


Pensieve header: Failed \bar{G} - \bar{G} experiments.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

```
In[ ]:= Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

 e: Symbol e appears in multiple contexts {KnotTheory`FastKh`Tangles`, Global`}; definitions in context KnotTheory`FastKh`Tangles` may shadow or be shadowed by other definitions.

Loading Rot.m from <http://drorbn.net/APAI> to compute rotation numbers.

```
In[ ]:= R1[s_, i_, j_] := s (gji (gj+,j + gj+,j* - gij) - gii (gj+,j* - 1) - 1 / 2);
rho[K_] := rho[K] = Module[{Cs, phi, n, A, s, i, j, k, Delta, G, rho1},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} => (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))];
  Delta = T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A];
  G = Inverse[A];
  rho1 = Sum[k=1 to n] R1 @@ Cs[[k]] - Sum[k=1 to 2^n] phi[[k]] (gkk - 1 / 2);
  Factor@{Delta, Delta^2 rho1 /. alpha_+ => alpha + 1 /. g_alpha_beta_ => G[[alpha, beta]]};
```

```
In[ ]:= GGbar[K_] := Module[{Cs, phi, n, A, alpha, beta, k, s, i, j, G},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  For[k = 1, k <= n, k++,
    {s, i, j} = Cs[[k]];
    A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 );
  ];
  G = Factor[T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A] x Inverse[A]];
  Cases[Cs, {s_, i_, j_} => If[s == 1 & i < j,
    Factor@{(G[[i, j]] /. T -> T^-1), G[[i, i]], G[[i, j]],
      G[[j, i]], G[[j, j]], G[[1, i]], G[[1, j]], G[[i, 2 n + 1]], G[[j, 2 n + 1]]},
    Nothing
  ]
];
GGbar[Ks_List] := Join@@ (GGbar /@ Ks)
```

```
In[*]:= mat = GGbar[Knot[10, 163]];
mat // MatrixForm
```

Out[*]//MatrixForm=

$$\begin{pmatrix} \frac{(1-T+T^2)(1-4T+7T^2-4T^3+T^4)}{T^4} & \frac{(1-T+T^2)(1-4T+7T^2-4T^3+T^4)}{T^3} & \frac{(1-T+T^2)(1-4T+7T^2-4T^3+T^4)}{T^2} & 0 & \\ \frac{1-T+T^2}{T} & -\frac{(1-T+T^2)(-1+3T-4T^2+T^3)}{T^4} & \frac{1-T+T^2}{T} & \frac{(-1+T)(1-T+T^2)(-1+3T-4T^2+T^3)}{T^4} & \frac{(1-4T+7T^2-4T^3+T^4)}{T^3} \\ \frac{1-4T+7T^2-4T^3+T^4}{T^3} & \frac{(2-2T+T^2)(1-3T+3T^2)}{T^3} & \frac{1-4T+7T^2-4T^3+T^4}{T} & \frac{(-1+T)^2(1-T+T^2)}{T^3} & \frac{(1-4T+7T^2-4T^3+T^4)}{T^3} \\ \frac{1-4T+9T^2-8T^3+3T^4}{T^3} & \frac{1-4T+7T^2-4T^3+T^4}{T^2} & \frac{3-8T+9T^2-4T^3+T^4}{T} & \frac{(-1+T)(1-4T+7T^2-4T^3+T^4)}{T^3} & \frac{1-5T+4T^2-4T^3+T^4}{T^3} \end{pmatrix}$$

