

```

SetDirectory["C:/drorbn/AcademicPensieve/Projects/Arrow_Diagrams_and_gl(N)"];
<< "Arrow_Diagrams_and_gl(N).m"

diag = Diag[ar[1, 8], ar[3, 5], ar[6, 4], ar[7, 2]]
Diag[ar[1, 8], ar[3, 5], ar[6, 4], ar[7, 2]]

FromDigits[Cases[diag, _Integer, Infinity], sbase]

18 356 472

SaveUSO[diag_Diag] := Module[
  {fname},
  fname = StringJoin[
    "USOFor",
    ToString[FromDigits[Cases[diag, _Integer, Infinity], sbase]],
    ".m"
  ];
  Print["Saving ", fname, " ..."];
  Put[{diag, USO[diag]}, fname];
]

BasisAArrow[2]

{Diag[ar[1, 3], ar[4, 2]], Diag[ar[2, 4], ar[3, 1]],
 Diag[ar[3, 1], ar[4, 2]], Diag[ar[1, 4], ar[2, 3]],
 Diag[ar[1, 4], ar[3, 2]], Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]]}

SaveUSO /@ BasisAArrow[2]

Saving USOFor1342.m ...
Saving USOFor2431.m ...
Saving USOFor3142.m ...
Saving USOFor1423.m ...
Saving USOFor1432.m ...
Saving USOFor2341.m ...
Saving USOFor3241.m ...

{Null, Null, Null, Null, Null, Null, Null}

BasisAArrow[3] // First
Diag[ar[4, 1], ar[5, 3], ar[6, 2]]

SaveUSO[Diag[ar[4, 1], ar[5, 3], ar[6, 2]]]

Saving USOFor415362.m ...

SaveUSO[Diag[ar[1, 8], ar[3, 5], ar[6, 4], ar[7, 2]]]

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```
Saving USOFor18356472.m ...  
Computing OrderTypes [8, {{1, 2}, {3, 4}, {5, 6}, {7, 8}}]  
Computing OrderTypes [8, {{3, 4}, {5, 6}, {7, 8}}]  
Computing OrderTypes [8, {{5, 6}, {7, 8}}]  
Computing OrderTypes [8, {{7, 8}}]  
Computing OrderTypes [7, {{1, 2}, {3, 4}, {5, 6}}]  
Computing OrderTypes [7, {{3, 4}, {5, 6}}]  
Computing OrderTypes [7, {{5, 6}}]  
Computing OrderTypes [5, {{1, 2}, {3, 4}}]  
Computing OrderTypes [5, {{3, 4}}]  
Computing OrderTypes [3, {{1, 2}}]  
Computing OrderTypes [6, {{1, 2}, {3, 4}}]  
Computing OrderTypes [6, {{3, 4}}]  
Computing OrderTypes [4, {{1, 2}}]  
Computing OrderTypes [5, {{1, 2}}]  
Computing OrderTypes [7, {{1, 2}, {3, 4}, {6, 7}}]  
Computing OrderTypes [7, {{3, 4}, {6, 7}}]  
Computing OrderTypes [7, {{6, 7}}]  
Computing OrderTypes [5, {{1, 2}, {4, 5}}]  
Computing OrderTypes [5, {{4, 5}}]  
Computing OrderTypes [3, {{2, 3}}]  
Computing OrderTypes [6, {{1, 2}, {5, 6}}]  
Computing OrderTypes [6, {{5, 6}}]  
Computing OrderTypes [4, {{3, 4}}]  
Computing OrderTypes [7, {{1, 2}, {4, 5}, {6, 7}}]  
Computing OrderTypes [7, {{4, 5}, {6, 7}}]  
Computing OrderTypes [5, {{2, 3}, {4, 5}}]  
Computing OrderTypes [6, {{3, 4}, {5, 6}}]  
Computing OrderTypes [7, {{2, 3}, {4, 5}, {6, 7}}]  
{1 390 450, 165 944}
```

