```
ij[257]
```

```
{2, 57}
```

```
b[e[102], e[203]]
```

```
e[103]
```

```
w[101, 203, ...]
```

```
CanonicalForm[W[102, 101, 101]]
```

```
RW[102] - 2 RW[101, 102] + RW[101, 101, 102]
```

```
diag = Diag[ar[1, 3], ar[4, 2]]
```

```
Diag[ar[1, 3], ar[4, 2]]
```

```
Expand[CanonicalForm[
  Distribute[diag /. ar[i_, j_] :>
      1 / 2 PutAt[i, j, 101, 101] + PutAt[i, j, 102, 201] + 1 / 2 PutAt[i, j, 202, 202]
   ] /. d_Diag :> Times[
     Times @@ (d /. _PutAt -> 1),
     (
      w = Table[0, {2 Length[d]}];
      d /. PutAt[i_, j_, alpha1_, alpha2_] :> (
         w[[i]] = alpha1; w[[j]] = alpha2
        );
      W @@ w
     )
    ]
 ]]
```

$$-\frac{1}{2}\,RW[101, 101] + RW[101, 202] + 2\,RW[102, 201] - \frac{1}{2}\,RW[202, 202] - \frac{1}{2}\,RW[101, 101, 101] + \frac{1}{2}\,RW[101, 101, 202] - 2\,RW[101, 102, 201] - \frac{1}{2}\,RW[101, 202, 202] + 2\,RW[102, 201, 202] + \frac{1}{2}\,RW[202, 202, 202] + \frac{1}{4}\,RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{2}\,RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + RW[102, 201, 202, 202] + \frac{1}{4}\,RW[202, 202, 202, 202]$$

```
UGL2[ (Diag[ar[1, 2], ar[3, 4]] + Diag[ar[1, 3], ar[2, 4]] - 2 Diag[ar[1, 4], ar[2, 3]])]
```

```
0
```

```
(rels = {Diag[ar[1, 2], ar[3, 4]] + Diag[ar[1, 3], ar[2, 4]] - 2 Diag[ar[1, 4], ar[2, 3]],
    Diag[ar[1, 2], ar[4, 3]] - Diag[ar[1, 3], ar[2, 4]] + Diag[ar[1, 4], ar[2, 3]] -
     Diag[ar[1, 4], ar[3, 2]], -Diag[ar[1, 3], ar[2, 4]] +
     Diag[ar[1, 4], ar[2, 3]] - Diag[ar[1, 4], ar[3, 2]] + Diag[ar[2, 1], ar[3, 4]],
    -Diag[ar[2, 1], ar[3, 4]] + Diag[ar[2, 3], ar[4, 1]] + Diag[ar[3, 1], ar[4, 2]] -
     Diag[ar[3, 2], ar[4, 1]], -Diag[ar[1, 2], ar[4, 3]] +
     Diag[ar[2, 3], ar[4, 1]] + Diag[ar[3, 1], ar[4, 2]] - Diag[ar[3, 2], ar[4, 1]],
    -Diag[ar[2, 1], ar[4, 3]] - Diag[ar[3, 1], ar[4, 2]] + 2 Diag[ar[3, 2], ar[4, 1]]}) // UGL2
```

```
{0, 0, 0, 0, 0, 0}
```

```
rels[[3]]
```

```
-Diag[ar[1, 3], ar[2, 4]] + Diag[ar[1, 4], ar[2, 3]] -
 Diag[ar[1, 4], ar[3, 2]] + Diag[ar[2, 1], ar[3, 4]]
```

```
rels[[3]] // UGL2
```

```
0
```

```
W[102, 101, 201] // CanonicalForm
```

```
-RW[102, 201] + RW[101, 102, 201]
```

```
(rels // UGL2) /. RW → W // CanonicalForm
```

```
{0, 0, 0, 0, 0, 0}
```

```
rels = R /@ Diagrams[R6T + 1 ar];
```

```
UGL2 /@ rels
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
diags = Diagrams[2 ar]
```

```
{Diag[ar[1, 2], ar[3, 4]], Diag[ar[1, 2], ar[4, 3]], Diag[ar[2, 1], ar[3, 4]],
 Diag[ar[2, 1], ar[4, 3]], Diag[ar[1, 3], ar[2, 4]], Diag[ar[1, 3], ar[4, 2]],
 Diag[ar[2, 4], ar[3, 1]], Diag[ar[3, 1], ar[4, 2]], Diag[ar[1, 4], ar[2, 3]],
 Diag[ar[1, 4], ar[3, 2]], Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]]}
```

```
outs = UGL2 /@ diags;
```

```
RWs = Union[Cases[outs, _RW, Infinity]]
```

```
{RW[101], RW[202], RW[101, 101], RW[101, 202], RW[102, 201], RW[202, 202],
 RW[101, 101, 101], RW[101, 101, 202], RW[101, 102, 201], RW[101, 202, 202],
 RW[102, 201, 202], RW[202, 202, 202], RW[101, 101, 101, 101], RW[101, 101, 102, 201],
 RW[101, 101, 202, 202], RW[102, 102, 201, 201], RW[102, 201, 202, 202], RW[202, 202, 202, 202]}
```

```
Normal[mat = SparseArray[
    Join @@ Table[
      t = outs[[i]];
      {i, Position[RWs, #][[1, 1]]} → Coefficient[t, #] & /@ Cases[{t}, _RW, Infinity],
      {i, Length[outs]}
     ],
    {Length[outs], Length[RWs]}
   ]] // MatrixForm
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 0 & 0 & 2 & 0 & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 0 & 0 & 2 & 0 & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 1 & -2 & 2 & 1 & -1 & 1 & -3 & -1 & 3 & 1 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & -\frac{1}{2} & 1 & 2 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & -\frac{1}{2} & 1 & 2 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
2 & -2 & 1 & -2 & 0 & 1 & -1 & 1 & -3 & -1 & 3 & 1 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
0 & 0 & 0 & 0 & 3 & 0 & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
-1 & 1 & 0 & 0 & 3 & 0 & -\frac{1}{2} & \frac{1}{2} & -2 & -\frac{1}{2} & 2 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4} \\
1 & -1 & 1 & -2 & 1 & 1 & -1 & 1 & -3 & -1 & 3 & 1 & \frac{1}{4} & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{4}
\end{pmatrix}$$

