

Pensieve header: With Joe Boninger: ρ_1 is not log-concave even on alternating knots. Continues pensieve://2023-03/ (for Alexander).

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/APAI> to compute rotation numbers.

```
In[ ]:= ConcaveQ[s_List] := And@@Thread[2 s[[2 ;; -2]] ≥ s[[1 ;; -3]] + s[[3 ;; -1]]];
```

```
In[ ]:= AlexanderLogConcaveQ[K_] := Module[{alex, T, cs},
alex = Alexander[K] [-T];
cs = CoefficientList[Sign[alex /. T → 1] TExponent[alex, T] alex, T];
If[Not[And@@(Positive /@ cs)], Return@False];
If[Length[cs] < 3, Return@True];
ConcaveQ[Log@cs]
]
```

```
In[ ]:= NLCs = Select[AllKnots[{3, 10}], Not[AlexanderLogConcaveQ[#]] &]
```

 KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[ ]:= {Knot[8, 19], Knot[9, 42], Knot[9, 43], Knot[9, 47], Knot[10, 124], Knot[10, 125],
Knot[10, 128], Knot[10, 134], Knot[10, 138], Knot[10, 139], Knot[10, 142], Knot[10, 145],
Knot[10, 152], Knot[10, 153], Knot[10, 154], Knot[10, 160], Knot[10, 161]}
```

```
In[ ]:= AlternatingQ /@ NLCs
```

```
Out[ ]:= {False, False, False, False, False, False, False, False,
False, False, False, False, False, False, False}
```

```
In[ ]:= Total[
AlternatingQ /@ Select[AllKnots[{3, 10}], Not[AlexanderLogConcaveQ[#]] &]
]
```

```
Out[ ]:= 17 False
```

ρ_1

```

In[*]:= R1[s_, i_, j_] := s (g_{j,i} (g_{j^+,j} + g_{j,j^+} - g_{i,j}) - g_{i,i} (g_{j,j^+} - 1) - 1 / 2);
\rho[K_] := \rho[K] = Module[{Cs, \varphi, n, A, s, i, j, k, \Delta, G, \rho1},
  {Cs, \varphi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} \rightrightarrows (A[[{i, j}, {i + 1, j + 1}]] += (
    -T^s T^s - 1
  ))];
  \Delta = T^{(-Total[\varphi] - Total[Cs[[All, 1]]) / 2} Det[A];
  G = Inverse[A];
  \rho1 = \sum_{k=1}^n R1 @@ Cs[[k]] - \sum_{k=1}^{2^n} \varphi[[k]] (g_{kk} - 1 / 2);
  Factor@{\Delta, \Delta^2 \rho1 / . \alpha_+ \rightrightarrows \alpha + 1 / . g_{\alpha, \beta} \rightrightarrows G[[\alpha, \beta]]};

```

```

In[*]:= K = Knot[8, 4]

```

```

Out[*]=
Knot[8, 4]

```

```

In[*]:= \rho[K][[2]] // Factor

```

```

Out[*]=
\frac{(-1 + T)^2 (3 - 8 T + 6 T^2 - 4 T^3 + 6 T^4 - 8 T^5 + 3 T^6)}{T^4}

```

```

In[*]:= \rho1 = Expand@Factor[\frac{T \rho[K][[2]]}{(-1 + T)^2} / . T \rightrightarrows -T]

```

```

Out[*]=
-4 - \frac{3}{T^3} - \frac{8}{T^2} - \frac{6}{T} - 6 T - 8 T^2 - 3 T^3

```

```

In[*]:= cs = CoefficientList[Sign[\rho1 / . T \rightrightarrows 1] T^{Exponent[\rho1, T]} \rho1, T]

```

```

Out[*]=
{3, 8, 6, 4, 6, 8, 3}

```

```

In[*]:= ConcaveQ[Log@cs]

```

```

Out[*]=
False

```


```

In[*]:= \rho1LogConcaveQ[K_] := Module[{rho1, cs},
  \rho1 = Expand[\rho[K][[2]] / . T \rightrightarrows -T];
  cs = CoefficientList[Sign[\rho1 / . T \rightrightarrows 1] T^{Exponent[\rho1, T]} \rho1, T];
  If[Not[And@@(Positive /@ cs)], Return@False];
  If[Length[cs] < 3, Return@True];
  ConcaveQ[Log@cs]
]

```

```
In[*]:= NLCs = Select[AllKnots[{3, 10}], Not[ $\rho$ 1LogConcaveQ[#]] &]
```

 **Infinity:** Indeterminate expression $T^{-\infty}$ encountered.

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 **General:** Further output of Infinity::indet will be suppressed during this calculation.

```
Out[*]=
```

```
{Knot[8, 4], Knot[8, 19], Knot[9, 17], Knot[9, 22], Knot[9, 43], Knot[9, 47], Knot[10, 4],
Knot[10, 8], Knot[10, 15], Knot[10, 19], Knot[10, 52], Knot[10, 54], Knot[10, 93],
Knot[10, 108], Knot[10, 128], Knot[10, 129], Knot[10, 130], Knot[10, 132], Knot[10, 134],
Knot[10, 136], Knot[10, 138], Knot[10, 139], Knot[10, 142], Knot[10, 145],
Knot[10, 152], Knot[10, 153], Knot[10, 154], Knot[10, 160], Knot[10, 161]}
```

```
In[*]:= AlternatingQ /@ NLCs
```

```
Out[*]=
```

```
{False, False, False, False, False, False, False, False,
False, False, False, False, False, False, False}
```

```
In[*]:= Total[
  AlternatingQ /@ Select[AllKnots[{3, 10}], Not[AlexanderLogConcaveQ[#]] &]
]
```

```
Out[*]=
```

```
17 False
```