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[Book] *Introduction to Vassiliev Knot Invariants*, by S. Chmutov, S. Duzhin, and J. Mostovoy, Cambridge University Press, Cambridge UK, 2012, xvi+504pp., hardback, \$70.00, ISBN 978-1-10702-083-2.

Merely 30 years ago, if you had asked even the best informed mathematician about the relationship between knots and Lie algebras, she would have laughed, for there isn't and there can't be. Knots are flexible, Lie algebras are rigid. Knots are irregular, Lie algebras are symmetric. The list of knots is a lengthy mess, the collection of Lie algebras is well-organized. Knots are useful for sailors, scouts, and hangmen, Lie algebras for navigators, engineers, and high energy physicists. Knots are blue collar, Lie algebras are white. They are as similar as worms and crystals.

Then in the 1980s came Jones, and Witten, and Reshetikhin and Turaev [Jo, Wi, RT] and showed that if you really are the best informed, and you know about quantum field theory and conformal field theory and quantum groups, then you know that the two disjoint fields are in fact intricately related. This "quantum" approach remains the most powerful way to get computable knot invariants out of (certain) Lie algebras (and representations thereof). Yet shortly later, in the late 80s and early 90s, an alternative perspective aroused, that of "finite-type" or "Vassiliev-Goussarov" invariants [Va1, Va2, Go1, Go2, BL, Ko1, Ko2, BN1], which made the surprising relationship between knots and Lie algebras appear simple and almost inevitable.

The reviewed book is about that alternative perspective, the one reasonable sounding but not entirely trivial theorem that is needed within it (the "fundamental theorem" or the "Kontsevich integral" REF HERE?), and the many threads that begin with that perspective. Let me start with a brief summary of the mathematics, and even before, a very brief summary.

The very brief summary is that in some combinatorial sense it makes sense to "differentiate" knot invariants, and hence it makes sense to talk about "polynomials" on the space of knots—these are functions on the set of knots (meaning, knot invariants) whose sufficiently high derivatives vanish. Such polynomials can be fairly conjectured to separate knots—elsewhere in math in lucky cases polynomials separate points, and in our case, specific computations are encouraging. Also, such polynomials are determined by their "coefficients", and each of these, by the one-side-easy "fundamental theorem", is a linear functional on some finite space of graphs modulo relations. These same graphs turn out to parameterize formulas that make sense in a wide class of Lie algebras, and these relations match exactly with the relations in the definition of a Lie algebra—anti-symmetry and the Jacobi identity. Hence what is more or less dual to knots (invariants), is also, after passing to the coefficients, more or less dual to Lie algebras. QED, and onto the brief summary <sup>1</sup>.

Let  $V$  be an arbitrary invariant of oriented knots in oriented space with values in some Abelian group  $A$ . Extend  $V$  to be an invariant of 1-singular knots, knots that may have a single singularity that locally looks like a double point, using the formula

$$(1) \quad V(\text{singularity}) = V(\text{!}) - V(\text{"}).$$

<sup>1</sup>2010 *Mathematics Subject Classification*. Primary 57M25.

$\TeX$  at <http://drorbn.net/AcademicPensieve/2013-01/CDMReview/>, copy left at <http://www.math.toronto.edu/~drorbn/Copyleft/>

<sup>1</sup>Partially self-plagiarized from [BN2].

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