```
MatrixRank[mat]
```

```
5
```

```
v = Wgl[#][n, k] & /@ diags;
mat1 = Transpose[Normal[mat]] ~Join~ Table[
    v /. {n → Random[], k → Random[]},
    {Length[v] + 1}
  ]
```

$$\left\{\{0, 0, 0, 0, 0, 0, 0, 2, 0, 0, -1, 1\}, \{0, 0, 0, 0, 0, 0, 0, -2, 0, 0, 1, -1\},\right.$$
$$\left\{0, 0, 0, 1, 0, -\frac{1}{2}, -\frac{1}{2}, 1, 0, 0, 0, 1\right\}, \{0, 0, 0, -2, 0, 1, 1, -2, 0, 0, 0, -2\},$$
$$\{2, 2, 2, 2, 0, 2, 2, 0, 1, 3, 3, 1\}, \left\{0, 0, 0, 1, 0, -\frac{1}{2}, -\frac{1}{2}, 1, 0, 0, 0, 1\right\},$$

$$\left\{0, -\frac{1}{2}, -\frac{1}{2}, -1, 0, -\frac{1}{2}, -\frac{1}{2}, -1, 0, -\frac{1}{2}, -\frac{1}{2}, -1\right\},$$

$$\left\{0, \frac{1}{2}, \frac{1}{2}, 1, 0, \frac{1}{2}, \frac{1}{2}, 1, 0, \frac{1}{2}, \frac{1}{2}, 1\right\}, \{-1, -2, -2, -3, -1, -2, -2, -3, -1, -2, -2, -3\},$$

$$\left\{0, -\frac{1}{2}, -\frac{1}{2}, -1, 0, -\frac{1}{2}, -\frac{1}{2}, -1, 0, -\frac{1}{2}, -\frac{1}{2}, -1\right\}, \{1, 2, 2, 3, 1, 2, 2, 3, 1, 2, 2, 3\},$$

$$\left\{0, \frac{1}{2}, \frac{1}{2}, 1, 0, \frac{1}{2}, \frac{1}{2}, 1, 0, \frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\},$$

$$\{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}, \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\},$$

$$\{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}, \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\},$$

$$\left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}, \{0.0002468, 0.000488012, 0.000488012,$$

$$0.000964973, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.125123, -0.124389, -0.12403, 0.125482\},$$

$$\{0.246502, 0.127908, 0.127908, 0.0663702, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.248251, 0.126158,$$

$$0.0360927, 0.158185\}, \{0.0183076, 0.0360172, 0.0360172, 0.0708579, \frac{1}{4}, \frac{1}{4},$$

$$\frac{1}{4}, \frac{1}{4}, 0.134154, -0.079829, -0.0535539, 0.160429\}, \{0.0089076, 0.0307256,$$

$$0.0307256, 0.105984, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.129454, -0.0898206, -0.0412826, 0.177992\},$$

$$\{0.0714444, -0.0335015, -0.0335015, 0.0157094, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.160722, -0.122779,$$

$$-0.150647, 0.132855\}, \{1.30369, -0.423069, -0.423069, 0.137292, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4},$$

$$0.776847, 0.103778, -0.479423, 0.193646\}, \{0.394171, -0.0980892, -0.0980892,$$

$$0.0244095, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.322085, -0.0260038, -0.210884, 0.137205\},$$

$$\{0.0673436, -0.0767951, -0.0767951, 0.0875731, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.158672, -0.168123,$$

$$-0.158009, 0.168787\}, \{0.0153506, 0.0584804, 0.0584804, 0.222789, \frac{1}{4}, \frac{1}{4},$$

$$\frac{1}{4}, \frac{1}{4}, 0.132675, -0.0588443, 0.044875, 0.236395\}, \{0.327966, -0.111359,$$

$$\{-0.111359, 0.037811, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.288983, -0.0723755, -0.217453, 0.143906\},$$
$$\{0.299123, 0.0515289, 0.0515289, 0.00887671, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.274561, 0.0760901,$$
$$-0.0690328, 0.129438\}, \{1.34341, -0.462182, -0.462182, 0.159008, \frac{1}{4}, \frac{1}{4},$$
$$\frac{1}{4}, \frac{1}{4}, 0.796703, 0.0845201, -0.507678, 0.204504\}, \{1.0634, -0.450482,$$
$$-0.450482, 0.190836, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0.656698, -0.0437841, -0.480064, 0.220418\}\}$$

MatrixRank[mat1]