Union[USO /@ BasisAArrow[2] /. n^4 -> 1 /. n -> 0]

$$\begin{aligned} & \{2 \text{RSW}[112, 121, 134, 143] + 2 \text{RSW}[112, 121, 234, 334] + 2 \text{RSW}[113, 124, 131, 142] + \\ & 2 \text{RSW}[113, 131, 224, 324] + 2 \text{RSW}[114, 123, 132, 141] + 2 \text{RSW}[114, 141, 223, 323] + \\ & 2 \text{RSW}[123, 132, 214, 314] + 2 \text{RSW}[124, 142, 213, 313] + 2 \text{RSW}[134, 143, 212, 312] + \\ & 2 \text{RSW}[212, 234, 312, 334] + 2 \text{RSW}[213, 224, 313, 324] + 2 \text{RSW}[214, 223, 314, 323], \\ & 16 \text{RSW}[111, 122] + 8 \text{RSW}[111, 133] - 4 \text{RSW}[111, 123, 132] - 4 \text{RSW}[111, 124, 142] - \\ & 4 \text{RSW}[111, 134, 143] - 4 \text{RSW}[111, 223, 323] - 4 \text{RSW}[111, 224, 324] - \\ & 4 \text{RSW}[111, 234, 334] - 4 \text{RSW}[112, 121, 133] - 4 \text{RSW}[113, 122, 131] - \\ & 4 \text{RSW}[114, 122, 141] - 4 \text{RSW}[122, 213, 313] - 4 \text{RSW}[122, 214, 314] - 4 \text{RSW}[133, 212, 312] + \\ & 2 \text{RSW}[112, 121, 134, 143] + 2 \text{RSW}[112, 121, 234, 334] + 2 \text{RSW}[113, 124, 131, 142] + \\ & 2 \text{RSW}[113, 131, 224, 324] + 2 \text{RSW}[114, 123, 132, 141] + 2 \text{RSW}[114, 141, 223, 323] + \\ & 2 \text{RSW}[123, 132, 214, 314] + 2 \text{RSW}[124, 142, 213, 313] + 2 \text{RSW}[134, 143, 212, 312] + \\ & 2 \text{RSW}[212, 234, 312, 334] + 2 \text{RSW}[213, 224, 313, 324] + 2 \text{RSW}[214, 223, 314, 323], \\ & -2 \text{RSW}[111, 123, 132] - 2 \text{RSW}[111, 124, 142] - 2 \text{RSW}[111, 134, 143] - 2 \text{RSW}[111, 223, 323] - \\ & 2 \text{RSW}[111, 224, 324] - 2 \text{RSW}[111, 234, 334] - 2 \text{RSW}[112, 121, 133] - 2 \text{RSW}[113, 122, 131] - \\ & 2 \text{RSW}[114, 122, 141] - 2 \text{RSW}[122, 213, 313] - 2 \text{RSW}[122, 214, 314] - 2 \text{RSW}[133, 212, 312] + \\ & 2 \text{RSW}[112, 121, 134, 143] + 2 \text{RSW}[112, 121, 234, 334] + 2 \text{RSW}[113, 124, 131, 142] + \\ & 2 \text{RSW}[113, 131, 224, 324] + 2 \text{RSW}[114, 123, 132, 141] + 2 \text{RSW}[114, 141, 223, 323] + \\ & 2 \text{RSW}[123, 132, 214, 314] + 2 \text{RSW}[124, 142, 213, 313] + 2 \text{RSW}[134, 143, 212, 312] + \\ & 2 \text{RSW}[212, 234, 312, 334] + 2 \text{RSW}[213, 224, 313, 324] + 2 \text{RSW}[214, 223, 314, 323]\} \end{aligned}$$