```
In[*]:= NullSpace[mat]
```

Out[*]=

$$\left\{ \begin{aligned} & \{ T^2 - 13 T^3 + 75 T^4 - 257 T^5 + 582 T^6 - 906 T^7 + 946 T^8 - 545 T^9 - 94 T^{10} + 522 T^{11} - 514 T^{12} + \\ & 275 T^{13} - 87 T^{14} + 15 T^{15} - T^{16}, 2 T - 26 T^2 + 158 T^3 - 582 T^4 + 1427 T^5 - 2392 T^6 + 2658 T^7 - \\ & 1588 T^8 - 400 T^9 + 2096 T^{10} - 2612 T^{11} + 2050 T^{12} - 1122 T^{13} + 428 T^{14} - 108 T^{15} + 16 T^{16} - T^{17}, \\ & -1 + 15 T - 108 T^2 + 471 T^3 - 1362 T^4 + 2687 T^5 - 3552 T^6 + 2769 T^7 - 333 T^8 - \\ & 2147 T^9 + 3070 T^{10} - 2415 T^{11} + 1266 T^{12} - 456 T^{13} + 110 T^{14} - 16 T^{15} + T^{16}, \\ & -2 T + 26 T^2 - 159 T^3 + 592 T^4 - 1480 T^5 + 2575 T^6 - 3084 T^7 + 2230 T^8 - 123 T^9 - \\ & 2113 T^{10} + 3180 T^{11} - 2722 T^{12} + 1531 T^{13} - 571 T^{14} + 135 T^{15} - 18 T^{16} + T^{17}, \\ & -2 T + 26 T^2 - 153 T^3 + 539 T^4 - 1247 T^5 + 1935 T^6 - 1892 T^7 + 746 T^8 + 909 T^9 - \\ & 1965 T^{10} + 1900 T^{11} - 1162 T^{12} + 467 T^{13} - 119 T^{14} + 17 T^{15} - T^{16}, 0, 0, 0, 0 \}, \\ & \{ -2 T^2 + 24 T^3 - 124 T^4 + 370 T^5 - 717 T^6 + 958 T^7 - 907 T^8 + 607 T^9 - 279 T^{10} + 83 T^{11} - 14 T^{12} + T^{13}, \\ & -4 T^3 + 24 T^4 - 61 T^5 + 70 T^6 - 13 T^7 - 55 T^8 + 69 T^9 - 37 T^{10} + 10 T^{11} - T^{12}, \\ & 2 - 22 T + 112 T^2 - 345 T^3 + 706 T^4 - 993 T^5 + 961 T^6 - 625 T^7 + 257 T^8 - 58 T^9 + 5 T^{10}, \\ & 2 T^2 - 14 T^3 + 43 T^4 - 69 T^5 + 51 T^6 + 10 T^7 - 62 T^8 + 64 T^9 - 33 T^{10} + 9 T^{11} - T^{12}, 0, \\ & -2 T^2 + 16 T^3 - 49 T^4 + 72 T^5 - 35 T^6 - 41 T^7 + 73 T^8 - 47 T^9 + 12 T^{10} - T^{11}, 0, 0, 0 \}, \\ & \{ -2 T^2 + 24 T^3 - 124 T^4 + 370 T^5 - 717 T^6 + 958 T^7 - 907 T^8 + 607 T^9 - 279 T^{10} + 83 T^{11} - 14 T^{12} + T^{13}, \\ & -4 T^3 + 24 T^4 - 61 T^5 + 70 T^6 - 13 T^7 - 55 T^8 + 69 T^9 - 37 T^{10} + 10 T^{11} - T^{12}, \\ & 2 - 22 T + 112 T^2 - 345 T^3 + 706 T^4 - 993 T^5 + 961 T^6 - 625 T^7 + 257 T^8 - 58 T^9 + 5 T^{10}, \\ & 2 T^2 - 14 T^3 + 43 T^4 - 69 T^5 + 51 T^6 + 10 T^7 - 62 T^8 + 64 T^9 - 33 T^{10} + 9 T^{11} - T^{12}, 0, \\ & 0, -2 T + 16 T^2 - 49 T^3 + 72 T^4 - 35 T^5 - 41 T^6 + 73 T^7 - 47 T^8 + 12 T^9 - T^{10}, 0, 0 \}, \\ & \{ T^2 - 7 T^3 + 17 T^4 - 19 T^5 + 11 T^6 - 14 T^7 + 26 T^8 - 21 T^9 + 8 T^{10} - T^{11}, \\ & 2 T - 16 T^2 + 56 T^3 - 103 T^4 + 90 T^5 + 7 T^6 - 97 T^7 + 101 T^8 - 49 T^9 + 12 T^{10} - T^{11}, \\ & -1 + 7 T - 24 T^2 + 50 T^3 - 66 T^4 + 48 T^5 - 2 T^6 - 33 T^7 + 29 T^8 - 10 T^9 + T^{10}, \\ & -2 T + 16 T^2 - 57 T^3 + 107 T^4 - 95 T^5 - 10 T^6 + 106 T^7 - 102 T^8 + 47 T^9 - 11 T^{10} + T^{11}, \\ & 0, 0, 0, -2 T + 16 T^2 - 49 T^3 + 72 T^4 - 35 T^5 - 41 T^6 + 73 T^7 - 47 T^8 + 12 T^9 - T^{10}, 0 \}, \\ & \{ T^2 - 7 T^3 + 17 T^4 - 19 T^5 + 11 T^6 - 14 T^7 + 26 T^8 - 21 T^9 + 8 T^{10} - T^{11}, \\ & 2 T - 16 T^2 + 56 T^3 - 103 T^4 + 90 T^5 + 7 T^6 - 97 T^7 + 101 T^8 - 49 T^9 + 12 T^{10} - T^{11}, \\ & -1 + 7 T - 24 T^2 + 50 T^3 - 66 T^4 + 48 T^5 - 2 T^6 - 33 T^7 + 29 T^8 - 10 T^9 + T^{10}, \\ & -2 T + 16 T^2 - 57 T^3 + 107 T^4 - 95 T^5 - 10 T^6 + 106 T^7 - 102 T^8 + 47 T^9 - 11 T^{10} + T^{11}, \\ & 0, 0, 0, 0, -2 T + 16 T^2 - 49 T^3 + 72 T^4 - 35 T^5 - 41 T^6 + 73 T^7 - 47 T^8 + 12 T^9 - T^{10} \} \end{aligned} \right\}$$

