

```

diag = Diag[ar[1, 3], ar[4, 6], ar[5, 2]]
Diag[ar[1, 3], ar[4, 6], ar[5, 2]]

p = Times @@ (diag /. ar[i_, j_] => eq[i, j-1] eq[i-1, j] ltheq[i-1, i])
eq[0, 3] eq[1, 2] eq[3, 6] eq[4, 2] eq[4, 5] eq[5, 1] ltheq[0, 1] ltheq[3, 4] ltheq[4, 5]

p = (p
  /. {rule = Cases[p, eq[i_, j_] => (i -> j), Infinity, 1];
    eq[ij_] => (eq[ij] /. rule),
    ltheq[ij_] => (ltheq[ij] /. rule)
  }
  /. {
    eq[i_, i_] -> 1
  }
)
ltheq[5, 5] ltheq[6, 5]^2

p = Times @@ (diag /. ar[i_, j_] => eq[i, j-1] eq[i-1, j] ltheq[i-1, i]);
While[!FreeQ[p, eq],
  p = (p
    /. Cases[p, eq[i_, j_] => (i -> j), Infinity, 1]
    /. {
      eq[i_, i_] -> 1
    }
  )
]
p

ltheq[5, 5] ltheq[6, 5]^5

l = Length[indices = Union@@ Cases[{p}, ltheq[i_, j_] => {i, j}, Infinity]]
2

indices
{5, 6}

ineqs = p /. e_ltheq -> (e /. Thread[indices -> Range[1]])

ltheq[1, 1] ltheq[2, 1]^5

```

```

Expand[Plus @@ (Times[
  ineqs /. ltheq[i_, j_] => Switch[
    Order#[[i]], #[[j]],
    1, 1,
    0, 1/2,
    -1, 0
  ],
  Binomial[n, Max[#]]
] & /@ OrderTypes[1])]

```

$$-\frac{15n}{64} + \frac{n^2}{4}$$

Wgl[diag]

$$\frac{n}{4}$$