Union[

(USO /@ BasisAArrow[2]) /. {h[i\_] -> xh[i], rsw\_RSW -> x^Length[rsw] rsw} /. x^4 -> 1 /. x -> 0]

$$\begin{aligned} & \left\{ -\frac{1}{2} n h[1]^2 \text{RSW}[111, 111] - \frac{1}{2} n^2 h[2]^2 \text{RSW}[111, 111] - n^2 h[1]^2 \text{RSW}[112, 121] - \right. \\ & n^2 h[2]^2 \text{RSW}[112, 121] - n^3 h[3]^2 \text{RSW}[112, 121] - n^3 h[2]^2 \text{RSW}[113, 131] - \\ & \frac{1}{2} n^2 h[1]^2 \text{RSW}[122, 122] - n^3 h[1]^2 \text{RSW}[123, 132] - n^2 h[1]^2 \text{RSW}[212, 312] - \\ & n^2 h[2]^2 \text{RSW}[212, 312] - n^3 h[3]^2 \text{RSW}[212, 312] - n^3 h[2]^2 \text{RSW}[213, 313] - n^3 h[1]^2 \text{RSW}[223, 323] + \\ & \frac{1}{4} n \text{RSW}[111, 111, 111, 111] + n^2 \text{RSW}[111, 111, 112, 121] + \frac{1}{2} n^2 \text{RSW}[111, 111, 122, 122] + \\ & n^3 \text{RSW}[111, 111, 123, 132] + n^2 \text{RSW}[111, 111, 212, 312] + n^3 \text{RSW}[111, 111, 223, 323] + \\ & n^2 \text{RSW}[112, 112, 121, 121] + 2 n^3 \text{RSW}[112, 113, 121, 131] + n^2 \text{RSW}[112, 121, 122, 122] + \\ & 2 n^3 \text{RSW}[112, 121, 123, 132] + n^3 \text{RSW}[112, 121, 133, 133] + 2 n^4 \text{RSW}[112, 121, 134, 143] + \\ & 2 n^2 \text{RSW}[112, 121, 212, 312] + 2 n^3 \text{RSW}[112, 121, 213, 313] + 2 n^3 \text{RSW}[112, 121, 223, 323] + \\ & 2 n^4 \text{RSW}[112, 121, 234, 334] + n^3 \text{RSW}[113, 122, 122, 131] + 2 n^3 \text{RSW}[113, 123, 131, 132] + \\ & 2 n^4 \text{RSW}[113, 124, 131, 142] + 2 n^3 \text{RSW}[113, 131, 212, 312] + 2 n^3 \text{RSW}[113, 131, 223, 323] + \\ & 2 n^4 \text{RSW}[113, 131, 224, 324] + 2 n^4 \text{RSW}[114, 123, 132, 141] + 2 n^4 \text{RSW}[114, 141, 223, 323] + \\ & n^2 \text{RSW}[122, 122, 212, 312] + n^3 \text{RSW}[122, 122, 213, 313] + 2 n^3 \text{RSW}[123, 132, 212, 312] + \\ & 2 n^3 \text{RSW}[123, 132, 213, 313] + 2 n^4 \text{RSW}[123, 132, 214, 314] + 2 n^4 \text{RSW}[124, 142, 213, 313] + \\ & n^3 \text{RSW}[133, 133, 212, 312] + 2 n^4 \text{RSW}[134, 143, 212, 312] + n^2 \text{RSW}[212, 212, 312, 312] + \\ & 2 n^3 \text{RSW}[212, 213, 312, 313] + 2 n^3 \text{RSW}[212, 223, 312, 323] + 2 n^4 \text{RSW}[212, 234, 312, 334] + \\ & \left. 2 n^3 \text{RSW}[213, 223, 313, 323] + 2 n^4 \text{RSW}[213, 224, 313, 324] + 2 n^4 \text{RSW}[214, 223, 314, 323] \right\} \end{aligned}$$

LeafCount[USO /@ BasisAArrow[2]]

6851

LeafCount[(USO /@ BasisAArrow[2]) /. rsw\_RSW /; Length[rsw] == 4 -> 0]