```
In[*]:= mat = GGbar[AllKnots[{3, 7}]];
mat // MatrixForm
```

Out[*]//MatrixForm=

$$\begin{pmatrix}
 -\frac{1-3T+T^2}{T^2} & -\frac{1-3T+T^2}{T} & -1+3T-T^2 & 0 & -\frac{-1+T}{T} \\
 \frac{1}{T} & \frac{-1+2T}{T} & T & 0 & -\frac{1-3T}{T} \\
 -\frac{(-2+T)(-1+2T)}{T^3} & -((-2+T)(-1+2T)) & -((-2+T)T(-1+2T)) & -\frac{2(-1+T)^3}{T} & -\frac{(-2+T)(-1+T)}{T} \\
 -\frac{1-3T+3T^2-3T^3+T^4}{T^4} & -\frac{1-3T+3T^2-3T^3+T^4}{T} & -1+3T-3T^2+3T^3-T^4 & -\frac{(-1+T)^3(1+T^2)}{T^2} & -\frac{1-3T+5T^2-6T}{T} \\
 \frac{1-3T+5T^2-3T^3+T^4}{T^3} & \frac{1-3T+5T^2-3T^3+T^4}{T^2} & \frac{1-3T+5T^2-3T^3+T^4}{T} & 0 & -\frac{-1+2T}{T} \\
 -\frac{-1+2T-3T^2+T^3}{T^2} & \frac{1}{T} & \frac{-1+3T-2T^2+T^3}{T} & -\frac{(-1+T)^2}{T^2} & \frac{1-4T+6T^2}{T^2} \\
 \frac{2-3T+3T^2-3T^3+2T^4}{T^3} & \frac{2-3T+3T^2-3T^3+2T^4}{T^2} & \frac{2-3T+3T^2-3T^3+2T^4}{T} & 0 & \frac{2-2T+2T^2}{T^2} \\
 \frac{2-3T+3T^2-2T^3+T^4}{T^2} & \frac{1-T+2T^2-3T^3+2T^4}{T^3} & \frac{1-2T+3T^2-3T^3+2T^4}{T^2} & \frac{(-1+T)^2(1-T+2T^2)}{T^3} & \frac{3-5T+6T^2}{T^2} \\
 \frac{1-T+T^2-T^3+T^4}{T^3} & \frac{2-2T+2T^2-2T^3+T^4}{T^2} & \frac{1-T+T^2-T^3+T^4}{T} & 0 & \frac{2-3T+3T^2}{T^2} \\
 \frac{(2-2T+T^2)(1-T+T^2)}{T^2} & \frac{1-T+2T^2-2T^3+T^4}{T^3} & \frac{(1-T+T^2)(1-2T+2T^2)}{T^2} & \frac{(-1+T)^2(1+T^2)}{T^3} & \frac{3-5T+5T^2}{T^2} \\
 \frac{4-7T+4T^2}{T^2} & \frac{4-7T+4T^2}{T} & 4-7T+4T^2 & 0 & \frac{4-5T}{T} \\
 \frac{2-2T+T^2}{T} & \frac{2-2T+T^2}{T^2} & \frac{1-2T+2T^2}{T} & \frac{2(-1+T)^2}{T^2} & \frac{5-8T}{T} \\
 \frac{2-3T+2T^2}{T^2} & \frac{4-5T+2T^2}{T} & 2-3T+2T^2 & 0 & \frac{4-7T}{T} \\
 \frac{3-4T+2T^2}{T} & \frac{3-5T+3T^2}{T^2} & \frac{2-4T+3T^2}{T} & -\frac{(-1+T)(3-4T+2T^2)}{T^2} & -\frac{(-2+T)(3-4T+2T^2)}{T} \\
 \frac{2-3T+2T^2}{T} & \frac{-1+4T-5T^2+3T^3}{T^3} & \frac{2-3T+2T^2}{T} & -\frac{(-1+T)(-1+4T-5T^2+3T^3)}{T^3} & -\frac{1-7T+12T^2}{T} \\
 \frac{1-5T+9T^2-5T^3+T^4}{T^4} & \frac{1-5T+9T^2-5T^3+T^4}{T} & 1-5T+9T^2-5T^3+T^4 & \frac{(-1+T)^5}{T^2} & \frac{1-5T+11T^2-12T^3}{T^2} \\
 -\frac{-1+5T-8T^2+3T^3}{T^4} & \frac{1-4T+8T^2-5T^3+T^4}{T} & T(-3+8T-5T^2+T^3) & \frac{(-1+T)^5}{T^2} & \frac{1-5T+10T^2-11T^3}{T^2}
 \end{pmatrix}$$

```
In[*]:= NullSpace[mat]
```

Out[*]=

$$\{ \{0, 0, 0, 0, 0, -T, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1, 1\} \}$$