

Pensieve header: Compute and store ρ , Kh, and HOMFLYPT for knots with up to 14 crossings

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APerturbedAlexanderInvariant"];
```

```
In[ ]:= Once[<< KnotTheory` ; << RVK.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:=  $\rho[K\_]$  :=  $\rho[K]$  = Module[{Cs, r, n, B, A, c, s, i, j,  $\Delta$ , G, g,  $\rho1$ },
  {Cs, r} = List@@RVK[K]; n = Length[Cs]; B = Table[0, 2 n, 2 n + 1];
  Do[{s, i, j} = c;
    B[[{i, j}, {i, j, i + 1, j + 1}]] =  $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & T^s & -1 - T^s \end{pmatrix}$ , {c, Cs}];
  A = B[[All, 2 ;;]];
   $\Delta$  = T^(Total[r] - Total[First/@Cs])/2 Det[A];
  G = Prepend[Table[0, 2 n]] [Inverse[A]];  $g_{\alpha, \beta}$  := G[[ $\alpha$ ,  $\beta$ ]];
   $\rho1$  =  $\Delta^2$  Sum[{s, i, j} = c;
    s ((1 - T^s)  $g_{ij}$  ( $g_{ij} - g_{jj}$ ) + 2  $g_{ii} g_{ij} - g_{ij} g_{ji} - g_{ii} g_{jj} - g_{ij} + g_{jj} - 1 / 2$ ), {c, Cs}];
   $\rho1$  +=  $\Delta^2$  Sum[r[[k]] ( $g_{kk} - 1 / 2$ ), {k, 2 n}];
  Factor@{ $\Delta$ ,  $\rho1$ };
```

```
In[ ]:= Do[Echo[K  $\rightarrow$   $\rho[K]$ ], {K, AllKnots[{3, 6}]}]
```

KnotTheory: Loading precomputed data in PD4Knots`

- » Knot[3, 1] \rightarrow $\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}$
- » Knot[4, 1] \rightarrow $\left\{ -\frac{1 - 3 T + T^2}{T}, 0 \right\}$
- » Knot[5, 1] \rightarrow $\left\{ \frac{1 - T + T^2 - T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2 T^4)}{T^4} \right\}$
- » Knot[5, 2] \rightarrow $\left\{ \frac{2 - 3 T + 2 T^2}{T}, \frac{(-1 + T)^2 (5 - 4 T + 5 T^2)}{T^2} \right\}$
- » Knot[6, 1] \rightarrow $\left\{ -\frac{(-2 + T) (-1 + 2 T)}{T}, \frac{(-1 + T)^2 (1 - 4 T + T^2)}{T^2} \right\}$
- » Knot[6, 2] \rightarrow $\left\{ -\frac{1 - 3 T + 3 T^2 - 3 T^3 + T^4}{T^2}, \frac{(-1 + T)^2 (1 - 4 T + 4 T^2 - 4 T^3 + 4 T^4 - 4 T^5 + T^6)}{T^4} \right\}$
- » Knot[6, 3] \rightarrow $\left\{ \frac{1 - 3 T + 5 T^2 - 3 T^3 + T^4}{T^2}, 0 \right\}$

```
In[ ]:= Timing[Length@Union[ $\rho$  /@AllKnots[{3, 10}]]]
```

```
Out[ ]:= {128.125, 249}
```

In[]:= **Timing**[**Length**@**Union**[{**Kh**[**PD**@#][**q**, **t**], **HOMFLYPT**[**PD**@#][**a**, **z**] & /@ **AllKnots**[{**3**, **10**}]]]

KnotTheory: The Khovanov homology program JavaKh-v2 is an update of Jeremy Green's program JavaKh-v1, written by Scott Morrison in 2008 at Microsoft Station Q.

KnotTheory: The HOMFLYPT program was written by Scott Morrison.

Out[]:= {6., 248}

In[]:= **Length**[**AllKnots**[{**3**, **13**}]]

Out[]:= 12965

In[]:= **Monitor**[**Timing**[**Tally** ρ **13** = **Tally**[**Last** /@ **Tally**@**Table**[ρ [**K**], {**K**, **AllKnots**[{**3**, **13**}]}]]], **K**]

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

Out[]:= {20270.5, {{1, 11140}, {2, 809}, {4, 33}, {3, 23}, {6, 1}}}

In[]:= **Total**[**Times**@@@**Rest**[**Tally** ρ **13**]]

Out[]:= 1825

In[]:= **Monitor**[**Timing**[**Tally**H**Kh****13** = **Tally**[**Last** /@ **Tally**@**Table**[{**Kh**[**PD**@**K**][**q**, **t**], **HOMFLYPT**[**PD**@**K**][**a**, **z**]}, {**K**, **AllKnots**[{**3**, **13**}]}]]], **K**]

Out[]:= {950., {{1, 9714}, {2, 1269}, {3, 150}, {4, 47}, {5, 10}, {6, 3}, {7, 1}}}

In[]:= **Total**[**Times**@@@**Rest**[**Tally**H**Kh****13**]]

Out[]:= 3251

In[]:= {**NumberOfKnots**[**14**], **NumberOfKnots**[**14**, **Alternating**], **NumberOfKnots**[**14**, **NonAlternating**]}

Out[]:= {46972, 19536, 27436}

In[]:= **Monitor**[**Timing**[**Tally** ρ **14** = **Tally**[**Last** /@ **Tally**@**Table**[ρ [**K**], {**K**, **AllKnots**[{**3**, **14**}]}]]], **K**]

KnotTheory: Loading precomputed data in KnotTheory/14A.dts.

KnotTheory: Loading precomputed data in KnotTheory/14N.dts.

Out[]:= {207320., {{1, 48336}, {2, 4814}, {3, 217}, {4, 291}, {6, 19}, {5, 4}, {8, 3}}}

In[]:= **Monitor**[**Timing**[**Tally**H**Kh****14** = **Tally**[**Last** /@ **Tally**@**Table**[{**Kh**[**PD**@**K**][**q**, **t**], **HOMFLYPT**[**PD**@**K**][**a**, **z**]}, {**K**, **AllKnots**[{**3**, **14**}]}]]], **K**]

Out[]:= {6727.34, {{1, 40661}, {2, 6969}, {3, 965}, {5, 85}, {4, 411}, {6, 43}, {8, 6}, {10, 1}, {9, 1}, {7, 7}}}

In[]:= {**Total**[**Times**@@@**Rest**[**Tally** ρ **14**]], **Total**[**Times**@@@**Rest**[**Tally**H**Kh****14**]]}

Out[]:= {11601, 19276}

```
Table[K →  $\rho$ [K], {K, AllKnots[{3, 14}]}] >> "Data/Rho3-14.m"  
In[ ]:= Monitor[Table[K → Kh[PD@K][q, t], {K, AllKnots[{3, 14}]}] >> "Data/Kh3-14.m", K]  
In[ ]:= Monitor[  
  Table[K → HOMFLYPT[PD@K][a, z], {K, AllKnots[{3, 14}]}] >> "Data/HOMFLYPT3-14.m", K]
```