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<< "Arrow_Diagrams_and_gl(N).m"

DescendingDiagram[p_List] := Module[
  {n = Length[p]},
  Diag @@ Table[ar[i, p[[i]] + n], {i, n}]
];

DescendingBasis[n_] := DescendingDiagram /@ Permutations[Range[n]]

bas3 = DescendingBasis[3]

{Diag[ar[1, 4], ar[2, 5], ar[3, 6]], Diag[ar[1, 4], ar[2, 6], ar[3, 5]],
 Diag[ar[1, 5], ar[2, 4], ar[3, 6]], Diag[ar[1, 5], ar[2, 6], ar[3, 4]],
 Diag[ar[1, 6], ar[2, 4], ar[3, 5]], Diag[ar[1, 6], ar[2, 5], ar[3, 4]]}

bas4 = DescendingBasis[4]

{Diag[ar[1, 5], ar[2, 6], ar[3, 7], ar[4, 8]], Diag[ar[1, 5], ar[2, 6], ar[3, 8], ar[4, 7]],
 Diag[ar[1, 5], ar[2, 7], ar[3, 6], ar[4, 8]], Diag[ar[1, 5], ar[2, 7], ar[3, 8], ar[4, 6]],
 Diag[ar[1, 5], ar[2, 8], ar[3, 6], ar[4, 7]], Diag[ar[1, 5], ar[2, 8], ar[3, 7], ar[4, 6]],
 Diag[ar[1, 6], ar[2, 5], ar[3, 7], ar[4, 8]], Diag[ar[1, 6], ar[2, 5], ar[3, 8], ar[4, 7]],
 Diag[ar[1, 6], ar[2, 7], ar[3, 5], ar[4, 8]], Diag[ar[1, 6], ar[2, 7], ar[3, 8], ar[4, 5]],
 Diag[ar[1, 6], ar[2, 8], ar[3, 5], ar[4, 7]], Diag[ar[1, 6], ar[2, 8], ar[3, 7], ar[4, 5]],
 Diag[ar[1, 7], ar[2, 5], ar[3, 6], ar[4, 8]], Diag[ar[1, 7], ar[2, 5], ar[3, 8], ar[4, 6]],
 Diag[ar[1, 7], ar[2, 6], ar[3, 5], ar[4, 8]], Diag[ar[1, 7], ar[2, 6], ar[3, 8], ar[4, 5]],
 Diag[ar[1, 7], ar[2, 8], ar[3, 5], ar[4, 6]], Diag[ar[1, 7], ar[2, 8], ar[3, 6], ar[4, 5]],
 Diag[ar[1, 8], ar[2, 5], ar[3, 6], ar[4, 7]], Diag[ar[1, 8], ar[2, 5], ar[3, 7], ar[4, 6]],
 Diag[ar[1, 8], ar[2, 6], ar[3, 5], ar[4, 7]], Diag[ar[1, 8], ar[2, 6], ar[3, 7], ar[4, 5]],
 Diag[ar[1, 8], ar[2, 7], ar[3, 5], ar[4, 6]], Diag[ar[1, 8], ar[2, 7], ar[3, 6], ar[4, 5]]}

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U[deg_, diag_] :=
  (UGLnBiAlg[diag] /. n^p_.*RW[l_] => EW[GL*n^p*FromDigits[{1}, base^2]] //.)
  h[i_]^p_.*EW[l_] => EW[h[i]^p]);
diags = Take[bas4, All];
mat = {};
EWRules = {};
Do[
  Print[{i, diags[[i]], MaxMemoryUsed[], TimeUsed[]}];
  vect = U[4, diags[[i]]] /. EWRules;
  Print["Length[vect] is ", Length[vect]];
  EWS = Union[Cases[vect, _EW, Infinity]];
  If[EWRules === {}, l = 0, l = Length[EWRules[[1]]]];
  Print["l is ", l];
  If[EWRules === {},
    EWRules = Dispatch[Thread[
      EWS -> Table[v[j+1], {j, Length[EWS]}]
    ]],
    EWRules = Dispatch[Join[EWRules[[1]], Thread[
      EWS -> Table[v[j+1], {j, Length[EWS]}]
    ]]]];
  l = Length[EWRules[[1]]];
  Print["l is ", l];
  Print[i];
  AppendTo[mat,
    Replace[#, c_.*v[j_] => ({i, j} -> c)] & /@ (List @@ vect /. EWRules)
  ],
  {i, Length[diags]}
];
mat = SparseArray[Flatten[mat], {Length[diags], l}];
Print["Computing Rank..."];
MatrixRank[mat]

{1, Diag[ar[1, 5], ar[2, 6], ar[3, 7], ar[4, 8]], 19 439 312, 6.56}

Computing OrderTypes[7, {{2, 3}, {4, 5}, {6, 7}}]

Computing OrderTypes[7, {{4, 5}, {6, 7}}]

Computing OrderTypes[7, {{6, 7}}]

Computing OrderTypes[8, {{1, 2}, {3, 4}, {5, 6}, {7, 8}}]