```
6
```

NullSpace[mat1]

```
{{0.0184657, 0.233546, 0.457301, 0.140272, -0.0614029, 0.0117712,
  -0.0117712, -0.470706, 0.0429372, -0.0798686, -0.610978, 0.330434},
 {-0.257362, -0.0349367, -0.455756, -0.227141, 0.273096, -0.00764141,
  0.00764141, -0.266908, -0.0157337, 0.530459, -0.0397663, 0.494049},
 {0.219086, -0.0353266, 0.0859409, -0.473371, -0.222512, -0.0558204,
  0.0558204, -0.0823872, 0.0034259, -0.441598, 0.390984, 0.555758},
 {0.0670048, 0.703197, -0.456963, 0.0658292, -0.113031, -0.340516,
  0.340516, -0.00036955, 0.0460258, -0.180035, -0.0661987, -0.0654596},
 {0.422537, 0.152974, -0.0621224, 0.00992405, 0.361611, 0.147214,
  -0.147214, -0.0200017, -0.784148, -0.060926, -0.0299257, 0.0100776},
 {0.0489663, -0.353696, 0.251203, 0.0147787, 0.16911, -0.59922, 0.59922,
  -0.00287209, -0.218076, 0.120143, -0.0176508, -0.0119066}}
```

Length[diags]

