Pensieve header: Alexander Experiments: Modifying the A Matrix.

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

In[ ]:=
```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

In[ ]:=
```
As[K_] := Module[{Cs, φ, n, A, A1, A2, A3, A4, A5, s, i, j, Δ},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A5 = A4 = A3 = A2 = A1 = A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (
     A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1 ; 0  -1 );
     A1[[{i, j}, {i + 1, j + 1}]] += ( -1  T^s - 1 ; 0  -T^s );
     A2[[{i, j}, {i + 1, j + 1}]] += ( -1  0 ; T^s - 1  -T^s );
     A3[[{i, j}, {i + 1, j + 1}]] += ( -1  0 ; T^-s - 1  -T^-s );
     A4[[{i, j}, {i + 1, j + 1}]] += T^-s ( -T^s  T^s - 1 ; 0  -1 );
     A5[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1 ; 0  -1 );
    )];
  Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
  Factor@{Δ, Δ^-1 Det[A1], Δ^-1 Det[A2], Δ^-1 Det[A3], Δ^-1 Det[A4]}
 ];
```

In[ ]:=
```
As /@ AllKnots[{3, 6}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\frac{1-T+T^2}{T} & \frac{1}{T^2} & \frac{1}{T} & T & -\frac{T\left(-2+T-T^2+T^3\right)}{1-T+T^2} \\
-\frac{1-3T+T^2}{T} & \frac{1}{T} & T & \frac{1}{T} & \frac{T\left(-1-T+T^2\right)}{1-3T+T^2} \\
\frac{1-T+T^2-T^3+T^4}{T^2} & \frac{1}{T^3} & \frac{1}{T^2} & T^2 & -\frac{T^2\left(-2+T-T^2+T^3-T^4+T^5\right)}{1-T+T^2-T^3+T^4} \\
\frac{2-3T+2T^2}{T} & \frac{1}{T^3} & \frac{1}{T^2} & T^2 & \frac{(-2+T)\,T^2\left(-2+T-2T^2+2T^3\right)}{2-3T+2T^2} \\
-\frac{(-2+T)(-1+2T)}{T} & \frac{1}{T^2} & 1 & 1 & \frac{T\left(4-5T+2T^2\right)}{-1+2T} \\
-\frac{1-3T+3T^2-3T^3+T^4}{T^2} & \frac{1}{T^2} & 1 & 1 & \frac{T\left(-4+7T-9T^2+9T^3-5T^4+T^5\right)}{1-3T+3T^2-3T^3+T^4} \\
\frac{1-3T+5T^2-3T^3+T^4}{T^2} & 1 & 1 & 1 & -\frac{1-3T+3T^2-3T^3+T^4}{1-3T+5T^2-3T^3+T^4}
\end{pmatrix}$$

In[ ]:=

$$\text{Eigenvalues}\left[\begin{pmatrix} -T^s & T^s-1 \\ 0 & -1 \end{pmatrix}\right]$$

Out[ ]=

$$\{-1, -T^s\}$$

In[ ]:=

$$\begin{aligned}
&\text{PAQ}[K_] := \text{Module}\Big[\{Cs, \varphi, n, P, A, Q, s, i, j, \Delta\},\\
&\quad \{Cs, \varphi\} = \text{Rot}[K];\ n = \text{Length}[Cs];\\
&\quad P = A = Q = \text{IdentityMatrix}[2n+1];\\
&\quad \text{Cases}\Big[Cs, \{s_, i_, j_\} :\to \Big(\\
&\qquad P[\![\{i, j\}, \{i, j\}]\!] = \begin{pmatrix} -T^{-s} & T^{-s}-1 \\ 0 & -1 \end{pmatrix};\\
&\qquad A[\![\{i, j\}, \{i+1, j+1\}]\!] \mathrel{+}= \begin{pmatrix} -T^{s} & T^{s}-1 \\ 0 & -1 \end{pmatrix};\\
&\qquad Q[\![\{i+1, j+1\}, \{i+1, j+1\}]\!] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\\
&\quad \Big)\Big];\\
&\quad \Delta = T^{(-\text{Total}[\varphi]-\text{Total}[Cs[\![All,1]\!]])/2}\,\text{Det}[A];\\
&\quad \text{Expand}@\{P, A, Q, P.A.Q\}\\
&\Big];
\end{aligned}$$

In[ ]:=
```
MatrixForm /@ PAQ[PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]]
MatrixForm /@ PAQ[Knot[4, 1]]
```

Out[ ]=

$$\left\{\begin{pmatrix} -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1+\frac{1}{T} & 0 & 0 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -T & 0 & 0 & -1+T & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -T & 0 & 0 & -1+T \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1+T & 0 & 1 & -T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{T} & 1 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T} & 1 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1+\frac{1}{T} & 0 & 0 & -\frac{1}{T} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right\}$$

Out[ ]=

$$\left\{\begin{pmatrix} -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -T & 0 & 0 & -1+T & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1+T & 0 & 0 & 0 & 0 & -T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},\right.$$

$$\begin{pmatrix} 1 & -T & 0 & 0 & -1+T & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -T & 0 & 0 & -1+T \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\left.\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{T} & 1 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -T & 1 & 0 & -1+T & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T} & 1 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1+T & 0 & 0 & 0 & 0 & -T & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right\}$$

```
AAs[K_] := Module[{Cs, φ, n, A, A1, A2, s, i, j, Δ},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A2 = A1 = A = Table[0, {2 n + 1}, {2 n + 1}];
  Cases[Cs, {s_, i_, j_} :> (
    A[[{i, j}, {i, j, i + 1, j + 1}]] += (1 0 -T^s T^s - 1
                                          0 1  0    -1  ); (* Original *)
    A1[[{i, j}, {i, j, i + 1, j + 1}]] += (1 0 -T^-s T^-s - 1
                                           0 1  0     -1   ); (* Horizontal mirror *)
    A2[[{i, j}, {i, j, i + 1, j + 1}]] += (1 0  -1       0
                                           0 1 T^-s - 1 -T^-s); (* Vertical mirror *)
  )];
  A2[[2 n + 1, 2 n + 1]] = A1[[2 n + 1, 2 n + 1]] = A[[2 n + 1, 2 n + 1]] = 1;
  Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
  Factor@{Δ, Δ^-1 Det[A1], Δ^-1 Det[A2]}
];
AAs /@ AllKnots[{3, 6}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1-T+T^2}{T} & T^2 & T \\ -\frac{1-3T+T^2}{T} & T & \frac{1}{T} \\ \frac{1-T+T^2-T^3+T^4}{T^2} & T^3 & T^2 \\ \frac{2-3T+2T^2}{T} & T^3 & T^2 \\ -\frac{(-2+T)(-1+2T)}{T} & T^2 & 1 \\ -\frac{1-3T+3T^2-3T^3+T^4}{T^2} & T^2 & 1 \\ \frac{1-3T+5T^2-3T^3+T^4}{T^2} & 1 & 1 \end{pmatrix}$$

In[ ]:=

$$\begin{pmatrix} -T^s & T^s - 1 \\ 0 & -1 \end{pmatrix}$$ `// Inverse // Simplify // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -T^{-s} & -1 + T^{-s} \\ 0 & -1 \end{pmatrix}$$