

**OrderTypes [3]**

```
{ {3, 2, 1}, {2, 1, 1}, {3, 1, 2}, {2, 1, 2}, {2, 1, 3}, {2, 2, 1},  
  {1, 1, 1}, {1, 1, 2}, {2, 3, 1}, {1, 2, 1}, {1, 3, 2}, {1, 2, 2}, {1, 2, 3}}
```

**? OrderTypes**

Global `OrderTypes

```
OrderTypes [0] = {{}}
```

```
OrderTypes [1] = {{1}}
```

```
OrderTypes [2] = {{2, 1}, {1, 1}, {1, 2}}
```

```
OrderTypes [3] = {{3, 2, 1}, {2, 1, 1}, {3, 1, 2}, {2, 1, 2}, {2, 1, 3},  
  {2, 2, 1}, {1, 1, 1}, {1, 1, 2}, {2, 3, 1}, {1, 2, 1}, {1, 3, 2}, {1, 2, 2}, {1, 2, 3}}
```

```
OrderTypes [l_List] :=
```

```
Module[{nl, snl}, ((snl = Union[nl = Append[1, #1]]; nl /. Thread[snl → Range[Length[snl]]]) &) /@  
  Range[ $\frac{1}{2}$ ,  $\frac{1}{2}$  + Max[1],  $\frac{1}{2}$ ]]
```

```
OrderTypes [n_Integer] := OrderTypes [n] = Join @@ OrderTypes /@ OrderTypes [n - 1]
```

```
diag = Diag[ar[5, 4], ar[6, 3], ar[7, 1], ar[8, 2]]
```

```
Diag[ar[5, 4], ar[6, 3], ar[7, 1], ar[8, 2]]
```

```
p = Times @@ (diag /. ar[i_, j_] => eq[i, j - 1] eq[i - 1, j] ltheq[i - 1, i]);
```

```
While[! FreeQ[p, eq],
```

```
  p = (p  
    /. Cases[p, eq[i_, j_] => (i → j), Infinity, 1]  
    /. {  
      eq[i_, i_] → 1,  
      ltheq[i_, i_] → 1/2  
    }  
  )
```

```
]
```

```
p
```

```
 $\frac{1}{4}$  ltheq[3, 2] ltheq[4, 3]
```

```
l = Length[indices = Union @@ Cases[{p}, ltheq[i_, j_] => {i, j}, Infinity]]
```

```
3
```

```
indices
```

```
{2, 3, 4}
```

```
ineqs = p /. e_ltheq => (e /. Thread[indices → Range[l]])
```

```
 $\frac{1}{4}$  ltheq[2, 1] ltheq[3, 2]
```

```

Expand[Plus @@ (Times[
  ineqs /. ltheq[i_, j_] => Switch[
    Order#[[i]], #[[j]],
    1, 1,
    0, 1/2,
    -1, 0
  ],
  Binomial[n, Max[#]]
] & /@ OrderTypes[1])]

```

$$\frac{n}{48} + \frac{n^3}{24}$$

Wgl[diag]

$$\frac{n}{48} + \frac{n^3}{24}$$