Pensieve header: The Lashings Matrix.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

```
In[•]:= Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

```
In[•]:= R1[s_, i_, j_] := s (gji (gj*,j + gj,j* - gij) - gii (gj,j* - 1) - 1 / 2);
ρ[K_] := ρ[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1
                                                                 0     -1 ))];
    Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
    G = Inverse[A];
    ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (gkk - 1 / 2);
    Factor@{Δ, Δ^2 ρ1 /. α_^+ :> α + 1 /. gα_,β_ :> G[[α, β]]}];
```

```
In[•]:= CompareMatrices[A_, B_] := Grid[
  MapThread[Column@*List, {A, B} /. 0 → "", 2],
  Frame → All, ItemSize → All
 ]
```

```
In[•]:= GMat[K_] := Module[{Cs, φ, n, A, k, s, i, j},
          {Cs, φ} = Rot[K]; n = Length[Cs];
          A = IdentityMatrix[2 n + 1];
          For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]];
           A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1
                                             0    -1 )];
          Factor@Inverse[A]
         ];
        MatrixForm[GMat[Knot[4, 1]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
1 & T & T^2 & T & 1 & T & T^2 & T & 1 \\
0 & 1 & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T(-1+2T)}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & 1 \\
0 & 0 & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T(-1+2T)}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & 1 \\
0 & 0 & -\frac{(-1+T)T^2}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & -\frac{T(-1+2T)}{1-3T+T^2} & -\frac{T^3}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & 1 \\
0 & 0 & -\frac{(-1+T)T^2}{1-3T+T^2} & -\frac{(-1+T)T}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & -\frac{T(-1+2T)}{1-3T+T^2} & -\frac{T^3}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & 1 \\
0 & 0 & -\frac{(-1+T)T}{1-3T+T^2} & -\frac{-1+T}{1-3T+T^2} & -\frac{-1+T}{1-3T+T^2} & -\frac{-1+2T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & 1 \\
0 & 0 & -\frac{(-1+T)T}{1-3T+T^2} & -\frac{-1+T}{1-3T+T^2} & -\frac{-1+T}{1-3T+T^2} & -\frac{(-1+T)T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

```
LashMat[K_] := Module[{Cs, φ, n, A, α, β, k, s, i, j, Δ, G, g, k1, k2, s1, s2, i1, i2, j1, j2},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]];
   A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1
                                     0    -1 )];
  Δ = T^(-Total[φ]-Total[Cs[[All,1]]])/2 Det[A];
  G = Inverse[A];
  Factor[Table[
    {s1, i1, j1} = Cs[[k1]]; {s2, i2, j2} = Cs[[k2]];
    (g_{i1,j2} - T^s1 g_{i1+1,j2} - T^s2 (g_{i1,j2+1} - g_{i1+1,j2+1})) /. g_{α_,β_} :> G[[α, β]],
    {k1, n}, {k2, n}]]
 ];

Column[MatrixForm /@ {LM = LashMat[Knot[8, 17]], Factor[LM /. T → T^-1 / LM^T]}]
```

Out[•]=

$$\begin{pmatrix}
-\frac{(-1+T)\,T^2\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^3\,(1-T+2T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,T^3}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,(1-T+T^2)^2}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1}{1-4} \\
\frac{T\,(-1+2T-T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)\,T^3}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,T\,(-1+3T-2T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,(1-3T+4T^2-2T^3+T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{}{1-} \\
-\frac{(-1+T)^2\,T}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,(-1+2T-3T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)\,(1-3T+3T^2-3T^3+T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{T\,(1-4T+4T^2-3T^3+T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(}{1-4} \\
\frac{(-1+T)^2\,(1-T+T^2)^2}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,(1-2T+4T^2-3T^3+T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^3\,T\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)\,T\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{}{1-} \\
-\frac{(-1+T)^2\,(1-3T+4T^2-2T^3+T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,T\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,(-1+2T-T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^3\,T\,(2-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{}{1-4} \\
\frac{(-1+T)^2\,T^2}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,T\,(-1+2T-3T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+2T)\,(-1+T-2T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^3\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(}{1-} \\
\frac{(-1+T)^3\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,T\,(-1+T-2T^2+T^3)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^4\,T}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^2\,T}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-}{1-4} \\
-\frac{(-1+T)^3\,T\,(1-T+T^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{T\,(1-3T+5T^2-6T^3+2T^4)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & \frac{(-1+T)^4\,T^2}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(-1+T)^2\,T^2}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} & -\frac{(}{1-}
\end{pmatrix}$$

$$\begin{pmatrix}
-\frac{1}{T} & -\frac{(-1+T)^3\,(2-T+T^2)}{-1+2T-T^2+T^3} & -1 & -1 & -1 \\
-\frac{T^2\,(-1+T-2T^2+T^3)}{(-1+T)^3\,(1-T+2T^2)} & -\frac{1}{T} & -1 & -1 & -1 \\
-1 & -1 & -T & \frac{1-3T+4T^2-4T^3+T^4}{(-1+T)^3\,(1-T+T^2)} & -T \\
-1 & -1 & -\frac{(-1+T)^3\,(1-T+T^2)}{T\,(1-4T+4T^2-3T^3+T^4)} & -T & -\frac{-1+2T-T^2+T^3}{(-1+T)^3\,(2-T+T^2)} \\
-1 & -1 & -T & -\frac{(-1+T)^3\,(1-T+2T^2)}{T^2\,(-1+T-2T^2+T^3)} & -T \\
-1 & -1 & \frac{(-2+T)\,T^2\,(-1+2T-T^2+T^3)}{(-1+T)^2\,(1-4T+4T^2-3T^3+T^4)} & -T & -\frac{(-1+T)^2\,(-1+2T-T^2+T^3)}{T\,(2-6T+5T^2-3T^3+T^4)} \\
-\frac{1}{T} & -\frac{1}{T} & -1 & -1 & -1 \\
-\frac{1}{T} & -\frac{T\,(2-6T+5T^2-3T^3+T^4)}{(-1+T)^2\,(-1+2T-T^2+T^3)} & -1 & -1 & -1
\end{pmatrix}$$