```
12
```

UGL2BiAlg[diags]

$$\{\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] - \frac{1}{2} h[1]^2 RW[101, 101] -$$
$$\frac{1}{2} h[2]^2 RW[101, 101] + 2 RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] -$$
$$\frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - RW[101, 102, 201] + RW[102, 201, 202] +$$
$$\frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] +$$

$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,h[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,h[1]^2\,h[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,h[2]^4\,\mathrm{RW}[] + \frac{1}{2}\,h[1]^2\,\mathrm{RW}[101] + \frac{1}{2}\,h[2]^2\,\mathrm{RW}[101] -$$
$$\frac{1}{2}\,h[1]^2\,\mathrm{RW}[202] - \frac{1}{2}\,h[2]^2\,\mathrm{RW}[202] - \frac{1}{2}\,h[1]^2\,\mathrm{RW}[101, 101] - \frac{1}{2}\,h[2]^2\,\mathrm{RW}[101, 101] +$$
$$2\,\mathrm{RW}[102, 201] - h[1]^2\,\mathrm{RW}[102, 201] - h[2]^2\,\mathrm{RW}[102, 201] - \frac{1}{4}\,h[1]^2\,\mathrm{RW}[202, 202] -$$
$$\frac{1}{4}\,h[2]^2\,\mathrm{RW}[202, 202] - \frac{1}{2}\,\mathrm{RW}[101, 101, 101] + \frac{1}{2}\,\mathrm{RW}[101, 101, 202] -$$
$$2\,\mathrm{RW}[101, 102, 201] - \frac{1}{4}\,\mathrm{RW}[101, 202, 202] + 2\,\mathrm{RW}[102, 201, 202] + \frac{1}{4}\,\mathrm{RW}[202, 202, 202] +$$
$$\frac{1}{4}\,\mathrm{RW}[101, 101, 101, 101] + \mathrm{RW}[101, 101, 102, 201] + \frac{1}{4}\,\mathrm{RW}[101, 101, 202, 202] +$$
$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,h[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,h[1]^2\,h[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,h[2]^4\,\mathrm{RW}[] + \frac{1}{2}\,h[1]^2\,\mathrm{RW}[101] + \frac{1}{2}\,h[2]^2\,\mathrm{RW}[101] -$$
$$\frac{1}{2}\,h[1]^2\,\mathrm{RW}[202] - \frac{1}{2}\,h[2]^2\,\mathrm{RW}[202] - \frac{1}{2}\,h[1]^2\,\mathrm{RW}[101, 101] - \frac{1}{2}\,h[2]^2\,\mathrm{RW}[101, 101] +$$
$$2\,\mathrm{RW}[102, 201] - h[1]^2\,\mathrm{RW}[102, 201] - h[2]^2\,\mathrm{RW}[102, 201] - \frac{1}{4}\,h[1]^2\,\mathrm{RW}[202, 202] -$$
$$\frac{1}{4}\,h[2]^2\,\mathrm{RW}[202, 202] - \frac{1}{2}\,\mathrm{RW}[101, 101, 101] + \frac{1}{2}\,\mathrm{RW}[101, 101, 202] -$$
$$2\,\mathrm{RW}[101, 102, 201] - \frac{1}{4}\,\mathrm{RW}[101, 202, 202] + 2\,\mathrm{RW}[102, 201, 202] + \frac{1}{4}\,\mathrm{RW}[202, 202, 202] +$$
$$\frac{1}{4}\,\mathrm{RW}[101, 101, 101, 101] + \mathrm{RW}[101, 101, 102, 201] + \frac{1}{4}\,\mathrm{RW}[101, 101, 202, 202] +$$
$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,h[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,h[1]^2\,h[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,h[2]^4\,\mathrm{RW}[] + h[1]^2\,\mathrm{RW}[101] + h[2]^2\,\mathrm{RW}[101] -$$
$$h[1]^2\,\mathrm{RW}[202] - h[2]^2\,\mathrm{RW}[202] + \mathrm{RW}[101, 101] - \frac{1}{2}\,h[1]^2\,\mathrm{RW}[101, 101] - \frac{1}{2}\,h[2]^2\,\mathrm{RW}[101, 101] -$$
$$2\,\mathrm{RW}[101, 202] + 2\,\mathrm{RW}[102, 201] - h[1]^2\,\mathrm{RW}[102, 201] - h[2]^2\,\mathrm{RW}[102, 201] + \mathrm{RW}[202, 202] -$$
$$\frac{1}{4}\,h[1]^2\,\mathrm{RW}[202, 202] - \frac{1}{4}\,h[2]^2\,\mathrm{RW}[202, 202] - \mathrm{RW}[101, 101, 101] + \mathrm{RW}[101, 101, 202] -$$

$$3\,\mathrm{RW}[101, 102, 201] - \frac{1}{2}\,\mathrm{RW}[101, 202, 202] + 3\,\mathrm{RW}[102, 201, 202] + \frac{1}{2}\,\mathrm{RW}[202, 202, 202] +$$
$$\frac{1}{4}\,\mathrm{RW}[101, 101, 101, 101] + \mathrm{RW}[101, 101, 102, 201] + \frac{1}{4}\,\mathrm{RW}[101, 101, 202, 202] +$$