4191

```
(BasisAArrow[2] // First // USO) /.
```

```
{h[i_] => x h[i], rsw_RSW => x^Length[rsw] rsw} /. x^4 -> 0 /. x -> 1
```

$$\frac{1}{4} n h[1]^2 \text{RSW}[] - n^2 h[1] \text{RSW}[111] + n^2 h[1]^2 \text{RSW}[111] + n^2 h[2]^2 \text{RSW}[111] + n^3 h[2]^2 \text{RSW}[111] + n^3 h[3]^2 \text{RSW}[111] + n^3 h[1]^2 \text{RSW}[122] - n^2 h[2] \text{RSW}[122] - n^2 \text{RSW}[111, 111] + 2 n^2 \text{RSW}[112, 121] - 2 n^3 \text{RSW}[112, 121] + n^3 \text{RSW}[113, 131] - n^2 \text{RSW}[122, 122] - 2 n^3 \text{RSW}[123, 132] + 2 n^2 \text{RSW}[212, 312] + 4 n^3 \text{RSW}[212, 312] + 2 n^3 \text{RSW}[213, 313] - n^2 \text{RSW}[111, 111, 111] - n^3 \text{RSW}[111, 111, 122] - 3 n^2 \text{RSW}[111, 112, 121] - 2 n^3 \text{RSW}[111, 112, 121] - 2 n^3 \text{RSW}[111, 113, 131] - n^2 \text{RSW}[111, 122, 122] - n^3 \text{RSW}[111, 122, 122] - 4 n^3 \text{RSW}[111, 123, 132] - 2 n^4 \text{RSW}[111, 123, 132] - 2 n^4 \text{RSW}[111, 124, 142] - n^3 \text{RSW}[111, 133, 133] - 2 n^4 \text{RSW}[111, 134, 143] - 3 n^2 \text{RSW}[111, 212, 312] - 2 n^3 \text{RSW}[111, 212, 312] - 2 n^3 \text{RSW}[111, 213, 313] - 4 n^3 \text{RSW}[111, 223, 323] - 2 n^4 \text{RSW}[111, 223, 323] - 2 n^4 \text{RSW}[111, 224, 324] - 2 n^4 \text{RSW}[111, 234, 334] + n^2 \text{RSW}[112, 121, 122] - 2 n^3 \text{RSW}[112, 121, 122] - 2 n^4 \text{RSW}[112, 121, 133] + n^3 \text{RSW}[112, 123, 131] - n^3 \text{RSW}[112, 223, 313] - n^3 \text{RSW}[113, 121, 132] - 2 n^3 \text{RSW}[113, 122, 131] - 2 n^4 \text{RSW}[113, 122, 131] + n^3 \text{RSW}[113, 223, 312] - 2 n^4 \text{RSW}[114, 122, 141] - n^3 \text{RSW}[121, 213, 323] - n^2 \text{RSW}[122, 212, 312] - 2 n^3 \text{RSW}[122, 212, 312] - 2 n^3 \text{RSW}[122, 213, 313] - 2 n^4 \text{RSW}[122, 213, 313] - 2 n^4 \text{RSW}[122, 214, 314] - n^3 \text{RSW}[123, 213, 312] + n^3 \text{RSW}[131, 212, 323] - n^3 \text{RSW}[132, 212, 313] - 2 n^4 \text{RSW}[133, 212, 312]$$

```
LeafCount [
```

```
(USO /@ BasisAArrow[2]) /. {h[i_] => x h[i], rsw_RSW => x^Length[rsw] rsw} /. x^4 -> 0 /. x -> 1]
```

```
3071
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```
LeafCount [USO[BasisAArrow[3][[1]]]]
```

```
42 297
```

```
LeafCount [
```

```
USO[BasisAArrow[3][[1]]] /. {h[i_] => x h[i], rsw_RSW => x^Length[rsw] rsw} /. x^6 -> 0 /. x -> 1]
```

```
28 337
```