Pensieve header: Invariance of ρ_1 in the language of perturbed Gaussian Integration.

Programs

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

In[∘]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ; CF[ℰ_EPD] := CF /@ ℰ;
CF[sd_SeriesData] := MapAt[CF, sd, 3];
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p)_, ∞] ⋃ {x, p}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) ⧴ CCF[c] (Times @@ vs^ps)]];
CF[eqp_EQP] := CF /@ eqp
```

In[∘]:=
```
EQP /: c_ * EQP[Q_, P_] := EQP[Q, CF[c P]];
EQP /: EQP[Q1_, P1_] EQP[Q2_, P2_] := CF@EQP[Q1 + Q2, P1 + P2];
```

In[∘]:=
```
{p*, x*} = {π, ξ}; (z_i_)* := (z*)_i; vs_List* := (v ↦ v*) /@ vs;
Zip_{}[ℰ_] := ℰ;
Zip_{z_,zs___}[ℰ_] := (Collect[ℰ // Zip_{zs}, z] /. f_. z^d_. ⧴ (D[f, {z*, d}])) /. z* → 0
```

In[∘]:=
```
FI[EQP[Q_, P_]] := FI[EQP[Q, P], Union@Cases[Q, p_, ∞], Union@Cases[Q, x_, ∞]];
FI[EQP[Q_, P_], ps_List, xs_List] := Module[{u, v},
  A = Table[∂_{u,v} Q, {u, ps}, {v, xs}];
  CF[Det[A]^-1 Series[Zip_{ps⋃xs}[Normal[ⅇ^P] ⅇ^(-xs*.Inverse[A].ps*)], {ϵ, 0, 1}]]
  ]]
```

In[∘]:=
```
(α_⁺)⁺ := α^"++";  (* this is for cosmetic reasons only *)
```

In[∘]:=
```
R0[s_, i_, j_] := EQP[-x_i (p_i - T^s p_i⁺ + (T^s - 1) p_j⁺) - x_j (p_j - p_j⁺), 0];
r1[s_, i_, j_] :=
  s (-1 + 2 p_i x_i - 2 p_j x_i + (-1 + T^s) p_i p_j x_i^2 + (1 - T^s) p_j^2 x_i^2 - 2 p_i p_j x_i x_j + 2 p_j^2 x_i x_j) / 2;
γ1[φ_, k_] := φ (1 / 2 - p_k x_k);
R1[s_, i_, j_] := EQP[-x_i (p_i - T^s p_i⁺ + (T^s - 1) p_j⁺) - x_j (p_j - p_j⁺), ϵ r1[s, i, j] + O[ϵ]^2];
```

In[∘]:=
```
eqp1[K_] := Module[{Cs, φ, n, eqp, s, i, j, k, α},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   eqp = EQP[-x_{2n+1} p_{2n+1}, O[ϵ]^2];
   Cases[Cs, {s_, i_, j_} :> (eqp **= R_1[s, i, j])];
   Do[eqp **= EQP[0, ϵ γ_1[φ⟦k⟧, k]], {k, 2 n}];
   CF[eqp /. (α_)^+ :> α + 1] ];
```

In[∘]:=
```
eqp1[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

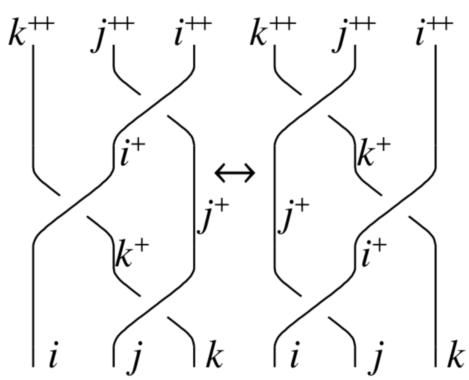
Out[∘]=

$$\text{EQP}\Big[-p_1 x_1 + p_2 x_1 - p_2 x_2 + \frac{p_3 x_2}{T} + \frac{(-1+T)\, p_6 x_2}{T} - p_3 x_3 + p_4 x_3 + \frac{(-1+T)\, p_2 x_4}{T} - p_4 x_4 + \frac{p_5 x_4}{T} - p_5 x_5 + p_6 x_5 + \frac{(-1+T)\, p_4 x_6}{T} - p_6 x_6 + \frac{p_7 x_6}{T} - p_7 x_7,$$
$$\Big(1 - p_2 x_2 + p_5 x_2 + \frac{(-1+T)\, p_2 p_5 x_2^2}{2T} - \frac{(-1+T)\, p_5^2 x_2^2}{2T} + p_1 x_4 - p_1^2 x_1 x_4 + p_1 p_4 x_1 x_4 - \frac{(-1+T)\, p_1^2 x_4^2}{2T} + \frac{(-1+T)\, p_1 p_4 x_4^2}{2T} + p_2 p_5 x_2 x_5 - p_5^2 x_2 x_5 + p_3 x_6 - p_6 x_6 - p_3^2 x_3 x_6 + p_3 p_6 x_3 x_6 - \frac{(-1+T)\, p_3^2 x_6^2}{2T} + \frac{(-1+T)\, p_3 p_6 x_6^2}{2T}\Big)\, \epsilon + O[\epsilon]^2\Big]$$

In[∘]:=
```
eqp1[Knot[5, 2]] // FI
```

Out[∘]=

$$-\frac{T^4}{2 - 3T + 2T^2} - \frac{(-1+T)^2\, T^4 \left(5 - 4T + 5T^2\right) \epsilon}{\left(2 - 3T + 2T^2\right)^3} + O[\epsilon]^2$$



In[∘]:=
```
lhs = R_0[1, j, k] R_0[1, i, k^+] R_0[1, i^+, j^+]
```

Out[∘]=

$$\text{EQP}\big[-p_i x_i + (1-T)\, p_{k^{++}} x_i + T\, p_{i^+} x_i - p_j x_j + T\, p_{j^+} x_j + (1-T)\, p_{k^+} x_j - p_k x_k + p_{k^+} x_k + T\, p_{i^{++}} x_{i^+} + (1-T)\, p_{j^{++}} x_{i^+} - p_{i^+} x_{i^+} + p_{j^{++}} x_{j^+} - p_{j^+} x_{j^+} + p_{k^{++}} x_{k^+} - p_{k^+} x_{k^+},\ 0\big]$$

In[ ]:= **rhs = R_0[1, i, j] R_0[1, i^+, k] R_0[1, j^+, k^+]**

Out[ ]=

$\text{EQP}[-p_i x_i + T p_{i^+} x_i + (1-T) p_{j^+} x_i - p_j x_j + p_{j^+} x_j - p_k x_k + p_{k^+} x_k + T p_{i^{++}} x_{i^+} - p_{i^+} x_{i^+} + (1-T) p_{k^+} x_{i^+} + T p_{j^{++}} x_{j^+} + (1-T) p_{k^{++}} x_{j^+} - p_{j^+} x_{j^+} + p_{k^{++}} x_{k^+} - p_{k^+} x_{k^+}, 0]$