$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,\mathrm{h}[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{h}[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,\mathrm{h}[2]^4\,\mathrm{RW}[] - \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[101, 101] -$$
$$\frac{1}{2}\,\mathrm{h}[2]^2\,\mathrm{RW}[101, 101] - \mathrm{h}[1]^2\,\mathrm{RW}[102, 201] - \mathrm{h}[2]^2\,\mathrm{RW}[102, 201] - \frac{1}{4}\,\mathrm{h}[1]^2\,\mathrm{RW}[202, 202] -$$
$$\frac{1}{4}\,\mathrm{h}[2]^2\,\mathrm{RW}[202, 202] - \mathrm{RW}[101, 102, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202] +$$
$$\frac{1}{4}\,\mathrm{RW}[101, 101, 101, 101] + \mathrm{RW}[101, 101, 102, 201] + \frac{1}{4}\,\mathrm{RW}[101, 101, 202, 202] +$$
$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,\mathrm{h}[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{h}[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,\mathrm{h}[2]^4\,\mathrm{RW}[] - \frac{1}{2}\,\mathrm{h}[1]\,\mathrm{RW}[101] + \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[101] +$$
$$\frac{1}{2}\,\mathrm{h}[2]\,\mathrm{RW}[101] + \frac{1}{2}\,\mathrm{h}[2]^2\,\mathrm{RW}[101] + \frac{1}{2}\,\mathrm{h}[1]\,\mathrm{RW}[202] - \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[202] - \frac{1}{2}\,\mathrm{h}[2]\,\mathrm{RW}[202] -$$
$$\frac{1}{2}\,\mathrm{h}[2]^2\,\mathrm{RW}[202] - \frac{1}{2}\,\mathrm{RW}[101, 101] - \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[101, 101] - \frac{1}{2}\,\mathrm{h}[2]^2\,\mathrm{RW}[101, 101] +$$
$$\frac{3}{4}\,\mathrm{RW}[101, 202] + 2\,\mathrm{RW}[102, 201] - \mathrm{h}[1]^2\,\mathrm{RW}[102, 201] - \mathrm{h}[2]^2\,\mathrm{RW}[102, 201] - \frac{1}{4}\,\mathrm{RW}[202, 202] -$$
$$\frac{1}{4}\,\mathrm{h}[1]^2\,\mathrm{RW}[202, 202] - \frac{1}{4}\,\mathrm{h}[2]^2\,\mathrm{RW}[202, 202] - \frac{1}{2}\,\mathrm{RW}[101, 101, 101] + \frac{1}{2}\,\mathrm{RW}[101, 101, 202] -$$
$$2\,\mathrm{RW}[101, 102, 201] - \frac{1}{4}\,\mathrm{RW}[101, 202, 202] + 2\,\mathrm{RW}[102, 201, 202] + \frac{1}{4}\,\mathrm{RW}[202, 202, 202] +$$
$$\frac{1}{4}\,\mathrm{RW}[101, 101, 101, 101] + \mathrm{RW}[101, 101, 102, 201] + \frac{1}{4}\,\mathrm{RW}[101, 101, 202, 202] +$$
$$\mathrm{RW}[102, 102, 201, 201] + \frac{1}{2}\,\mathrm{RW}[102, 201, 202, 202] + \frac{1}{16}\,\mathrm{RW}[202, 202, 202, 202],$$

$$\frac{1}{4}\,\mathrm{h}[1]^4\,\mathrm{RW}[] + \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{h}[2]^2\,\mathrm{RW}[] + \frac{1}{4}\,\mathrm{h}[2]^4\,\mathrm{RW}[] + \frac{1}{2}\,\mathrm{h}[1]\,\mathrm{RW}[101] + \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[101] -$$
$$\frac{1}{2}\,\mathrm{h}[2]\,\mathrm{RW}[101] + \frac{1}{2}\,\mathrm{h}[2]^2\,\mathrm{RW}[101] - \frac{1}{2}\,\mathrm{h}[1]\,\mathrm{RW}[202] - \frac{1}{2}\,\mathrm{h}[1]^2\,\mathrm{RW}[202] + \frac{1}{2}\,\mathrm{h}[2]\,\mathrm{RW}[202] -$$

$$\frac{1}{2} h[2]^2 RW[202] - \frac{1}{2} RW[101, 101] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] + \frac{3}{4} RW[101, 202] + 2 RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] - \frac{1}{4} RW[202, 202] - \frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - \frac{1}{2} RW[101, 101, 101] + \frac{1}{2} RW[101, 101, 202] - 2 RW[101, 102, 201] - \frac{1}{4} RW[101, 202, 202] + 2 RW[102, 201, 202] + \frac{1}{4} RW[202, 202, 202] + \frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2} RW[102, 201, 202, 202] + \frac{1}{16} RW[202, 202, 202, 202],$$