Computing OrderTypes[8, {{3, 4}, {5, 6}, {7, 8}}]

Computing OrderTypes[8, {{5, 6}, {7, 8}}]

Computing OrderTypes[8, {{7, 8}}]

Length[vect] is 9400

l is 0

l is 9400

1

{2, Diag[ar[1, 5], ar[2, 6], ar[3, 8], ar[4, 7]], 167 517 400, 93.8537}

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Length[vect] is 9431
1 is 9400
1 is 9470
2
{3, Diag[ar[1, 5], ar[2, 7], ar[3, 6], ar[4, 8]], 192888800, 174.01}
Length[vect] is 9414
1 is 9470
1 is 9470
3
{4, Diag[ar[1, 5], ar[2, 7], ar[3, 8], ar[4, 6]], 200494064, 257.585}
Length[vect] is 9451
1 is 9470
1 is 9485
4
{5, Diag[ar[1, 5], ar[2, 8], ar[3, 6], ar[4, 7]], 207579064, 340.929}
Length[vect] is 9449
1 is 9485
1 is 9492
5
{6, Diag[ar[1, 5], ar[2, 8], ar[3, 7], ar[4, 6]], 207579064, 424.786}
Length[vect] is 9272
1 is 9492
1 is 9492
6
{7, Diag[ar[1, 6], ar[2, 5], ar[3, 7], ar[4, 8]], 207579064, 508.481}
Length[vect] is 9431
1 is 9492
1 is 9492
7
{8, Diag[ar[1, 6], ar[2, 5], ar[3, 8], ar[4, 7]], 207579064, 590.539}
Length[vect] is 9361
1 is 9492
1 is 9492
8
{9, Diag[ar[1, 6], ar[2, 7], ar[3, 5], ar[4, 8]], 207579064, 674.077}

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Length[vect] is 9451
1 is 9492
1 is 9496
9
{10, Diag[ar[1, 6], ar[2, 7], ar[3, 8], ar[4, 5]], 216 381 328, 761.003}
Length[vect] is 9387
1 is 9496
1 is 9496
10
{11, Diag[ar[1, 6], ar[2, 8], ar[3, 5], ar[4, 7]], 216 381 328, 832.005}
Length[vect] is 9365
1 is 9496
1 is 9499
11
{12, Diag[ar[1, 6], ar[2, 8], ar[3, 7], ar[4, 5]], 219 843 912, 917.063}
Length[vect] is 9315
1 is 9499
1 is 9499
12
{13, Diag[ar[1, 7], ar[2, 5], ar[3, 6], ar[4, 8]], 219 843 912, 989.895}
Length[vect] is 9449
1 is 9499
1 is 9499
13
{14, Diag[ar[1, 7], ar[2, 5], ar[3, 8], ar[4, 6]], 220 158 912, 1073.63}
Length[vect] is 9365
1 is 9499
1 is 9499
14
{15, Diag[ar[1, 7], ar[2, 6], ar[3, 5], ar[4, 8]], 225 139 600, 1157.55}
Length[vect] is 9272
1 is 9499
1 is 9499
15
{16, Diag[ar[1, 7], ar[2, 6], ar[3, 8], ar[4, 5]], 225 139 600, 1239.32}
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Length[vect] is 9315
1 is 9499
1 is 9499
16
{17, Diag[ar[1, 7], ar[2, 8], ar[3, 5], ar[4, 6]], 225 139 600, 1311.12}
Length[vect] is 8939
1 is 9499
1 is 9499
17
{18, Diag[ar[1, 7], ar[2, 8], ar[3, 6], ar[4, 5]], 225 139 600, 1382.29}
Length[vect] is 8858
1 is 9499
1 is 9499
18
{19, Diag[ar[1, 8], ar[2, 5], ar[3, 6], ar[4, 7]], 225 139 600, 1451.88}
Length[vect] is 9366
1 is 9499
1 is 9500
19
{20, Diag[ar[1, 8], ar[2, 5], ar[3, 7], ar[4, 6]], 225 139 600, 1525.95}
Length[vect] is 9285
1 is 9500
1 is 9500
20
{21, Diag[ar[1, 8], ar[2, 6], ar[3, 5], ar[4, 7]], 233 802 968, 1610.45}
Length[vect] is 9285
1 is 9500
1 is 9500
21
{22, Diag[ar[1, 8], ar[2, 6], ar[3, 7], ar[4, 5]], 235 462 808, 1695.58}
Length[vect] is 8927
1 is 9500
1 is 9500
22
{23, Diag[ar[1, 8], ar[2, 7], ar[3, 5], ar[4, 6]], 235 462 808, 1767.32}

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Length[vect] is 8808
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1 is 9500
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1 is 9500
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23
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{24, Diag[ar[1, 8], ar[2, 7], ar[3, 6], ar[4, 5]], 235 462 808, 1838.26}
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Length[vect] is 7983
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1 is 9500
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1 is 9500
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24
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Computing Rank...
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24
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