag, s1, s2},
   diag = Norm[ImageDimensions[shot]];
   s1 = ColorConvert[shot, "Grayscale"];
   s2 = ImageAdjust[
      ImageApply[Max[#1 - #2, 0] &, {s1, Blur[s1, 0.01 diag]}] ] // ColorNegate;
   ImageAdjust[s2, 0.05]
  ];
```

```
origs = Import /@ fnames;
imps = BBImprove /@ origs;
```

```
ImageAssemble[Transpose[{origs, imps}]] // Rasterize
```




```
HomologyBBS = ImageAssemble[List /@ imps]
```

$$\mathcal{C}: \to C^{n-1} \xrightarrow{d^{n-1}} C^n \xrightarrow{d^n} C^{n+1} \xrightarrow{d^{n+1}} \cdots$$

$d^{n-1} /\!/ d^n = 0 \quad \forall n$

$dd = d^2 = 0.$

"Chain complex"

$\operatorname{im} d^{n-1} \subset \ker d^n$

$B^n \qquad Z^n$

n-boundaries ⊂ n-cycles

$$H^n(\mathcal{C}) = Z^n/B^n = \frac{\ker d^n}{\operatorname{im} d^{n-1}}$$

Why common? It's a restatement of one of the most basic quests of mathematics:

soln of $x^2+y^2-1=0$

"$\ker(x^2+y^2-1)$"

$\operatorname{im} \theta \mapsto \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$

"Homology is 0"

$\frac{\ker d^n}{\operatorname{im} d^{n-1}}$ restatement -sic quests $+y^2-1=0$ $^2+y^2-1)$ $\cos\theta$ $\sin\theta$ is 0"

$$\mathbb{Z} \xrightarrow{n \mapsto 2\pi n} \mathbb{R} \xrightarrow{\theta \mapsto \binom{\cos\theta}{\sin\theta}} \mathbb{R}^2 \xrightarrow{x^2+y^2-1} \mathbb{R} \to \cdots$$

$$\left\{\begin{matrix}\text{Functions} \\ \text{of} \\ \text{1-var}\end{matrix}\right\} \xrightarrow{d^1} \left\{\begin{matrix}\text{Functions} \\ \text{of} \\ \text{2 vars.}\end{matrix}\right\} \xrightarrow{d^2} \left\{\begin{matrix}\text{Functions} \\ \text{of} \\ \text{3 vars}\end{matrix}\right\} \to \cdots$$

$f \mapsto (d^1 f)(x,y) = f(y) - f(x+y) + f(x)$

$g \mapsto d^2 g(x,y,z) = g(y,z) - g(x+y,z) + g(x,y+z) - g(x,y)$

$$C^{n,d} = \left\{ \text{Functions on } \mathbb{R}^n_{x_1 \ldots x_n} \,/\, \text{polys of deg } d \right\}$$

$$d^n: C^n \to C^{n+1} \text{ by } d^n = \sum_{k=0}^{n+1} (-1)^k d^{n,k}$$

$$(d^{n,k} f)(x_1 \ldots x_{k+1}) = \begin{cases} f(x_2 \ldots x_{n+1}) & k=0 \\ f(x_1 \ldots x_n) & k=n+1 \\ f(x_1 \ldots x_{k-1}, x_k + x_{k+1}, x_{k+2} \ldots x_{n+1}) & \text{otherwise} \end{cases}$$

Goal: Compute $H^{n,d}$.

if $k=0$ ....
if $k=n+1$ ....

$$d_{n,k}(\mathcal{E}) := \mathcal{E} /.\ x_i \mapsto \begin{cases} x_i & i<k \\ x_k + x_{k+1} & i=k \\ x_{i+1} & i>k \end{cases}$$

$$C_{n,d} = \bigcup_{k=0}^{d} x_n^k \cdot C_{n-1,d-k}$$

$$C_{0,d} = \begin{cases} \{\{\,\}\} & d=0 \\ \{\} & d>0 \end{cases}$$

def $\mathcal{C}$ is a complex, $\beta_n = \dim H^n(\mathcal{C})$ is "n-th Betti number of $\mathcal{C}$".

$$M(d^n) = \begin{pmatrix} da_1 & \cdots & da_m \end{pmatrix} = \begin{pmatrix} M_{ba} \end{pmatrix}$$

$C^n = \langle a_1 \ldots a_m \rangle$

M_{ba} = coeff of b in $d(a)$.

$a \in$ basis C^n, $b \in$ basis of C^{n+1}

```
Export["HomologyBBS.png", HomologyBBS]
HomologyBBS.png
```