$$\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] + 2 RW[101] + h[1]^2 RW[101] + h[2]^2 RW[101] - 2 RW[202] - h[1]^2 RW[202] - h[2]^2 RW[202] + RW[101, 101] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] - \frac{5}{2} RW[101, 202] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] + \frac{3}{2} RW[202, 202] - \frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - RW[101, 101, 101] + RW[101, 101, 202] - 3 RW[101, 102, 201] - \frac{1}{2} RW[101, 202, 202] + \frac{7}{2} RW[102, 201, 202] + \frac{1}{2} RW[202, 202, 202] + \frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2} RW[102, 201, 202, 202] + \frac{1}{16} RW[202, 202, 202, 202],$$

$$\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] + \frac{3}{4} RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] - \frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - RW[101, 102, 201] + \frac{1}{2} RW[102, 201, 202] + \frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2} RW[102, 201, 202, 202] + \frac{1}{16} RW[202, 202, 202, 202],$$

$$\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] + \frac{1}{2} h[1]^2 RW[101] + \frac{1}{2} h[2]^2 RW[101] - \frac{1}{2} h[1]^2 RW[202] - \frac{1}{2} h[2]^2 RW[202] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] + \frac{11}{4} RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] - \frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - \frac{1}{2} RW[101, 101, 101] + \frac{1}{2} RW[101, 101, 202] - 2 RW[101, 102, 201] - \frac{1}{4} RW[101, 202, 202] + \frac{3}{2} RW[102, 201, 202] + \frac{1}{4} RW[202, 202, 202] + \frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2} RW[102, 201, 202, 202] + \frac{1}{16} RW[202, 202, 202, 202],$$

$$\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] - \frac{3 RW[101]}{4} + \frac{1}{2} h[1]^2 RW[101] + \frac{1}{2} h[2]^2 RW[101] + \frac{3 RW[202]}{4} - \frac{1}{2} h[1]^2 RW[202] - \frac{1}{2} h[2]^2 RW[202] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] - \frac{1}{2} RW[101, 202] + \frac{11}{4} RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] + \frac{1}{2} RW[202, 202] - \frac{1}{4} h[1]^2 RW[202, 202] - \frac{1}{4} h[2]^2 RW[202, 202] - \frac{1}{2} RW[101, 101, 101] + \frac{1}{2} RW[101, 101, 202] - 2 RW[101, 102, 201] - \frac{1}{4} RW[101, 202, 202] + \frac{5}{2} RW[102, 201, 202] + \frac{1}{4} RW[202, 202, 202] + \frac{1}{4} RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4} RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2} RW[102, 201, 202, 202] + \frac{1}{16} RW[202, 202, 202, 202],$$

$$\frac{1}{4} h[1]^4 RW[] + \frac{1}{2} h[1]^2 h[2]^2 RW[] + \frac{1}{4} h[2]^4 RW[] + \frac{5 RW[101]}{4} + h[1]^2 RW[101] + h[2]^2 RW[101] - \frac{5 RW[202]}{4} - h[1]^2 RW[202] - h[2]^2 RW[202] + RW[101, 101] - \frac{1}{2} h[1]^2 RW[101, 101] - \frac{1}{2} h[2]^2 RW[101, 101] - \frac{5}{2} RW[101, 202] + \frac{3}{4} RW[102, 201] - h[1]^2 RW[102, 201] - h[2]^2 RW[102, 201] + \frac{3}{2} RW[202, 202] - \frac{1}{4} h[1]^2 RW[202, 202] -$$

$$\frac{1}{4}\,h[2]^2\,RW[202, 202] - RW[101, 101, 101] + RW[101, 101, 202] - 3\,RW[101, 102, 201] - \frac{1}{2}\,RW[101, 202, 202] + \frac{7}{2}\,RW[102, 201, 202] + \frac{1}{2}\,RW[202, 202, 202] + \frac{1}{4}\,RW[101, 101, 101, 101] + RW[101, 101, 102, 201] + \frac{1}{4}\,RW[101, 101, 202, 202] + RW[102, 102, 201, 201] + \frac{1}{2}\,RW[102, 201, 202, 202] + \frac{1}{16}\,RW[202, 202, 202, 202]\Big\}$$

```
rels = R /@ Diagrams[R6T + 0 ar];

UGL2BiAlg /@ rels
```

```
{0, 0, 0, 0, 0, 0}
```

```
rels = R /@ Diagrams[R6T + 1 ar];

UGL2BiAlg /@ rels
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```